Integrated Pathway

Mathematics III

Program Overview

TEACHER RESOURCE

BW WALCH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

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PROGRAM OVERVIEW

Introduction to the Program

Introduction

The *New Mexico Integrated Pathway Math III* program is a complete set of materials developed around the Common Core State Standards (CCSS), the overview of the Integrated Pathway for the Common Core State Mathematics Standards, and the Mathematics III content map found in Appendix A of the Common Core State Standards. Topics are built around accessible core curricula, ensuring that the *New Mexico Integrated Pathway Math III* program is useful for striving students and diverse classrooms.

This program realizes the benefits of exploratory and investigative learning and employs a variety of instructional models to meet the learning needs of students with a range of abilities.

The *New Mexico Integrated Pathway Math III* program includes components that support problem-based learning, instruct and coach as needed, provide practice, and assess students' skills. Instructional tools and strategies are embedded throughout.

The program includes:

- More than 150 hours of lessons, addressing the six units of CCSS IP: Mathematics III
- Essential Questions for each instructional topic
- Vocabulary
- Instruction and Guided Practice
- Problem-based Tasks and Coaching questions
- Step-by-step graphing calculator instructions for the TI-Nspire and the TI-83/84
- Station activities to promote collaborative learning and problem-solving skills

Purpose of Materials

The *New Mexico Integrated Pathway Math III* program has been organized to coordinate with the CCSS Integrated Pathway: Mathematics III content map and specifications from Appendix A of the Common Core State Standards.

Each topic includes activities that offer opportunities for exploration and investigation. These activities incorporate concept and skill development and guided practice, then move on to the application of new skills and concepts in problem-solving situations. Throughout the lessons and activities, problems are contextualized to enhance rigor and relevance.

This program includes all the topics addressed in the CCSS Integrated Pathway: Mathematics III content map. These include:

- Inferences and Conclusions from Data
- Polynomial Relationships
- Rational and Radical Relationships
- Trigonometry of General Triangles and Trigonometric Functions
- Mathematical Modeling of Inverse, Logarithmic, and Trigonometric Functions
- Mathematical Modeling and Choosing a Model

The eight Mathematical Practices described in the Common Core are infused throughout:

- CCSS.MP.1: Make sense of problems and persevere in solving them.
- CCSS.MP.2: Reason abstractly and quantitatively.
- CCSS.MP.3: Construct viable arguments and critique the reasoning of others.
- CCSS.MP.4: Model with mathematics.
- CCSS.MP.5: Use appropriate tools strategically.
- CCSS.MP.6: Attend to precision.
- CCSS.MP.7: Look for and make use of structure.
- CCSS.MP.8: Look for and express regularity in repeated reasoning.

Structure of the Teacher Resource

The *New Mexico Integrated Pathway Math III* program is completely reproducible. Online materials can be provided in your Learning Management System (such as Canvas or Schoology) or in Walch's proprietary course management platform, the Curriculum Engine. The nested folder organization in the Curriculum Engine allows you to access the materials quickly and easily. The digital format also facilitates printing and copying student pages and/or making assignments online.

The Program Overview is the first section. This section helps you to navigate the materials, offers a collection of research-based Instructional Strategies along with their literacy connections and implementation suggestions, and shows the correlation between the Common Core State Standards and the CCSS Integrated Pathway: Mathematics III content map found in Appendix A of the Common Core State Standards.

The remaining materials focus on content, knowledge, and application of the six units in the CCSS Integrated Pathway Mathematics III curriculum: Inferences and Conclusions from Data; Polynomial Relationships; Rational and Radical Relationships; Trigonometry of General Triangles and Trigonometric Functions; Mathematical Modeling of Inverse, Logarithmic, and Trigonometric Functions; and Mathematical Modeling and Choosing a Model. The units in the *New Mexico Integrated Pathway Math III* program are designed to be flexible so that you can mix and match activities as the needs of your students and your instructional style dictate.

The Station Activities correspond to the content in the units and provide students with the opportunity to apply concepts and skills, while you have a chance to circulate, observe, speak to individuals and small groups, and informally assess and plan.

Each topic begins with a pre-assessment and ends with a progress assessment. These allow you to assess students' progress as you move from topic to topic, enabling you to gauge how well students have understood the material and to differentiate as appropriate.

***Please note*:** Throughout the Teacher Resource, page references are provided on the lower, inner corner of some pages. These references indicate the corresponding page(s) in the Student Workbook (e.g., SWB p. U1-SWB) as appropriate. These page references have been included here to facilitate assigning materials to students.

Glossary

The Glossary contains vocabulary terms and formulas from throughout the program, organized alphabetically. Each listing provides the term and the definition in both English and Spanish. The listings include the lesson number(s) where the terms can be found in the Words to Know.

Correspondence to Standards for Mathematical Practice

How Do Walch's New Mexico Mathematics Resources Address the Common Core Standards for Mathematical Practice?

Walch's programs for the *New Mexico Integrated Pathway Math I, II,* and *III* courses employ a problem-based model of instruction that supports and reinforces the eight Standards for Mathematical Practice. Although the following table focuses on Problem-Based Tasks, Walch's full programs also include hundreds of additional problems in warm-ups and practices. The Implementation Guide for each PBT highlights two or more SMPs to focus on during implementation and discussion.

CCSS Standards for Mathematical Practice		Relevant Attributes of Walch's New Mexico Math Resources
1	Make sense of problems and persevere in solving them.	Each lesson is built around a Problem-Based Task (PBT) that requires students to "make sense of problems and persevere in solving them."
2	Reason abstractly and quantitatively.	Each PBT uses a meaningful real-world context that requires students to reason both abstractly about the situation/relationships and quantitatively about the values representing the elements and relationships.
3	Construct viable arguments and critique the reasoning of others.	Since the PBT provides opportunities for multiple problem-solving approaches and varied solutions, students are required to construct viable arguments to support their approach and answer. This, in turn, provides other students the opportunity to analyze and critique their classmates' reasoning.
4	Model with mathematics.	Each PBT represents a real-world situation and requires students to model it with mathematics.
5	Use appropriate tools strategically.	PBTs require students to make choices about using appropriate tools, such as calculators, spreadsheets, graph paper, manipulatives, protractors, and compasses. The tasks do not prescribe specific tools, but instead provide opportunities for their use.
6	Attend to precision.	The real-world contexts of the PBTs require students to be precise in their solutions, both in the ways that the solutions are stated, labeled, and explained, and in the degree of precision necessary given the context (e.g., tripling chili for a crowd vs. machining a part for an airplane engine).
7	Look for and make use of structure.	The PBTs present students with complicated scenarios that must be analyzed to discern patterns and significant mathematical features.
8	Look for and express regularity in repeated reasoning.	PBTs require multiple steps, providing opportunities for students to note repeated calculations, monitor their process, and continually evaluate reasonableness of intermediate results before arriving at a solution.

Correspondence to CCSS Publishers' Criteria

How Do Walch's New Mexico Mathematics Resources Address the High School Publishers' Criteria for CCSS Mathematics?

Walch's programs for the *New Mexico Integrated Pathway Math I, II,* and *III* courses were designed to the specifications of Appendix A and the Publishers' Criteria. Focus, Coherence, and Rigor represent the fundamental assumptions of the resources.

Publishers' Criteria	Relevant Attributes of Walch's New Mexico Math Resources
FOCUS Focus strongly where the standards focus.	Each course is built around the units specified in the CCSS for Mathematics Appendix A. Units are organized around important topics in mathematics and reflect the focus of the standards. Units range from 22 lessons for Math I Unit 2, "Linear and Exponential Relationships," which address 21 Common Core standards, to three lessons for Math I Unit 6, "Connecting Algebra and Geometry Through Coordinates," which address only three Common Core standards.
COHERENCE Think across grades/courses, and link to major topics in each course.	Walch Integrated Math programs include two features contributing to coherence. Each lesson identifies necessary prerequisite skills (our forthcoming Support Supplement will point to where they were introduced, including in elementary or middle school, and provide instructional materials for remediation). Warm-Ups tie to these prerequisites in order to activate prior knowledge. Also, the major topics of each course are mapped forward to show where they are revisited and expanded upon in future courses.
RIGOR In major topics, pursue with equal intensity • conceptual understanding, • procedural skill and fluency, and • applications.	The instructional components of Walch's Integrated Math programs guide students from conceptual understanding and procedural skill developed through modeling and guided practice to the application opportunities of the PBT. The resources move students on to additional practice to refine procedural skill and fluency, and later to station activities to crystalize conceptual understanding.

Correspondence to NCTM *Principles to Actions* Teaching Practices

How Do Walch's New Mexico Mathematics Resources Address the NCTM *Principles to Actions* Mathematics Teaching Practices?

Walch's programs for the *New Mexico Integrated Pathway Math I, II,* and *III* courses were designed by experienced educators and curriculum developers, informed by best-practice research, and refined through an iterative process of implementation and feedback. Together with professional development, these materials support and sustain good teaching practices.

NCTM Mathematics Teaching Practices	Relevant Attributes of Walch's New Mexico Math Resources
Establish mathematics goals to focus learning.	Each lesson in Walch's programs addresses specified standards which can be used as goals to focus learning. Essential Questions offer further focus.
Implement tasks that promote reasoning and problem solving.	Each lesson in Walch's programs is built around a Problem-Based Task (PBT), set in a meaningful real-world context and designed to promote reasoning and problem solving. The courses include dozens of PBTs as well as warm-up and practice problems.
Use and connect mathematical representations.	Walch's High School Math programs make frequent use of, and connections among and between, equations, tables, and graphs. PBTs often require students to use and connect two or more of these representations, and the representations are modeled through guided practice.
Facilitate meaningful mathematical discourse.	Several features of the programs support mathematical discourse, including warm-up debriefs with connections to the upcoming lesson, implementation guides and optional coaching questions for the PBTs, and discussion guides for Station Activities. Explanations of PBT solutions are another opportunity for discourse. Please note: Mathematical discourse is an important topic for professional development, in conjunction with implementation of these materials.
Pose purposeful questions.	The implementation guides, coaching questions and discussion guides provide samples of purposeful questions. Note that this is another important topic for professional development.
Build procedural fluency from conceptual understanding.	The programs develop conceptual understanding through modeling, guided practice, and application, and then provide additional opportunities to practice and develop fluency.
Support productive struggle in learning mathematics.	The PBTs require "productive struggle;" implementation guides include suggestions for facilitation and monitoring, and coaching questions provide an option for additional support as appropriate, allowing students to proceed through the task and ensuring that the struggle remains productive rather than too frustrating.
Elicit and use evidence of student thinking.	Various discussions and PBTs require students to display their thinking. Implementation guides offer specific prompts and suggestions for eliciting and responding to student thinking. Professional development supports teachers in using that evidence to respond in instructionally appropriate ways.

Unit Structure

All of the instructional units have common features. Each unit begins with a table of Standards Correlations for the lessons in the unit, as well as a list of relevant online educational resources.

Each topic begins with a pre-assessment, followed by a Topic Overview listing the standards addressed by the lessons in the topic; Essential Questions; vocabulary (titled "Words to Know"); and a list of recommended websites to be used as additional resources. This Topic Overview is followed by the lesson(s) for that topic.

Each lesson begins with Warm-Up activity, followed by a list of identified prerequisite skills that students need to have mastered in order to be successful with the new material in the upcoming lesson. This is followed by an introduction, key concepts, common errors/misconceptions, scaffolded practice problems, guided practice examples, a problem-based task with coaching questions and sample responses, a closure activity, and practice.

Each topic ends with a progress assessment to evaluate students' learning. Selected topics include a Conceptual Activity before the progress assessment. The Unit Assessment, answer keys, and one or more station activities round out the unit.

All of the components are described below and on the following pages for your reference.

Standards Correlations and Connections to Future Courses

In this section, you'll find a comprehensive list of the Common Core State Standard(s) addressed in each lesson, followed by a map detailing where and how future courses will build upon the topics introduced in the unit.

Pre-Assessment

This can be used to gauge students' prior knowledge and to inform instructional planning.

Common Core State Standards for the Topic

All standards that are addressed in the entire topic are listed.

Essential Questions

These are intended to guide students' thinking as they proceed through the topic. By the end of each topic, students should be able to respond to the questions.

Words to Know

Vocabulary terms and formulas are provided as background information for instruction or to review key concepts that are addressed in the topic.

Recommended Resources

This is a list of websites that can be used as additional resources. Some websites are games; others provide additional examples and/or explanations. (*Note*: Links will be monitored and repaired or replaced as necessary.) Each Recommended Resource is also accessible through Walch's cloud-based Curriculum Engine Learning Object Repository as a separate learning object that can be assigned to students.

Conceptual Activities

Conceptual understanding serves as the foundation on which to build deeper understanding of mathematics. In an effort to build conceptual understanding of mathematical ideas and to provide more than procedural fluency and application, links to interactive open education and Desmos resources are included. (*Note*: These website links will be monitored and repaired or replaced as necessary.) These and many other open educational resources (OERs) are also accessible through the Learning Object Repository as separate objects that can be assigned to students.

Warm-Up

Each warm-up takes approximately 5 minutes and addresses either prerequisite and critical-thinking skills or previously taught math concepts.

Common Core State Standards for the Lesson

When topics are broken down into lessons, the specific standard or standards that are addressed are presented at the beginning of the instructional portion of the lesson.

Warm-Up Debrief

Each debrief provides the answers to the warm-up questions, and offers suggestions for situations in which students might have difficulties. A section titled Connection to the Lesson is also included in the debrief to help answer students' questions about the relevance of the particular warm-up activity to the upcoming instruction. Warm-Ups with debriefs are also provided in PowerPoint presentations.

Identified Prerequisite Skills

This list cites the skills necessary to be successful with the new material.

Introduction

This brief paragraph gives a description of the concepts about to be presented and often contains some Words to Know.

Key Concepts

Provided in bulleted form, this instruction highlights the important ideas and/or processes for meeting the standard.

Graphing Calculator Directions

Step-by-step instructions for using a TI-Nspire and a TI-83/84 are provided whenever graphing calculators are referenced.

Common Errors/Misconceptions

This is a list of the common errors students make when applying Key Concepts. This list suggests what to watch for when students arrive at an incorrect answer or are struggling with solving the problems.

Scaffolded Practice (Printable Practice)

This set of 10 printable practice problems provides introductory level skill practice for the lesson. This practice set can be used during instruction time.

Guided Practice

This section provides step-by-step examples of applying the Key Concepts. The three to five examples are intended to aid during initial instruction, but are also for individuals needing additional instruction and/or for use during review and test preparation.

Enhanced Instructional PowerPoint (Presentation)

Each lesson includes an instructional PowerPoint presentation with the following components: Warm-Up, Key Concepts, and Guided Practice. Selected Guided Practice examples include links to GeoGebra applets. These instructional PowerPoints are downloadable and editable.

Problem-Based Task

This activity can serve as the centerpiece of a problem-based lesson, or it can be used to walk students through the application of the standard, prior to traditional instruction or at the end of instruction. The task makes use of critical-thinking skills.

Optional Problem-Based Task Coaching Questions with Sample Responses

These questions scaffold the task and guide students to solving the problem(s) presented in the task. They should be used at the discretion of the teacher for students requiring additional support. The Coaching Questions are followed by answers and suggested appropriate responses to the coaching questions. In some cases answers may vary, but a sample answer is given for each question.

Recommended Closure Activity

Students are given the opportunity to synthesize and reflect on the lesson through a journal entry or discussion of one or more of the Essential Questions.

Problem-Based Task Implementation Guide

This instructional overview, found with selected Problem-Based Tasks in each unit, highlights connections between the task and the lesson's key concepts and Mathematical Practices. The Implementation Guide also offers suggestions for facilitating and monitoring, and provides alternative solutions.

Printable Practice (Sets A and B) and Interactive Practice (Set A)

Each lesson includes two sets of practice problems to support students' achievement of the learning objectives. They can be used in any combination of teacher-led instruction, cooperative learning, or independent application of knowledge. Each Practice A is also available as an interactive Learnosity activity with Technology-Enhanced Items.

Progress Assessment

Each lesson ends with 10 multiple-choice questions, as well as one extended-response question that incorporates critical thinking and writing components. This can be used to document the extent to which students grasp the concepts and skills addressed during instruction.

Unit Assessment

Each unit ends with 12 multiple-choice questions and three extended-response questions that incorporate critical thinking and writing components. This can be used to document the extent to which students grasped the concepts and skills of each unit.

Answer Key

Answers for all of the Warm-Ups and practice problems are provided at the end of each unit.

Station Activities

Most units include a collection of station-based activities to provide students with opportunities to practice, reinforce, and apply mathematical skills and concepts. The debriefing discussions after each set of activities provide an important opportunity to help students reflect on their experiences and synthesize their thinking.

Conceptual Tasks

These engaging tasks provide opportunities for students to deepen their understanding and develop their conceptual knowledge of math concepts. These tasks provide multiple entry points and are accessible for ALL learners.

Standards Correlations

Each lesson in this program was written specifically to address the Common Core State Standards. Each topic lists the standards covered in all the lessons, and each lesson lists the standards addressed in that particular lesson. In this section, you'll find a comprehensive list mapping the lessons to the CCSS.

Guide to Common Core State Standards Annotation

As you use this program, you will come across a symbol included with the Common Core standards for some of the lessons and activities. These symbols are explained below.

Symbol: ⋆

Denotes: Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (⋆).

From https://www.walch.com/CCSS/00003

Symbol: (+)

Denotes: College and Career Readiness Standards

Advanced mathematics standards that are required in higher-level courses such as advanced statistics may also be included in lower-level courses. These additional standards are denoted by (+). According to the Common Core State Standards Initiative, "the evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6–8."

From https://www.walch.com/CCSS/00004

Connections to Future Courses

This section provides a map between topics introduced in each unit of this course and subsequent courses where each topic is revisited and built upon.

Unit 1: Inferences and Conclusions from Data			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Using the Normal Curve**		
	1.1	Normal Distributions and the 68–95–99.7 Rule	S–ID.A.4★
	1.2	Standard Normal Calculations	S–ID.A.4★
	1.3	Assessing Normality	S–ID.A.4★
Topic B	**Populations Versus Random Samples and Random Sampling**		
	1.4	Differences Between Populations and Samples	S–IC.A.1★
	1.5	Simple Random Sampling	S–IC.A.2★
	1.6	Other Methods of Random Sampling	S–IC.A.2★
Topic C	**Surveys, Experiments, and Observational Studies**		
	1.7	Identifying Surveys, Experiments, and Observational Studies	S–IC.B.3★
	1.8	Designing Surveys, Experiments, and Observational Studies	S–IC.B.3★
Topic D	**Estimating Sample Proportions and Sample Means**		
	1.9	Estimating Sample Proportions	S–IC.B.4★
	1.10	The Binomial Distribution	S–IC.B.4★
	1.11	Estimating Sample Means	S–IC.B.4★
	1.12	Estimating with Confidence	S–IC.B.4★
Topic E	**Comparing Treatments and Reading Reports**		
	1.13	Evaluating Treatments	S–IC.B.5★
	1.14	Designing and Simulating Treatments	S–IC.B.5★
	1.15	Reading Reports	S–IC.B.6★
Topic F	**Making and Analyzing Decisions**		
	1.16	Making Decisions	S–MD.B.6★ (+)
	1.17	Analyzing Decisions	S–MD.B.7★ (+)

Unit 2: Polynomial Relationships			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Polynomial Structures and Operating with Polynomials**		
	2.1	Structures of Expressions	A–SSE.A.1a★
	2.2	Adding and Subtracting Polynomials	A–APR.A.1
	2.3	Multiplying Polynomials	A–APR.A.1
Topic B	**Proving Identities**		
	2.4	Polynomial Identities	A–SSE.A.1b★ A–SSE.A.2 A–APR.C.4
	2.5	Complex Polynomial Identities	N–CN.C.8 (+) A–SSE.A.1b★ A–SSE.A.2 A–APR.C.4
	2.6	The Binomial Theorem	A–SSE.A.1a★ A–SSE.A.1b★ A–SSE.A.2 A–APR.C.4 A–APR.C.5 (+)
Topic C	**Graphing Polynomial Functions**		
	2.7	Describing End Behavior and Turns	F–IF.C.7c★
	2.8	The Remainder Theorem	A–APR.B.2
	2.9	Finding Zeros	A–APR.B.3 N–CN.C.9 (+) F–IF.C.7c★
	2.10	The Rational Root Theorem	A–APR.B.3
Topic D	**Solving Systems of Equations with Polynomials**		
	2.11	Solving Systems of Equations Graphically	A–REI.D.11★
Topic E	**Geometric Series**		
	2.12	Geometric Sequences	A–SSE.B.4★
	2.13	Sum of a Finite Geometric Series	A–SSE.B.4★
	2.14	Sum of an Infinite Geometric Series	A–SSE.B.4★

Unit 3: Rational and Radical Relationships			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Operating with Rational Expressions**		
	3.1	Structures of Rational Expressions	A–SSE.A.1a★ A–SSE.A.1b★ A–SSE.A.2
	3.2	Adding and Subtracting Rational Expressions	A–APR.D.7 (+) A–SSE.A.2
	3.3	Multiplying Rational Expressions	A–APR.D.7 (+) A–SSE.A.2
	3.4	Dividing Rational Expressions	A–APR.D.6 A–APR.D.7 (+) A–SSE.A.2
Topic B	**Solving Rational and Radical Equations**		
	3.5	Solving Rational Equations	A–REI.A.2
	3.6	Solving Radical Equations	A–REI.A.2
	3.7	Solving Systems of Equations	A–REI.D.11★

Unit 4: Trigonometry of General Triangles and Trigonometric Functions			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Radians and the Unit Circle**		
	4.1	Radians	F–TF.A.1 F–TF.A.2
	4.2	The Unit Circle	F–TF.A.2
	4.3	Special Angles in the Unit Circle	F–TF.A.2
	4.4	Evaluating Trigonometric Functions	F–TF.A.2
Topic B	**Trigonometry of General Angles**		
	4.5	Proving the Law of Sines	G–SRT.D.9 (+) G–SRT.D.10 (+)
	4.6	Proving the Law of Cosines	G–SRT.D.10 (+)
	4.7	Applying the Laws of Sines and Cosines	G–SRT.D.11 (+)
Topic C	**Graphs of Trigonometric Functions**		
	4.8	Periodic Phenomena and Amplitude, Frequency, and Midline	F–TF.B.5★
	4.9	Using Trigonometric Functions to Model Periodic Phenomena	F–TF.B.5★

Unit 5: Mathematical Modeling of Inverse, Logarithmic, and Trigonometric Functions			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Inverses of Functions**		
	5.1	Determining Inverses of Quadratic Functions	F–BF.B.4a
	5.2	Determining Inverses of Other Functions	F–BF.B.4a
Topic B	**Modeling Logarithmic Functions**		
	5.3	Logarithmic Functions as Inverses	F–BF.B.4a F–LE.A.4★
	5.4	Common Logarithms	F–IF.C.8 F–LE.A.4★
	5.5	Natural Logarithms	F–IF.C.8 F–LE.A.4★
	5.6	Graphing Logarithmic Functions	F–IF.C.7e★
	5.7	Interpreting Logarithmic Models	F–IF.B.4★ F–IF.B.5★ F–IF.B.6★
Topic C	**Modeling Trigonometric Functions**		
	5.8	Graphing the Sine Function	F–IF.C.7e★
	5.9	Graphing the Cosine Function	F–IF.C.7e★

Unit 6: Mathematical Modeling and Choosing a Model			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Creating Equations**		
	6.1	Creating Equations in One Variable	A–CED.A.1★
	6.2	Representing and Interpreting Constraints	A–CED.A.3★
	6.3	Rearranging Formulas	A–CED.A.4★
Topic B	**Transforming a Model and Combining Functions**		
	6.4	Transformations of Parent Graphs	F–BF.B.3
	6.5	Recognizing Odd and Even Functions	F–BF.B.3
	6.6	Combining Functions	F–BF.A.1b★
Topic C	**Comparing Properties Within and Between Functions**		
	6.7	Reading and Identifying Key Features of Real-World Situation Graphs	F–IF.B.4★ F–IF.B.5★ F–IF.B.6★
	6.8	Calculating Average Rates of Change	F–IF.B.6★
	6.9	Comparing Functions	F–IF.B.6★ F–IF.C.9
Topic D	**Choosing a Model**		
	6.10	Linear, Exponential, and Quadratic Functions	A–CED.A.2★ F–IF.B.4★ F–IF.B.5★ F–BF.B.3
	6.11	Piecewise, Step, and Absolute Value Functions	F–IF.B.4★ F–IF.B.5★ F–IF.C.7b★ F–BF.B.3
	6.12	Square Root and Cube Root Functions	F–IF.B.4★ F–IF.B.5★ F–IF.C.7b★ F–BF.B.3
Topic E	**Geometric Modeling**		
	6.13	Two-Dimensional Cross Sections of Three-Dimensional Objects	G–GMD.B.4 G–MG.A.1★
	6.14	Density	G–MG.A.2★
	6.15	Design	G–MG.A.3★

TOPICS FROM FUTURE COURSES: NEW MEXICO INTEGRATED PATHWAY MATH III

Math III	Topic introduced	Course/unit where addressed	How addressed
Unit 1	Methods of gathering data for statistical analysis; making and analyzing decisions and making inferences about sample means and proportions; normal and approximately normal distributions	AP Statistics or Math IV	calculating the expected value of distributions other than normal and approximately normal distributions is applied to making inferences and decisions, including decision-making based on payoffs calculated from expected value
Unit 2	Interpreting structures of higher-order polynomial expressions; systems of equations, including linear-polynomial, polynomial-polynomial, and absolute value-polynomial systems; sequences as infinite and finite series	Precalculus, Calculus, or Math IV	extended to systems of equations, including larger, more complex systems (e.g., 3×3 and larger systems) using matrices
Unit 3	Rational expressions; solving rational and radical equations; linear-rational and linear-radical systems	Precalculus, Calculus, or Math IV	solving systems that can be extended to larger, more complex systems (e.g., 3×3 and larger) using vectors and matrices; graphing more complex rational functions and exploring features of the graphs
Unit 4	Calculating the sine, cosine, and tangent for any triangle using the laws of sines and cosines; trigonometric ratios, including angles measured in radians; the unit circle	Precalculus, Calculus, or Math IV	representing and operating with vectors using trigonometric ratios, trigonometric functions, and complex numbers; evaluating special angles in right triangles using the unit circle and concepts of periodicity

Math III	Topic introduced	Course/unit where addressed	How addressed
Unit 5	Finding the inverses of quadratic and other functions, including rational and trigonometric functions; making the connection between exponential functions and their inverses, logarithmic functions, as the method for solving exponential equations	Precalculus, Calculus, or Math IV	extended to complex logarithms and verifying inverses using compositions of functions
	Exploring the intervals where trigonometric functions are increasing and decreasing	Precalculus, Calculus, or Math IV	creating inverses of trigonometric functions
Unit 6	Using knowledge of function families to generate models that fit real-world phenomena	Precalculus, Calculus, or Math IV	exploring odd and even functions with respect to trigonometric functions; studying and graphing more complex logarithmic and trigonometric functions

Conceptual Activities

Use these interactive open education and/or Desmos resources to build conceptual understanding of mathematical ideas. (*Note*: Activity links will be monitored and repaired or replaced as necessary.)

Unit 1

- Illustrative Mathematics. "The Titanic 3."

  https://www.walch.com/ca/10005

  This task poses an open-ended question that forces students to think about how they can answer it. Students will analyze data given in the two-way table to find meaningful probabilities and increase their understanding of conditional probability and independence.

- Illustrative Mathematics. "Should We Send Out a Certificate?"

  https://www.walch.com/ca/10006

  This task allows students to practice calculating normal distributions and further encourages them to draw conclusions from their results based on the properties of normal distributions. Students will communicate their findings in a narrative form within the context of the problem rather that reporting a simple computed number.

Unit 2

- Desmos. "Constructing Polynomials."

  https://www.walch.com/ca/01054

  In this activity, students will consider properties of polynomial functions such as end behavior, leading terms, and properties of roots. They will explore connections between those properties and the factored forms of the equations of the polynomials.

- Desmos. "Polygraph: Polynomial Functions."

  https://www.walch.com/ca/01055

  This activity is designed to spark vocabulary-rich conversations about polynomial functions. Key vocabulary terms that may appear in student questions include *degree*, *roots*, *end behavior*, *limit*, *quadrant*, *axis*, *increasing*, *decreasing*, *maximum*, *minimum*, *extrema*, *concave up*, and *concave down*.

- Desmos. "Polynomial Equation Challenges."

  https://www.walch.com/ca/01056

  In this activity, students will create polynomial equations (of degree 2, 3, and 4) to match given zeros and points. Students will explore how the factored form of the equations relates to the zeros and the order of those zeros.

Unit 3

- Desmos. "Marbleslides: Rationals."

  https://www.walch.com/ca/01057

  In this activity, students will transform rational functions to send marbles through stars.

- Desmos. "Polygraph: Rational Functions."

  https://www.walch.com/ca/01058

  This activity is designed to spark vocabulary-rich conversations about rational functions. Key vocabulary terms that may appear in student questions include *asymptote*, *vertical*, *horizontal*, *quadrant*, *axis*, *increasing*, *decreasing*, *discontinuity*, and *hole*.

Unit 4

- Desmos. "Burning Daylight."

  https://www.walch.com/ca/01059

  In this activity, students use sinusoids to model daylight data for two U.S. cities. They predict which city has more total daylight during a given year, and then use their model to calculate an answer to that question.

- Desmos. "Graphing the Sine Function Using Amplitude, Period, and Vertical Translation."

  https://www.walch.com/ca/01060

  Students will build a visual understanding of amplitude, period, and phase shift in this introduction to trigonometric graphing. They will use this understanding to find models for given graphs of the sine function.

- Desmos. "Marbleslides: Periodics."

  https://www.walch.com/ca/01061

  In this activity, students will transform periodic functions to send marbles through stars.

- Desmos. "Polygraph: Sinusoids."

  https://www.walch.com/ca/01062

  This activity is designed to spark vocabulary-rich conversations about sinusoids. Key vocabulary terms that may appear in student questions include *amplitude*, *periods*, *maximum*, *minimum*, and *shift*.

- Desmos. "Polygraph: Sinusoids with Vertical Transformations."

  https://www.walch.com/ca/01063

  This activity is designed to spark vocabulary-rich conversations about vertical transformations of sinusoids. Key vocabulary terms that may appear in student questions include *translation*, *dilation*, *amplitude*, *midline*, and *sinusoidal axis*.

- Desmos. "Trigonometric Graphing: Introduction to Amplitude and Vertical Shift."

  https://www.walch.com/ca/01064

  In this activity, students will informally explore range, midline, and amplitude of trigonometric functions. They'll use what they learn about the relationships to write equations of sine and cosine graphs.

Unit 5

- Desmos. "Polygraph: Exponential & Logarithmic Functions."

  https://www.walch.com/ca/01045

  This activity is designed to spark vocabulary-rich conversations about exponential and logarithmic functions. Key vocabulary terms that may appear in student questions include *exponential*, *asymptote*, *logarithmic*, and *quadrant*.

Unit 6

- Desmos. "Card Sort: Exponentials."

  https://www.walch.com/ca/01044

  In this activity, students practice what they've learned about exponential functions by matching equations to properties of the graphs the functions will produce. Students will then use their knowledge of transforming exponential functions to pair equations with graphs.

- Desmos. "Polygraph: Exponential Functions."

  https://www.walch.com/ca/01046

  This activity is designed to spark vocabulary-rich conversations about exponential functions. Key vocabulary terms that may appear in student questions include *increasing*, *decreasing*, *asymptote*, *quadrant*, and *axis*.

- Desmos. "Polygraph: Parent Functions."

  https://www.walch.com/ca/01051

  This activity is designed to spark vocabulary-rich conversations about graphs of parent functions. Key vocabulary terms that may appear in student questions include *increasing*, *decreasing*, *linear*, *quadratic*, *cubic*, *absolute value*, *exponential*, *logarithmic*, *rational*, *radical*, *axis*, *intercept*, and *coordinate*.

- Desmos. "Polygraph: Twelve Functions."

  https://www.walch.com/ca/01053

  This activity is designed to spark vocabulary-rich conversations about various functions. Key vocabulary terms that may appear in student questions include *linear*, *quadratic*, *exponential*, *cubic*, *absolute value*, *rational*, *radical*, *sinusoid*, and *step*.

- Desmos. "Writing Rules: Linear, Quadratic, and Exponential."

  https://www.walch.com/ca/01047

  In this activity, students have an opportunity to deepen their understanding of linear, quadratic, and exponential functions by making connections between their tables, graphs, and equations.

Station Activities Guide

Introduction

Each unit includes a collection of station-based activities to provide students with opportunities to practice and apply the mathematical skills and concepts they are learning. You may use these activities in addition to the instructional lessons, or, especially if the pre-test or other formative assessment results suggest it, instead of direct instruction in areas where students have the basic concepts but need practice. The debriefing discussions after each set of activities provide an important opportunity to help students reflect on their experiences and synthesize their thinking. Debriefing also provides an additional opportunity for ongoing, informal assessment to guide instructional planning.

Implementation Guide

The following guidelines will help you prepare for and use the activity sets in this section.

Setting Up the Stations

Each activity set consists of four or five stations. Set up each station at a desk, or at several desks pushed together, with enough chairs for a small group of students. Place a card with the number of the station on the desk. Each station should also contain the materials specified in the teacher's notes, and a stack of student activity sheets (one copy per student). Place the required materials (as listed) at each station.

When a group of students arrives at a station, each student should take one of the activity sheets to record the group's work. Although students should work together to develop one set of answers for the entire group, each student should record the answers on his or her own activity sheet. This helps keep students engaged in the activity and gives each student a record of the activity for future reference.

Forming Groups of Students

All activity sets consist of four or five stations. You might divide the class into four or five groups by having students count off from 1 to 4 or 5. If you have a large class and want to have students working in small groups, you might set up two identical sets of stations, labeled A and B. In this way, the class can be divided into eight groups, with each group of students rotating through the "A" stations or "B" stations.

Assigning Roles to Students

Students often work most productively in groups when each student has an assigned role. You may want to assign roles to students when they are assigned to groups and change the roles occasionally. Some possible roles are as follows:

- Reader—reads the steps of the activity aloud
- Facilitator—makes sure that each student in the group has a chance to speak and pose questions; also makes sure that each student agrees on each answer before it is written down
- Materials Manager—handles the materials at the station and makes sure the materials are put back in place at the end of the activity
- Timekeeper—tracks the group's progress to ensure that the activity is completed in the allotted time
- Spokesperson—speaks for the group during the debriefing session after the activities

Timing the Activities

The activities in this section are designed to take approximately 10 minutes per station. Therefore, you might plan on having groups change stations every 10 minutes, with a two-minute interval for moving from one station to the next. It is helpful to give students a "5-minute warning" before it is time to change stations.

Since each activity set consists of four or five stations, the above time frame means that it will take about 50 to 60 minutes for groups to work through all stations.

Guidelines for Students

Before starting the first activity set, you may want to review the following "ground rules" with students. You might also post the rules in the classroom.

- All students in a group should agree on each answer before it is written down. If there is a disagreement within the group, discuss it with one another.
- You can ask your teacher a question only if everyone in the group has the same question.
- If you finish early, work together to write problems of your own that are similar to the ones on the activity sheet.
- Leave the station exactly as you found it. All materials should be in the same place and in the same condition as when you arrived.

Debriefing the Activities

After each group has rotated through every station, bring students together for a brief class discussion. At this time, you might have the groups' spokespersons pose any questions they had about the activities. Before responding, ask if students in other groups encountered the same difficulty or if they have a response to the question. The class discussion is also a good time to reinforce the essential ideas of the activities. The questions that are provided in the teacher's notes for each activity set can serve as a guide to initiating this type of discussion.

You may want to collect the student activity sheets before beginning the class discussion. However, it can be beneficial to collect the sheets afterward so that students can refer to them during the discussion. This also gives students a chance to revisit and refine their work based on the debriefing session. If you run out of time to hold class discussions, you might want to have students journal about their experiences and follow up with a class discussion the next day.

Digital Enhancements Guide

Introduction

With this program, you have access to the following digital components, described here with guidelines and suggestions for implementation.

Digital Instruction PowerPoints (Presentations)

These optional versions of the Warm-Ups, Warm-Up Debriefs, Introductions, Key Concepts, and Guided Practices for each lesson run on PowerPoint. (*Please note*: Computers may render PowerPoint images differently. For best viewing and display, use a PowerPoint Viewer and adjust your settings to optimize images and text.)

Each PowerPoint begins with the lesson's Warm-Up and is followed by the Warm-Up Debrief, which reveals the answers to the Warm-Up questions.

In the notes section of the last Warm-Up slide, you will find the "Connections to the Lesson," which describes concepts students will glean or skills they will need in the upcoming lesson. The "Connections" help transition from the Warm-Up to instruction.

GeoGebra Applets (Interactive Practice Problems)

One or two interactive GeoGebra applets are provided for most lessons. The applets model the mathematics in the Guided Practice examples for these lessons. Links to these applets are also embedded within the Instructional PowerPoints. With an Internet connection, simply click on the "Play" button slide that follows selected examples.

Once you've accessed the GeoGebra applet, please adjust your view to maximize the image. Each applet illustrates the specific problem addressed in the Guided Practice example. The applets allow you to walk through the solution by visually demonstrating the steps, such as defining points and drawing lines. Variable components of the applets (usually fill-in boxes or sliders) allow you to substitute different values in order to explore the mathematics. For example, "What happens to the line when we increase the amount of time?" or "What if we cut the number of students in half?" This experimentation and discussion supports development of conceptual understanding.

GeoGebra for PC/MAC

GeoGebra is not required for using the applets, but can be downloaded for free for further exploration at the following link:

https://www.geogebra.org/download

Curriculum Engine Learning Object Repository

Walch's Curriculum Engine comes loaded with thousands of curated learning objects that can be used to build formative and summative assessments, as well as practice worksheets, instructional components, and an item bank. District leaders and teachers can search for items by standard and create assessments or worksheets in minutes using the three-step assessment builder.

For more information about the Curriculum Engine, or for additional support, please contact us at (207) 828-8800 or success@bwwalch.com.

Standards for Mathematical Practice Implementation Guide

Introduction

The eight Standards for Mathematical Practice describe features of lesson design, teaching pedagogy, and student actions that will lead to a true conceptual understanding of the mathematics standards. The Walch lessons, practice problems, and Problem-Based Tasks lend themselves to teaching through this framework. When the Walch resources are combined with high-level questioning and engaging teacher decisions in the classroom, it will lead to high-level math instruction and student achievement.

Here is a brief description of the SMPs and how they can be applied in the classroom:

CCSS.MP.1: Make sense of problems and persevere in solving them.

Students will read, interpret, and understand complicated mathematical and real-world problems, and they will be willing to try multiple methods with the ultimate goal of determining the correct answer. Strategies such as annotation and student discourse can lead to improvement on this standard. Presenting students with higher-level problems is essential to ensuring students achieve maximum understanding. Teacher prompts that can enhance this standard include:

- What is the problem asking you to solve?
- What are some (other) strategies you could use to solve this problem?
- Compare your answer with a classmate's answer. Who is correct? Why?

CCSS.MP.2: Reason abstractly and quantitatively.

Mathematical reasoning with numbers and variables is essential to understanding the connections among the standards. Students must be able to discover and formalize general rules using numbers and variables, and apply them to determine numerical quantities in other situations. Teacher prompts that can enhance this standard include:

- Substitute realistic numbers into the situation.
- What operation/strategy would you use?
- Will your strategy work for any number?
- For which categories of numbers (negative integers, all real numbers, etc.) will your strategy work?

CCSS.MP.3: Construct viable arguments and critique the reasoning of others.

Many students are most concerned with the "what" aspects of mathematics, i.e. "what" do we do or "what" is the answer. However, math educators must develop the "why" of mathematics. Students must learn to question algorithms, challenge answers, and justify their reasoning in order to truly understand the concepts behind their answers. Teacher prompts that can enhance this standard include:

- How did you determine your answer?
- Why did you choose that strategy?
- Defend your answer based on a real-world situation.

CCSS.MP.4: Model with mathematics.

An important goal of mathematics instruction is for students to be able to apply mathematics to the world around them. Students should be able to link a real problem to a mathematical concept, identify quantities that are modeled well with mathematics, and use mathematics to find a solution. Emphasizing this standard will help students represent and interpret information using physical, visual, and abstract models. Encourage students to use any or all of their learning experiences to gain a deep and flexible understanding of mathematics. Teacher prompts that can enhance this standard include:

- Can you represent this situation with a visual model?
- How will it help you solve the problem?
- What information is needed to solve this problem?
- Is there another way to solve this problem?
- While working to solve this problem, what do you notice/wonder?

CCSS.MP.5: Use appropriate tools strategically.

There are many available tools suitable for mathematics, such as calculators, manipulatives, formulas, rulers, computers, and developed mathematical strategies. Choosing and using the correct tool to work through a problem is an important skill for mathematicians. Teacher prompts that can enhance this standard include:

- Can you graph this equation in the calculator to see a relationship?
- What formula or strategy might help you determine the answer to this question?
- How can you represent the situation using handheld tools (rulers, protractors, etc.) to determine an answer?

CCSS.MP.6: Attend to precision.

When using mathematics to solve problems, an answer can be considered correct only if it is sufficiently precise and accurate for the situation to which it pertains. When applying mathematics, it is vital to clearly define the question, the reasoning, the answer, and the explanation. Vocabulary, units, numerical responses, and pictures must be represented precisely in questions and answers to ensure that the mathematical solutions represent the true answer to a question. Teacher prompts that can enhance this standard include:

- What does your answer represent in a real-world context?
- Is your answer reasonable based on your initial estimate?
- What units of measure help describe your numerical answer?

CCSS.MP.7: Look for and make use of structure.

Structure, whether geometric, algebraic, statistical, or numerical, is an important aspect of mathematical reasoning that students often overlook. Teachers often explicitly refer to geometric and other visual structures as explanations of mathematical concepts, but algebraic and numerical structures can often be just as important in analyzing and interpreting mathematical situations. These structures yield clues as to the meaning of expressions, equations, graphs, and other representations. As students interpret these structures, they will gain a greater understanding of the mathematical concepts. Teacher prompts that can enhance this standard include:

- What do the characteristics of the graph tell us about the situation?
- What do each of the variables and numbers in the equation/formula represent?
- How are these situations the same and different based on their representations?

CCSS.MP.8: Look for and express regularity in repeated reasoning.

Just as patterns appear in real life, patterns appear throughout the subject of mathematics. Recognizing and applying these patterns, and applying the reasoning contained within, is one of the most important skills teachers can instill in their students. Rather than teaching isolated algorithms to determine answers, have students discover relationships, create their own algorithms, and apply the reasoning to other situations. These skills can be applied throughout their education and will enrich their lives after high school. Teacher prompts that can enhance this standard include:

- What relationship do you notice in the graph/table/numbers?
- Why did you choose to use this process to solve this word problem/equation?
- How can you apply this process in other situations?

Instructional Strategies

Ensuring Access for All Students

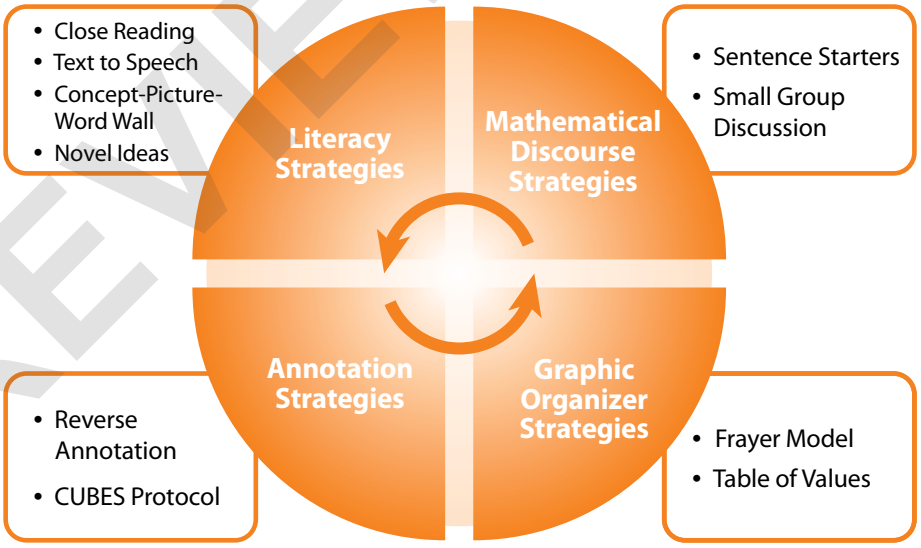
Introduction

The increased focus on literacy in math instruction can help some students navigate mathematical contexts, but for struggling readers, it can further complicate calculations. English language learners struggle to master difficult mathematical concepts while simultaneously processing a new language. Students with learning and behavioral disabilities struggle with the math concepts in their own contexts. This is where teachers and the strategies they select for their classrooms become essential.

The strategies presented here can help all students succeed in math, literacy, school, and, ultimately, in life. These instructional strategies provide teachers with a wide range of instructional support to aid English as a Second Language (ESL) students, students with disabilities (SWD), and struggling readers. These strategies provide support for the Mathematics Standards and the Standards of Mathematical Practice (SMP), English Language Development (ELD) Standards, English Language Arts Standards, and WIDA English Language Development Standards.

Within each lesson throughout this course, you will find suggested instructional strategies. These instructional strategies are research-based strategies and best practices that work well for all students.

The instructional strategies detailed here fall into four main categories: Literacy, Mathematical Discourse, Annotation, and Graphic Organizers. These strategies provide teachers with research-based strategies to address the needs of all students.

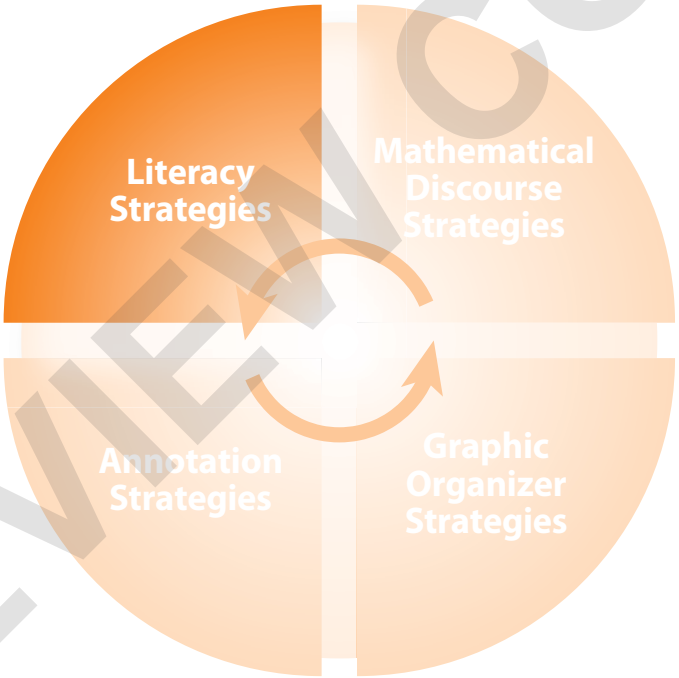


Source

- WIDA: https://www.walch.com/rr/09052

Understanding the Language of Mathematics: Literacy

Mathematics has its own language consisting of words, notations, formulas, and visuals. In education, the language of mathematics is often regarded solely in the context of word problems and articles. This neglects the vocabulary and other mathematical representations students must be able to interpret. The strategies presented here help students navigate the language of mathematics so that they can understand text and feel confident speaking in and listening to mathematical discussions. For students with disabilities, the stress on repetition and different representations in this approach is essential to their ability to grasp the math concepts. For ESL students, repetition and different representations can strip out some of the English language barriers to understanding the language of mathematics, as well as provide multiple means of accessing the content. Literacy strategies include Close Reading, Text-to-Speech, Concept-Picture-Word Walls, and Novel Ideas.



Literacy Strategies

Close Reading with Guiding Questions

What is Close Reading with Guiding Questions?

Close Reading with Guiding Questions is a process that allows students to preview mathematical reading and problems by answering questions related to the text in advance and reviewing their responses during and/or after reading. Multiple reading protocols can be used in conjunction with guiding questions to enhance their effectiveness.

How do you implement Close Reading with Guiding Questions in the classroom?

When utilizing a textbook, task, or article in a math class, literacy struggles are often a strong barrier to entry into the mathematical ideas. Asking students to answer accessible questions before and/or as they read can lead them to the key information.

Prior to implementation, the teacher should determine the most important information students need to obtain from a text, whether it is a math problem to solve, a task to complete, or an informational lesson or article to read. Then, the teacher should come up with some questions to guide students before they read. These questions can:

- assess and relate prior knowledge
- define key vocabulary words
- discuss non-mathematical concepts in the text

The teacher should also prepare some questions to guide students as they read. These questions can:

- point out key concepts within the text
- relate the text and concepts to future learning
- assist students in identifying key facts in the text
- highlight the importance of text features (graphics, headings, etc.) in the text

To ensure the questions are accessible for students and to encourage reflection and debate after reading, many of these questions should be designed as either "True/False" or "Always True/Sometimes True/Never True." Students can represent their reasoning for their answer in writing, numbers, or graphic/pictorial representations. Students should complete the guiding questions and reading individually, with discussion to follow.

After students complete the reading, they should be given some time to individually evaluate their initial answers. Then, in partners or in groups, they can discuss their answers and come to final conclusions that will help them find the important information initially identified by the teacher. After deciphering the text through close reading, students will be able to complete the given activity.

When would I use Close Reading with Guiding Questions in the classroom?

Close Reading with Guiding Questions can be used for any activity in which literacy could be a barrier to learning or demonstrating mastery of mathematical concepts. The number of questions and length of the discussions can be altered based on the length, importance, and difficulty of the text and concept. As students become more accustomed to mathematical literacy, the text complexity can be increased, but the adherence to close reading strategies must be maintained to ensure students can access the mathematical concepts. The length of time spent on the literacy aspect can be shortened as students become more skilled, but the questioning and discussions must occur to ensure students are properly interpreting the text in the mathematical context.

How can I use Close Reading with Guiding Questions with students needing additional support?

For struggling readers, including ESLs, Close Reading with Guiding Questions can help make an intimidating lesson, word problem, or task much more accessible. Questions focusing more on Tier 2 and Tier 3 vocabulary, text features, and real-world concepts can help struggling readers relate to the text and learn how to decipher the text in context. Discussions around the questions will help students grasp the math concepts.

Allowing struggling readers to explain their answers using words, numbers, or graphics/pictures ensures that they can express their opinion and rationale despite a potential lack of vocabulary. Through these representations and the ensuing discussion, students will begin to learn the necessary vocabulary to be successful.

What other standards does Close Reading with Guiding Questions address?

Standards of Mathematical Practice:

- SMP.1
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7

Sources

- Anne Adams, Jerine Pegg, and Melissa Case. "Anticipation Guides: Reading for Mathematics Understanding."

  https://www.walch.com/rr/09053

- Diane Staehr Fenner and Sydney Snyder. "Creating Text Dependent Questions for ELLs: Examples for 6th to 8th Grade."

  https://www.walch.com/rr/09054

Literacy Strategies

Text-to-Speech Technology

What is Text-to-Speech Technology?

Text-to-Speech Technology is an adaptive technology that reads text aloud from a text source for students. It is usually accessed through an application or program on a computer, smartphone, or tablet. Some new programs utilize Mathematical Markup Language (MathML) to read mathematical notation in a common, understandable manner for students. Many programs also highlight the words and notation on the screen as the audio plays, which helps students relate the written representation to the words they hear. The use of Text-to-Speech Technology allows students who struggle with literacy to hear the words and notation and access the text in a different way.

How do you implement Text-to-Speech Technology?

A classroom community focused on everyone's learning and a growth mindset is the first step in implementing Text-to-Speech Technology. One of the main barriers to implementation is encouraging students to use the program. Once they do, they will realize how the audio can help them understand the difficult mathematical texts and interpret the math content within them. After students realize the benefits of Text-to-Speech Technology, it can become part of the regular routine for group and independent work.

The use of headphones can be very important for effective use of Text-to-Speech Technology. Students can use the technology to listen to lessons and texts at their own pace. Extra noise from other students working or other students listening at different paces can confuse students attempting to use Text-to-Speech Technology, and headphones can help mitigate these distractions. Many teachers are nervous about the potential disruption headphones can cause in class. However, well-managed use of headphones can help students successfully utilize the technology to learn.

When would I use Text-to-Speech Technology in the classroom?

Text-to-Speech Technology can be used at any time throughout the year, and if the program speaks in MathML, it can be used with any lesson. Without MathML, effective use could be limited to word problems without unusual notation. For example, if x^2 is read as "x-two" instead of "x-squared" or "x to the second power," that could confuse students more.

During a lesson or small group discussion, Text-to-Speech Technology could detract from students' ability to listen, question, and process information. However, during warm-ups, independent work, or assessments, Text-to-Speech Technology can help students process the information and access the activity. It can become a routine for students to automatically listen to the question, problem, or directions first, and then attempt the activity.

How can I use Text-to-Speech Technology with students needing additional support?

Text-to-Speech Technology is an important adaptation and accommodation for struggling readers. Students who have read-aloud accommodations sometimes don't receive them because they are either embarrassed to accept them or because of staffing restrictions. These students can use Text-to-Speech Technology to supplement their math instruction by having text automatically read to them in a manner in which they can process it.

Additionally, for ESL students, hearing the English mathematical language, especially referring to mathematical representations and notation, can help put English words to the ideas they see. Some Text-to-Speech Technology can translate written and mathematical text into other languages, so students can hear the text in their natural language and see the English highlighted on the screen as they hear it. In this way, students are learning English vocabulary as well as learning the mathematical content in a language they can understand.

What other standards does Text-to-Speech Technology address?

Standards of Mathematical Practice:

- SMP.1
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7

Source

- Steve Noble. "Using Mathematics eText in the Classroom: What the Research Tells Us." https://www.walch.com/rr/09055

Literacy Strategies

Concept-Picture-Word Wall

What is a Concept-Picture-Word Wall?

A Concept-Picture-Word Wall is a classroom display, often a bulletin board or a set of posters, that exposes students to important vocabulary words they will use in math class.

Posting vocabulary words in class helps reinforce the words students will see in textbooks, videos, websites, and test questions on math concepts. These Tier 3 vocabulary words are often not used in everyday language, and the exposure to the words visually through Concept-Picture-Word Walls can help students connect them to the math content.

How do you implement Concept-Picture-Word Walls in the classroom?

Just seeing the vocabulary on a Concept-Picture-Word Wall by itself will help students; more importantly, referring to the words as the teacher uses them in class helps students connect the visual to the application. A simple gesture to the wall makes a very explicit reference to the word as it is used and allows students to connect the unfamiliar word to its meaning in context. Additionally, students can be taught to refer to the wall as they use the words in class, and they can be asked to make sure they say at least 3 words from the wall during each class period in small-group discourse or as answers to whole-class questions. The comfort gained from using these Tier 3 words will help students to use appropriate math vocabulary while solving problems and will help students connect concepts more explicitly.

Postings on the Concept-Picture-Word Wall can be arranged strategically to connect concepts, units of study, or groups of words where appropriate. Having three sections of the Concept-Picture-Word Wall—for example, an "In the Future" section, a "Live in the Present" section, and a "Remember the Past" section——can help students see and remember the vocabulary throughout the entire course. Even without regular use of some words, just seeing the words before a unit can help instill a familiarity with the vocabulary. Leaving the words on the Concept-Picture-Word Wall after a unit is taught can help students connect "old" concepts to the current lesson and ensure that students still have access to the vocabulary.

When would I use Concept-Picture-Word Walls in the classroom?

Concept-Picture-Word Walls can be used for the entire year. The actual words might have to change, or at least be moved to different areas of the Concept-Picture-Word wall. The more exposure students have to the words, the more familiar and comfortable they will become. The constant exposure to the math context is beneficial for students throughout the entire course, especially for words with multiple meanings (bias, tangent, etc.) that could exist as Tier 2 words in everyday conversation but are Tier 3 words in the math classroom.

How can I use Concept-Picture-Word Walls with students needing additional support?

For all students learning mathematics, knowing and using the math vocabulary is often a major barrier. This is a problem especially for ESL students, who are learning the English language along with math content. If teachers try to simplify the words too much for students, it does them a disservice as they seek out information from other teachers, textbooks, and online sources that use the proper vocabulary. Most tests, especially state tests, will expect students to have knowledge of the Tier 3, math-specific vocabulary. The more students see these words, the more familiarity they will have when they apply them.

Concept-Picture-Word Walls can also be written in multiple languages. Especially for students who are on-grade-level in their native language, a multi-lingual Concept-Picture-Word Wall can help students connect the content they already know in another language to the English vocabulary necessary for success on English-language math activities and tests.

This website can help you get started on an English-Spanish Concept-Picture-Word Wall: https://www.walch.com/rr/09056

What other standards do Concept-Picture-Word Walls address?

Standards of Mathematical Practice:

- SMP.1
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7

Source

- Janis M. Harmon, Karen D. Wood, Wanda B. Hedrick, Jean Vintinner, and Terri Willeford. "Interactive Word Walls: More Than Just Reading the Writing on the Walls."
  https://www.walch.com/rr/09057

Literacy Strategies

Novel Ideas

What is Novel Ideas?

Novel Ideas is a classroom activity that explores students' understanding of important Tier 2 vocabulary words they will use in math class. Instead of asking students to look up vocabulary words in the dictionary, Novel Ideas allows students to have conversations with their peers about vocabulary words in class. This reinforces the mathematical vocabulary students will see in textbooks, videos, websites, and test questions. These Tier 2 vocabulary words are often used in everyday language, but have specific meaning in mathematics. Exposure to the words through Novel Ideas can help students connect them to the math content.

How do you implement Novel Ideas in the classroom?

While building a rich representation of math content words and connecting the words to other words and concepts has inherent merit, it is more important to consider that pre-teaching the words before they are used in class helps students connect to the application. The understanding gained from discussing these Tier 2 words will help students apply them in a mathematical context to solve problems and connect concepts.

Here is a step-by-step process for implementing Novel Ideas:

1. Students separate into groups of four.
2. Students copy the teacher generated prompt/sentence starters and number their papers 1–8.
3. One student offers an idea, another echoes it, and all write it down.
4. After three minutes, students draw a line under the last item in the list.
5. All students stand, and the teacher calls one student from a group to read the group's list.
6. The student starts by reading the prompt/sentence starters, "We think a _____ called _____ may be about ... ," and then adds whatever ideas the team has agreed on.
7. The rest of the class must pay attention because after the first group has presented all their ideas, the teacher asks them to sit down and calls on a student from another team to add that team's "novel ideas only." Ideas that have already been presented cannot be repeated.
8. As teams complete their turns and sit down, each seated student should record novel ideas from other groups below the line that marks the end of his or her team's ideas.

When would I use Novel Ideas in the classroom?

Novel Ideas can be used for the entire year. The more students are exposed to mathematical vocabulary, the more familiar and comfortable they become, leading to increased usage of these math terms in their conversation and writing. Using math vocabulary in context is beneficial for students throughout the entire course, especially for words with multiple meanings (bias, tangent, etc.) that could exist as Tier 2 words in everyday conversation but are Tier 3 words in the math classroom.

How can I use Novel Ideas with students needing additional support?

Most tests, especially state tests, will expect students to have knowledge of the Tier 3, math-specific vocabulary. The more students use these words in conversation, the more familiarity they will have when they apply them. Understanding Tier 2 words also helps students avoid misconceptions in mathematics. Twice a week before the start of a lesson, allow students to use sentence starters in small groups that include all students. Prepare the sentence starter "When I hear the word _____, I think about _____" to share out with whole class. This will allow students who know the vocabulary words to share their knowledge, and will allow other students to hear the meaning of the vocabulary words. This strategy is particularly helpful for ESL students.

What other standards does Novel Ideas address?

Standards of Mathematical Practice:

- SMP.1
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7

Sources

- Colorín Colorado. "Selecting Vocabulary Words to Teach English Language Learners."

  https://www.walch.com/rr/09058

- Elsa Billings and Peggy Mueller, WestEd. "Quality Student Interactions: Why Are They Crucial to Language Learning and How Can We Support Them?"

  https://www.walch.com/rr/09059

Novel Ideas Sentence Starters

Slope

- When I hear the word climb, I think about ...
- When I hear the word steep, I think about ...

Volume

- When I hear the word filling, I think about ...

Equations

- When I hear the word balance, I think about ...
- When I hear the word equal, I think about ...

Graphing

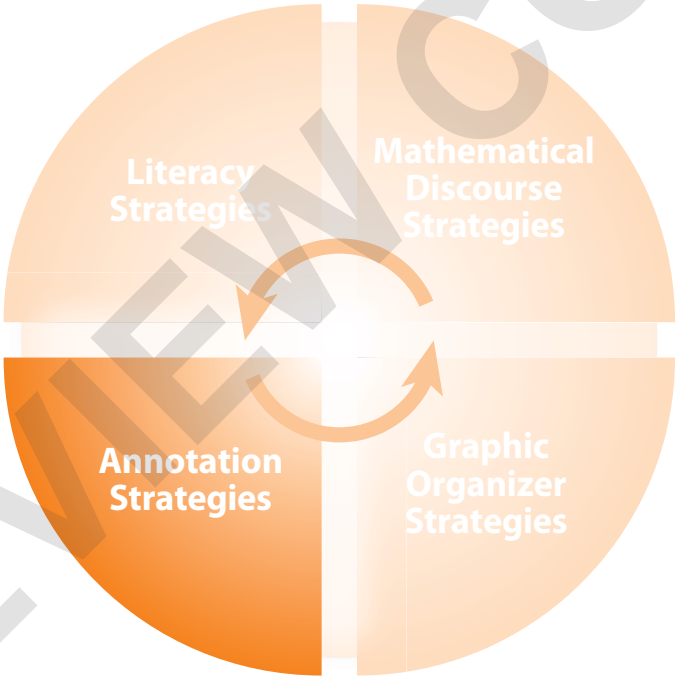
- When I hear the word grid, I think about ...
- When I hear the word graph, I think about ...

Scatter Plotsf

- When I hear the word scattered, I think about ...

Understanding Mathematical Content: Annotation

Understanding mathematical content is an extremely important skill, both in the math classroom and in life. When students read word problems, articles, charts, graphs, equations, tables, or other forms of mathematical text, they must be able to decode and extract meaning from the text. Annotation can help. The strategies presented here help students identify and focus on key characteristics and facts from various forms of text while ignoring the non-essential information. For students with disabilities, many of whom struggle with the distractions inherent in many high-school level texts, making notes and drawing pictures to explain a problem can help them focus. ESL students will be pointed to certain Tier 3 vocabulary words and determine which Tier 2 vocabulary words they must learn to be proficient in math class and in the English language. Annotation strategies include Reverse Annotation and CUBES protocol.



Annotation Strategies

Reverse Annotation Protocol

What is Reverse Annotation?

Reverse Annotation is a strategy that asks students to identify and write down key information from math problems. This is especially helpful for problems given on a computer or tablet, where students can't annotate directly on the problem. A template is given at the end of this section.

How do you implement Reverse Annotation in the classroom?

Many annotation strategies ask students to write, underline, or mark directly on the text of a problem. While those forms of annotation are also beneficial, they are not always possible with technology. Whether the problem is given on paper or using technology, having students write the answers to these questions will ensure that they are thinking strategically and specifically about the strategies and information needed to solve the problem.

The three questions at the top of the Reverse Annotation template are the key to understanding mathematical problems. For every problem given in class, ask students:

1. What is the problem asking us to solve?
2. What key words tell us the mathematical steps we need to perform?
3. What information in the problem can help us figure it out?

After answering the initial questions, students should make a guess, or estimate, of what they think the answer will be. This helps grow their number sense, and provides an initial, reasonable solution to guide their work. Students can then use the strategies they selected to solve the problem and evaluate their solution using the questions at the bottom of the template.

When students first begin to use Reverse Annotation, the teacher should walk them through the steps individually to ensure they can accurately identify the question, key words, and important information. Teachers can also lead students through the estimation process, making a game out of which student has the closest estimate.

Work through each step individually for several "easy" problems first, so that difficult math doesn't interfere with the process. Increase the problem difficulty incrementally as students begin to master the process. This may seem like a long process at first, but the ultimate result is worth the time investment.

When would I use Reverse Annotation in the classroom?

Reverse Annotation can be used to solve any math problem, and is especially helpful for word problems. When Reverse Annotation is initially implemented, the steps should be discussed in detail. As students become accustomed to Reverse Annotation and begin thinking about problems in

this manner automatically, the individual steps become less important and can be scaffolded out to improve efficiency. Students should reach the point where they immediately ask themselves the three initial questions when they first see a problem. However, the teacher should ensure that students are truly evaluating all the key information before routine discussions of the individual steps are removed.

How can I use Reverse Annotation with students needing additional support?

Annotation strategies can help students identify key information, even when certain vocabulary words are not known. As teachers introduce the content-specific Tier 3 vocabulary to their classes, annotation strategies such as reverse annotation can help students use these words to apply appropriate strategies while problem solving. Answering the three initial questions can help students organize the key facts and vocabulary, and the identification of key information can simplify the problem. This strategy is especially beneficial for ESL students.

Using reverse annotation with graphic organizers benefits ESL students by removing a lot of the confusing wording and allowing them to focus on the important pieces of a problem. When using Reverse Annotation, all students, including ESL students, will begin to think about problem solving in a way that encourages them to use the appropriate information to find a solution.

What other standards does the Reverse Annotation Protocol address?

Standards of Mathematical Practice:

- SMP.1
- SMP.2
- SMP.5
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.SL.9–10.3
- ELA–LITERACY.SL.9–10.2
- ELA–LITERACY.RST.9–10.4

Source

- Alliance for Excellent Education. “Six Key Strategies for Teachers of English Language Learners.” https://www.walch.com/rr/09060

Reverse Annotation Template

Name: _____ Problem/Assignment: _____

Analyze the Problem

What is the problem asking us to solve?	
What key words will tell us the mathematical steps we need to perform?	
What information in the problem can help us figure it out?	

Initial estimate of solution:

Work Space

Remember to box in your solution!

Name: _____ Problem/Assignment: _____

Check It Over

How close was your estimate?	
Does your answer make sense? Is it reasonable? How do you know?	
Did you perform the calculations correctly?	
What does your answer mean in context?	

Annotation Strategies

CUBES Protocol

What is the annotation strategy CUBES?

CUBES is an annotation strategy in which students use different written designs to highlight the key aspects of word problems. It can help them choose the correct mathematical strategy to solve the problem accurately.

How do you implement CUBES in the classroom?

The steps for CUBES are:

1. **C**: **C**ircle all the key numbers.
2. **U**: **U**nderline the question.
3. **B**: **B**ox in the key words that will determine the operation(s) necessary and write the mathematical symbol for the operation(s).
4. **E**: **E**valuate the information given to determine the strategy needed. **E**liminate any unnecessary information.
5. **S**: **S**olve the problem, **s**how your work, and check your answer.

As students learn to use CUBES, walk them through the steps individually to ensure they can accurately identify the key numbers, question, key words, unnecessary information, and strategy. Work through each step individually for several "easy" problems first, so that difficult math doesn't interfere with the process. Increase the problem difficulty incrementally as students begin to master the process. This may seem like a long process at first, but the ultimate result is worth the time investment.

A graphic organizer can help students master the process, especially when problems are given on a computer or tablet where students can't always annotate directly on the problem. Students can write down the key numbers and circle them, write down the question and underline it, and so on. This will encourage students to truly think about the different pieces of the problem they are identifying, and how these pieces will guide the strategy and affect the solution.

When would I use CUBES in the classroom?

CUBES can be used to solve any math problem, and is especially helpful for word problems. When CUBES is initially implemented, the steps should be discussed in detail. As students become accustomed to using CUBES and begin thinking about problems in this manner automatically, the individual steps become less important and can be scaffolded out to improve efficiency. However, the teacher should ensure that students are truly evaluating all the key information before routine discussions of the individual steps are removed.

How can I use CUBES with students needing additional support?

Design features can help students identify key words and features, even when certain vocabulary words are not known. As teachers introduce the content-specific Tier 3 vocabulary to their classes, annotation strategies such as CUBES can help students use these words to apply appropriate strategies while problem solving. Using circles, underlines, and boxes can help students organize the key facts and vocabulary, and the elimination of unnecessary information can simplify the problem. This strategy is especially beneficial for ESL students.

Combining CUBES with graphic organizers also benefits ESL students by removing a lot of the confusing wording and allowing them to focus on the important facts of a problem. When using CUBES with a graphic organizer, all students, including ESL students, will begin to think about problem solving in a way that helps encourage them to use the appropriate information to find a solution.

What other standards does the CUBES Protocol address?

Standards of Mathematical Practice:

- SMP.1
- SMP.2
- SMP.5
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

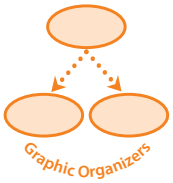
- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.SL.9–10.3
- ELA–LITERACY.SL.9–10.2
- ELA–LITERACY.RST.9–10.4

Source

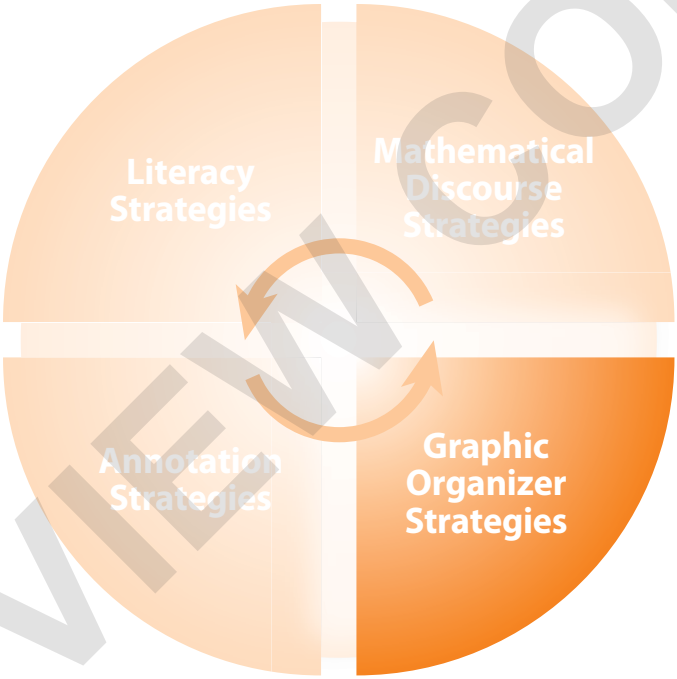
- Margaret Tibbett. “Comparing the effectiveness of two verbal problem solving strategies: Solve It! and CUBES.”

  https://www.walch.com/rr/09061

Organizing Mathematical Content: Graphic Organizers



Organizing mathematical content is a crucial skill for problem solving, exploring other possible methods for finding solutions, and managing math content. All students need strategies for organizing content to build conceptual understanding. For students with disabilities, visual representations and graphic organizers can help them clarify their thoughts and focus on the math. ESL students also benefit from visual representations and graphic organizers. Organizing mathematical knowledge with visuals can help ESL students navigate math content while learning the language. Graphic organizers include Frayer Models and Tables of Values.



Graphic Organizers

Frayer Models

What is a Frayer Model?

A Frayer Model is a graphic organizer that can help students understand new vocabulary words and concepts by exploring their characteristics. A Frayer model lists the definition of a word or concept, describes some key facts, and gives examples and non-examples. Examples and non-examples can come from a mathematical or real-world context.

How do you implement Frayer Models in the classroom?

Students can learn to create Frayer Models the first week of school, and the process can be used throughout the year each time students experience a new word or concept.

While it is important for teachers to give students precise mathematical definitions with appropriate content vocabulary, it is maybe more important for students to understand the application of mathematical words and concepts in their own context. As students learn new information, small group discussions and think-pair-share activities are great ways for students to formulate their own definitions, review the characteristics and facts they have learned, and discuss examples and non-examples.

Discussions of the examples and non-examples can help lead to the mathematical definition. For example, if students use a Frayer Model to define a quadratic function, they would notice that all examples have a highest exponent of 2, and all non-examples would not have a highest exponent of 2. All examples would have parabolic graphs, and all non-examples would have other graphs. Through these comparisons, students will understand the definition of quadratics using different representations, and they will be able to apply it in different contexts.

When would I use Frayer Models in the classroom?

Frayer Models can be used at different points during instruction. They are appropriate as introductions to new concepts, summaries to ensure understanding of new concepts, or as note-organizers throughout the lesson for students to fill in as they learn new concepts. At first, students might need help figuring out how to list and differentiate between the definition, facts and characteristics, examples, and non-examples. As students adapt to the process, they will be able to categorize information on their own or in small groups. As they compare newer Frayer Models to previous models, they will also be able to see how concepts build upon each other.

How can I use Frayer Models with students needing additional support?

Frayer Models can be a point of reference for students as they progress throughout the year. As students determine their own definitions for math-specific words and concepts, and use the examples

and non-examples to determine the key facts, they will be able to put them in their own context and apply them to solve complicated problems. As math concepts build upon each other both within a unit and throughout the year, the use of Frayer Models to remind students of their initial definitions of words or concepts can help solidify their understanding. Using Frayer Models as part of a Word Wall or Concept Wall, or having a consistent notebook process to reference past Frayer models, can help consistently reinforce learning.

What other standards do Frayer Models address?

Standards of Mathematical Practice:

- SMP.1
- SMP.2
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.1
- ELA–LITERACY.SL.9–10.1
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4

Source

- Deborah K. Reed. "Building Vocabulary and Conceptual Knowledge Using the Frayer Model." https://www.walch.com/rr/09062

Frayer Model

Definition	Characteristics
WORD	
Examples from Life	Non-Examples

Graphic Organizers

Tables of Values

What is a Table of Values?

A Table of Values is an organized way to list numbers that represent different categories of values. These values can be represented as ordered pairs, graphs, word problems, or lists. Tables can help students see and compare values in a different way.

How do you implement Tables of Values in the classroom?

Tables can be used throughout the year to support various mathematical standards. Some standards mention tables specifically, and in others, tables can be an effective support to help students organize and understand the meaning and application of values.

Tables can be set up with numerical values in rows or columns. The key to understanding the values lies in the headings. The headings must be specific enough to show students the meaning and/or application of the numerical values, but not so wordy that they interfere with the clarity of the numbers in the table. For example:

x (year)	*y* (population in millions)
1960	219
1970	230
1980	258
1990	312
2000	342

Mean (statistical average)	50	45
Median (middle value)	52	43
Quartile 1 (median of the lower 50%)	40	38
Quartile 3 (median of the upper 50%)	72	80
Range (difference of max and min values)	80	61
Interquartile Range (difference of quartiles)	32	42
Standard Deviation (measure of spread of data)	7.24	10.23

When would I use Tables of Values in the classroom?

Various mathematical topics can be represented by tables. For example:

- An (x, y) table of values to represent coordinates on a graph or independent and dependent variables for a given context
- A table to represent coefficients and/or constants in an equation
- A table to show different statistical measures when comparing sets of data
- A table to compare output values for the same input given different functions

Each time numbers or values are being listed, compared, or graphed, a table can help students differentiate between the values. Tables are easy to create, and students can be encouraged to create them as another representation to clarify and compare numbers for nearly any topic.

How can I use Tables of Values with students needing additional support?

Tables of Values can help students focus on numerical values and their meaning in context without distraction. They clarify what each number represents, what numbers can be compared, and what ordered pairs can be graphed to give a visual representation. Additionally, headings can be used to either highlight the relevant facts from a context or to describe mathematical vocabulary.

In general, graphic organizers benefit students by removing much of the confusing wording and focusing on the important facts and numbers of a problem.

What other standards do Tables of Values address?

Standards of Mathematical Practice:

- SMP.1
- SMP.2
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

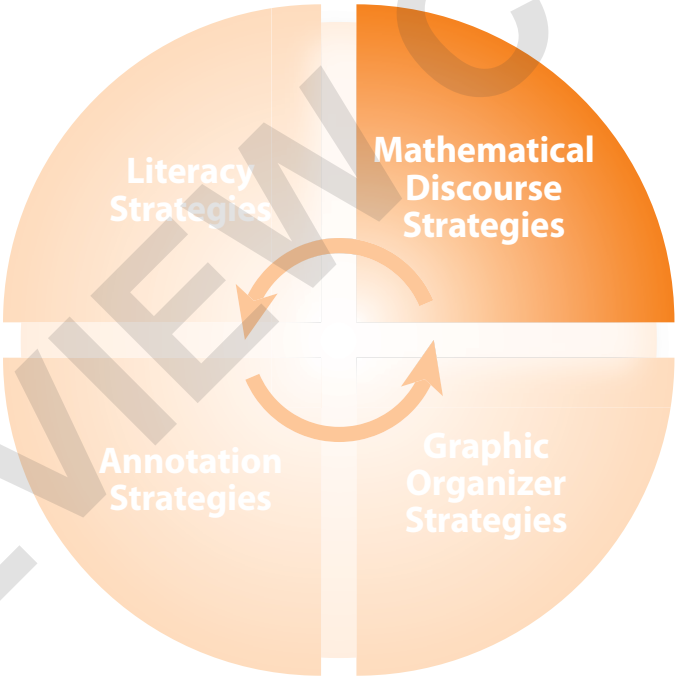
- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.1
- ELA–LITERACY.SL.9–10.1
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4

Source

- Alliance for Excellent Education. "Six Key Strategies for Teachers of English Language Learners." https://www.walch.com/rr/09060

Communicating Mathematical Content: Mathematical Discourse

Reading, writing, speaking, and listening are all important ways to learn and express information, but the last two ways are often slighted in the math classroom. The mathematical discourse strategies presented here promote speaking and listening in a math-focused literacy context. Working these strategies into the daily routine of a classroom can help students become comfortable speaking and listening in a mathematical context, which will help them become comfortable with the mathematical content. Routines and structures are essential to support students with disabilities, as they often benefit from following a routine. This can lead to developing capability in their mathematical skills. These strategies also remove the barrier to entry for many ESL students, as structure and routine can help them focus on the math content rather than English language deficiencies. Mathematical Discourse strategies include Sentence Starters and Small Group Discussion.



Mathematical Discourse Strategies

Sentence Starters

What is a Sentence Starter?

A Sentence Starter is a common phrase or mathematical sentence frame that can help students begin and sustain academic conversations around mathematical content. It helps guide students through the discussion and bring out pertinent ideas that can lead to greater understanding.

How do you implement Sentence Starters in the classroom?

Many people view math class as a place to calculate solutions to math problems. However, to ensure the conceptual understanding and proper application of a math concept, students need to be able to explain the concepts and reasoning behind a solution to a problem. As many students are not accustomed to having academic conversations about math, sentence starters can help begin and continue these conversations in a productive manner.

There are two main types of sentence starters for mathematical discussions: discourse starters and math starters. For example, a poster with these or other sentence starters can be displayed from the beginning of the year, and the expectation can be set that any answer to a question or comment in a discussion should be framed using one of these starters. As students become accustomed to framing mathematical conversations in this way, they can expand on the given sentence starters and create some of their own. They will begin to realize how these statements ensure that their conversations revolve around math, enhance understanding of the concept, and force them not only to state, but also to explain their thinking. They will gain confidence from the ability to engage, as the first step has already been taken for them.

When would I use Sentence Starters in the classroom?

Sentence Starters can be used throughout the entire school year with any concept. However, they are most important to use at the beginning of the school year to build a mathematical community in the classroom centered on a comfort with mathematical discourse. Especially at the beginning of the year, students should be encouraged to use these sentence starters for every math statement. Appropriate settings include during small group discussion, while responding to whole class questions, and when writing explanations for problem solutions.

Modifications can be introduced so that students must use certain mathematical vocabulary within the sentences, or must use certain sentence starters at different points in conversations or for different conversation types and situations. However the starters are implemented, it is important for students to realize that these are intended to enhance and focus their conversations, not limit them.

How can I use Sentence Starters with students needing additional support?

Often, students are reluctant to talk about math concepts because they either lack confidence in their knowledge, are afraid to be "wrong," or don't know how to start or continue the conversation. Sentence starters can help students overcome this reluctance. The non-threatening, easy-to-interpret sentence starters remove the barrier to entry for students who don't know how to engage, and the respectful, mathematical focus promoted by sentence starters can help build confidence and provide a structure so that students will not fear being wrong.

For ESL students specifically, sentence starters can provide the English language support to help students engage with and discuss the math. The support of sentence structure removes language barriers to entry for students who don't fully understand English sentence structure.

Discourse Starters	Math Starters
I agree/disagree with ... because ...	My answer was ... because ...
I understand/don't understand ...	The next step is ... because ...
First/Next/Finally I ... because ...	I used (insert formula/equation/concept) because ...
I noticed that ...	
I wonder ...	My answer is right/reasonable because ...

What other standards do Sentence Starters address?

WIDA English Language Development Standards

- ELD Standard 3

Standards of Mathematical Practice:

- SMP.1
- SMP.3
- SMP.6

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.1
- ELA–LITERACY.SL.9–10.1
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4

Source

- AVID. "Sentence Starters." https://www.walch.com/rr/09064

Mathematical Discourse Strategies

Small Group Discussion

What is Small Group Discussion?

Small Group Discussion is a structured way for students to verbalize their mathematical thinking in a comfortable setting to solve a problem, build conceptual understanding, or summarize a concept.

How do you implement Small Group Discussion?

Small Group Discussion in math class depends on a trusting relationship between the teacher and the students. From there, students can build trusting relationships among themselves. Once this trust has been built, students will feel free to explore mathematical topics in groups, take risks, and engage in a productive struggle toward understanding or a solution.

Once these relationships have been established, certain structures should be established for Small Group Discussion to be effective. Discussion norms can be set by the class to ensure discussions are respectful and productive, and discussions should have predetermined time limits. The group composition is also important and should be based on instructional measures. For different activities, homogeneous groups, heterogeneous groups, or groups based on specific data by standard could be appropriate. Students should always be aware that the groups were chosen to maximize their learning.

Another structure that can be effective for Small Group Discussion is assigning group roles. These roles can include group leader, note taker, timekeeper, resource manager, culture keeper, or other roles determined to be appropriate for the classroom context. During the discussion, assigning each student a letter within the group (A, B, C, D, etc.) can help structure the discussion. Different roles can specify certain time limits for talk, which sentence starters to use, or other structured aspects of the discussion.

When implementing a Small Group Discussion, the question or task should inspire students to think in different ways about a concept. Through the structured format of the discussion, students will compare their ideas and arrive at an answer or explanation of the concept. Within the trusting framework of the class and group, students can focus on the common goal of the discussion and develop their thinking around the math concept. These rich discussions will enhance their understanding.

When would I use Small Group Discussion in the classroom?

Small Group Discussion can be used for nearly any topic, and it can be used at a variety of times in the classroom. The questions and tasks may need to change depending on when it is used. Opening activities for lessons can be Small Group Discussions where students explore properties of new math concepts or review/build upon their prior learning. Turn and talks throughout the lesson can be structured as Small Group Discussions if a consistent framework is in place. At the end of class, a Small Group Discussion can be used to come to a common understanding about an essential question from the lesson.

Depending on when the Small Group Discussion is used in class, and what the goal of the discussion is, the discussion reporting may vary. For a warm-up, each group might be asked to share their thinking. For a guided practice, recording answers on chart paper and a gallery walk could be appropriate. For a closing activity, individual written responses to a question could be appropriate.

How can I use Small Group Discussion with students needing additional support?

As discussed in other Mathematical Discourse strategies, struggling students are reluctant to talk about math concepts because they lack confidence in their knowledge and don't always have the needed vocabulary in their toolbox. Structured discussions with effective grouping can help students through these barriers. After a trusting and respectful classroom environment has been established, struggling students often feel more comfortable sharing their ideas with just a few classmates rather than the whole class. Additionally, adding structure can help students engage by providing the expectation that they participate in the process.

The intentional grouping of students can also help them succeed using Small Group Discussion. At times, heterogeneous groups could be appropriate so that stronger students can help struggling students, and at other times, homogeneous groups could be appropriate so the teacher can work with an entire group of struggling students. ESL students can be grouped with other students with the same dominant language to help remove the language barrier from the conversation.

What other standards does Small Group Discussion address?

WIDA English Language Development Standards:

- ELD Standard 3

Standards of Mathematical Practice:

- SMP.1
- SMP.3
- SMP.6

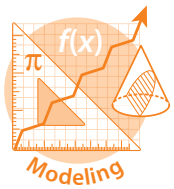
Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.1
- ELA–LITERACY.SL.9–10.1
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4

Source

- Jessie C. Store. "Developing Mathematical Practices: Small Group Discussions."
  https://www.walch.com/rr/09065

Modeling Strategies



Mathematical Modeling

What is Mathematical Modeling?

Mathematical modeling is generally understood as the process of applying mathematics to a real-world problem with a view of understanding the connection. According to the CCSSM, mathematical modeling is the ability to apply concepts learned in class to real-world applications and to use the model to analyze a situation, draw conclusions, and make predictions.

How do you implement Mathematical Modeling in the classroom?

Modeling can be implemented by demonstrating how to make or generate mathematical representations or models, how to validate them, and how to use them to solve real-world problems. There are many ways to show understanding in a math classroom, such as using words, drawings or sketches, physical models, computer programs, or math formulas.

The following is a list of questions and answers suggested in order to create a mathematical modeling classroom environment:

- **Why?** What are we looking for? Identify the need for the model.
- **Find?** What do we want to know? List the data we are seeking.
- **Given?** What do we know? Identify the available relevant data.
- **Assume?** What can we assume? Identify the circumstances that apply.
- **How?** How should we look at this model? Identify the parameters.
- **Predict?** What will our model predict? Identify the equations that will be used, the calculations that will be made, and the answers that will result.
- **Valid?** Are the predictions valid? Identify tests that can be made to validate the model; i.e., is it consistent with its principles and assumptions?
- **Verified?** Are the predictions good? Identify tests that can be made to verify the model; i.e., is it useful in terms of the initial reason it was done? *(inspired by Carson and Cobelli, 2001)*

Teachers should expect these questions to recur often during the modeling process, and should regard this list as a fairly general approach to ways of thinking about mathematical modeling.

In a classroom where mathematical modeling is the expectation, teachers will need to establish that students are responsible for coming up with methods for solving the problems presented and that the teacher will only assist and facilitate.

When would I use Mathematical Modeling in the classroom?

It should come as no surprise that many students find mathematics boring. The most common question posed to any mathematics teacher is "When will I ever need to use this?" Often teachers fail to find problems in which students are interested or to even take student interest into account when planning a lesson. Problems that spark students' interest and curiosity will increase their attention and desire to learn. These types of real-world problems provide students an opportunity to think and respond as a mathematician. Students should be exposed to rigorous learning tasks that allow opportunities for mathematical modeling in the classroom.

How can I use Mathematical Modeling with struggling students?

When struggling readers, which includes ELLs and students with learning disabilities, are exposed to rigorous math learning tasks, there must be a level of scaffolding that includes coaching and guided questions that help to make a word problem or learning task much more accessible. Teachers should come up with questions to guide the students before and during the engagement of the task. Teachers should also:

- assess prior knowledge;
- define Tier 2 and 3 vocabulary words;
- discuss non-mathematical concepts in the task; and
- assist students in identifying key concepts and facts within the tasks.

Allowing struggling readers to explain their answers using words, numbers, or graphics/pictures ensures that they can express their opinion and rationale despite a potential lack of vocabulary. Through these representations and the ensuing discussion, students will begin to learn the necessary math concepts to be successful.

What other standards does Mathematical Modeling address?

WIDA English Language Development Standards:

- ELD Standard 3

Standards of Mathematical Practice:

- SMP.1
- SMP.2
- SMP.4
- SMP.5

English Language Development for Mathematics:

- ELD–A.9–12: Explain (Interpretive)
- ELD–MA.9–12: Explain (Expressive)

English Language Arts standards:

- ELA–LITERACY.SL.9–10.2
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7
- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9

Sources

- Carson, Ewart, and Cobelli, Claudio. *Modeling Methodology for Physiology and Medicine*. Academic Press, San Diego, CA, 2001.
- Dym, Clive L. *Principles of Mathematical Modeling*. Amsterdam: Elsevier Academic Press, 2008.
- MathisFun.com. "Mathematical Models." https://www.walch.com/rr/09066
- Oswalt, Selena. "Mathematical Modeling in the High School Classroom." LSU Digital Commons. Accessed February 13, 2024. https://www.walch.com/rr/09067

PROGRAM OVERVIEW

Mathematical Modeling Implementation Guide

Introduction

Walch resources support the framework of the Standards for Mathematical Practice (SMPs) and the NCTM Principles of Teaching Practices. Implementing strategies and support from both practices lead to true conceptual understanding of the math standards. One of which includes mathematical modeling, the process of designing and revising representations to solve a problem.

Mathematical modeling is essential to building a deep conceptual understanding of math concepts for students. Teaching students to model boosts engagement, builds student confidence in math concepts, helps them to make sense of problems, and allows them to make connections to the world around them for better understanding. Students then make decisions about the information, create models, interpret the results, and form conclusions.

A Mathematical Modeling Framework

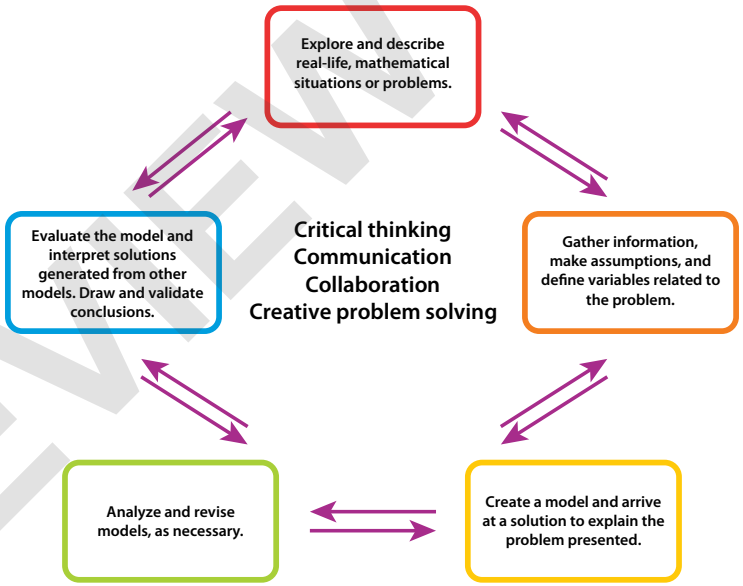


Image adapted from: Suh, Matson, Seshaiyer, 2017

The following is a brief description of how this framework can be applied in the classroom.

Critical Thinking

Students will explore and describe real-life mathematical situations or problems. We want students to discover new ways of thinking and ideas in mathematics. Students do this by developing questions to ask, gathering information, and coming up with solutions. Fostering critical thinking in the classroom

not only makes students better at math, but also prepares them for the real world. Below are some ideas and probing questions teachers may use to implement critical thinking.

- Allow for pair-share and small group discussions.
- Encourage students to think and form their own conclusions.
- Encourage the revision of their own thinking and the thinking of others.
- Ask students to think out loud as they work.
- Create a classroom environment that embraces and values student ideas.

Ask students:

- What is the problem asking you to solve?
- Can you think of other strategies you could use to solve this problem?
- What conclusions can you make from this particular problem?
- Will this strategy work in all problems like this? Why or why not? How can we test that?
- Explain how you got to your answer.
- Explain your reasoning.
- How would you respond to a different answer to the same problem?

Communication

When students gather information, make assumptions, and define variables related to the problem, communication allows for them to show their understanding of the math content. Encourage discourse by allowing students to explain their thinking and challenge each other. This encourages students to justify their reasoning. If students communicate their thinking in various ways (including written and oral responses) while doing math, it will improve their understanding of math concepts.

Teachers can do the following to foster communication in the classroom:

- Ask open-ended questions.
- Encourage oral and visual (written and pictorial) communication through journal writing.
- Provide students with detailed feedback.

Ask students:

- Can you explain your thinking?
- How did you get your answer?
- What strategies did you use?
- What information was necessary for you to solve this problem?

Collaboration

Collaboration is an essential component of student success. It allows students to rely on each other during their problem solving. During collaboration, students work in groups, share ideas, ask questions, and discuss math concepts and additional solution strategies while supporting and defending their thinking. Collaboration is most beneficial to students with the use of effective grouping strategies such as assigning students to heterogeneous groups or random grouping.

The following procedures and probing questions can help you implement collaboration in your classroom.

- Establish a classroom culture where all ideas are valued.
- Establish expectations and routines of collaborative learning.
- Discuss "math talk" passages with students.
- Allow students to teach each other.
- Incorporate an accountability piece for students.
- Arrange student seating to support collaboration (group seating).
- Create heterogeneous student groups with varying skill levels.
- Randomize student groups.
- Keep group sizes between 3 and 6 students.
- Assign group roles.

Ask students:

- Come up with as many strategies to solve the problem as you can.
- Explain how you made your calculations.
- Why did you choose that strategy? Why did that strategy work?
- Describe in your own words how your peer-solved the problem.
- Can you make any connections between your strategies?
- Were there any methods that were better than others when solving this problem? Why or why not?
- What did you learn from your group?
- Defend your reasoning behind that solution.

Creative Problem Solving

Creative problem solving is the ability for students to perform math tasks that allow for challenges that increase their conceptual understanding. While performing these tasks, we want students to use mathematical modeling. We want students to evaluate their models and to interpret solutions from other models.

In creative problem solving, students solve problems using different approaches and models, draw on prior knowledge, and justify their thinking. This results in students becoming better problem solvers and increases their understanding of math concepts. Problem solving should be integrated into their math learning and should not be separated.

Here are some tips for implementing creative problem solving.

- Encourage students to challenge different approaches and strategies from their peers as well as the teacher.
- Encourage discourse.
- Allow appropriate wait time for student responses.
- Refrain from telling students how to solve the problem. Instead, allow students to engage and come to their own solutions.
- Allow students to struggle productively.

Ask students:

- How is the information in the problem important to determining the solution?
- How did you go about solving this?
- Can you explain why you chose that model and strategy?
- Are there other ways to model this particular problem? Can you model the problem another way?
- Why did you make that calculation?
- Justify your solution.
- What generalizations can you make about the math concepts based on this particular problem?

Recommended Resource

- Georgia Department of Education. "Scaffolding Instruction for English Learners: A Georgia Mathematics Instructional Resource Guide."

  https://www.walch.com/rr/09047

  The purpose of this document is to provide mathematics teachers and leaders with evidence-based, pragmatic scaffolds and supports for English Learners (ELs). This guide is a useful tool to help teachers provide high-quality instruction aligned to Georgia's K-12 Mathematics Standards.

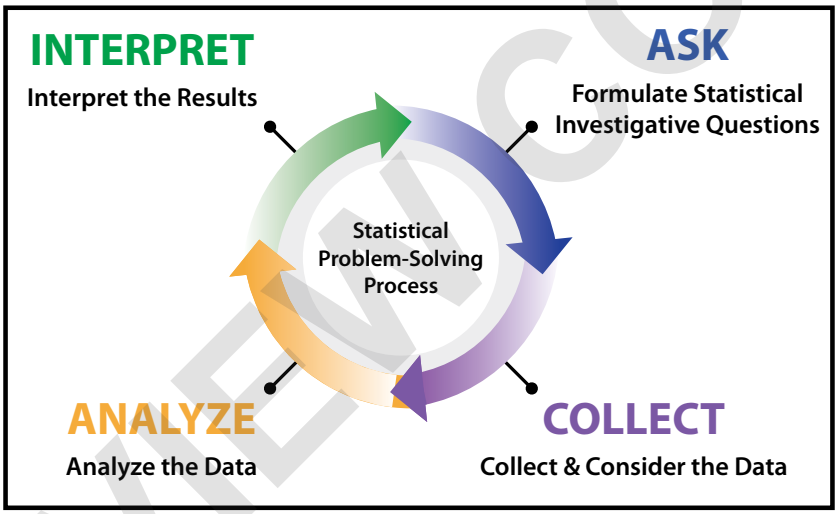
Source

National Council of Teachers of Mathematics. "Problem Solving." Accessed January 11, 2023. https://www.walch.com/rr/09048.

Statistical Reasoning Implementation Guide

Introduction

Statistical reasoning allows students to make sense of ideas, information, and the changing world through questioning and exploration. It provides the foundation necessary for students to fully understand the concept. Statistical reasoning is a continuous cycle consisting of students asking questions, collecting, analyzing, and interpreting data. In order to guide students in this sense-making process, Walch resources support this four-step statistical problem-solving strategy to help students develop their understanding in statistical reasoning.



Source: Georgia Department of Education

Here is a brief description of how this framework can be applied in the classroom.

Formulate Statistical Investigative Questions

Students will form and ask investigative questions that allow for various answers. These questions will clarify the problem and lead to questions that can be answered with the data. Best practices and teacher prompts that can foster this framework include:

- Using a student-centered approach.
- Having students prepare ahead of time with an assigned reading to familiarize themselves with words and techniques.

Ask students:

- What do you think?
- What do you notice? What do you wonder?
- What criteria need to be met in order for the question to be statistical?
- How did you determine your question?
- What changes would you make to the question?

Collect and Consider the Data

Students will collect data by creating a plan in order to collect real and relevant data. Making sure the data is relevant to students will increase engagement and lead to more math talk and discussion. Strategies include:

- Refraining from presenting students with procedures.
- Allowing students to use real data sets and to generate their own data.
- Encouraging students to discuss the questions and possible ideas.

Ask students:

- What do you notice about the data?
- In what other ways can the data be collected?
- What are some other methods you can use to collect the data? How do these different methods affect your data collection?
- How can you represent your data? Can you represent it with a visual?
- Are there representations better fit for particular findings? Justify your answer.

Analyze the Data

Students will analyze the data by selecting methods that are appropriate. Exploration of various methods will allow for students to make connections and draw conclusions based on the data. This will deepen their understanding of statistical reasoning. Strategies include:

- Allowing students to use technology tools to explore and analyze their findings.
- Refraining from giving students all the information. Allow students to form their own analysis of the data.
- Creating a classroom environment in which student ideas are valued.

Ask students:

- What conclusions can you draw from the data?
- Do you notice any trends in the data? How can you tell?
- What is the relationship between the data points?
- What evidence may help you distinguish between results?
- Do you agree or disagree? Justify your thinking.
- How can we test that conclusion?
- What do you do about outliers in your data? What do they tell you?
- If extreme values are removed, what happens to the data representation?
- Compare your data with a classmate's. What do you notice?

Interpret the Results

Students will interpret and discuss the results by relating all findings to the original question. Students will discuss these findings and justify their reasoning. Best practices and teacher prompts include:

- Encouraging discourse. Encourage students to present their ideas, answer classmates' questions, and support their responses.
- Focusing on key ideas instead of procedures and calculated answers.
- Making sure students have answered their "I wonder" questions.

Ask students:

- What do the results tell you about the original question?
- Have your "I wonder" questions been answered?
- What conclusions can you make from the results?
- Compare your interpretations to those of your classmates. What connections can you make?
- What do your interpretations represent in a real-world context?

Source

Garfield, Joan and Ben-Zvi, Dani. "Helping Students Develop Statistical Reasoning: Implementing a Statistical Reasoning Learning Environment." Accessed Jan. 11, 2023. https://www.walch.com/rr/09049

z-scores Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

z-scores Table

z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
–3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
–3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
–3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
–3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
–3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
–2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
–2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
–2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
–2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
–2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
–2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
–2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
–2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
–2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
–2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
–1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
–1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
–1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
–1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
–1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
–1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
–1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
–1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
–1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
–1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
–0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
–0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
–0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
–0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
–0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
–0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
–0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
–0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
–0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
–0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

t-distribution Table

One-tailed	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
Two-tailed	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
Confidence level	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%

Suggested Pacing Guide

Overview

The lessons in this program are designed to be covered over one, two, or three days, depending on the depth and complexity of the content. The pacing guide that follows provides *suggested* pacing, but this should be viewed as a flexible guideline only. If students need an extra day to solidify understanding, a one-day lesson may be spread over two days, or a two-day lesson may be spread over three days. Conversely, if students grasp a concept more quickly than suggested by the pacing guide, it is fine to shorten the duration of the lesson and move on to the next.

Each lesson features several components. The estimated time for each component is given in parentheses. Components include: a Warm-Up, Instruction (including an Introduction, Key Concepts, Scaffolded Practice, and Guided Practice), a Problem-Based Task, and Practice (Sets A and B).

Please note: PowerPoint versions of each lesson's Warm-Up and Instruction are provided. These PowerPoints include the Warm-Up and Warm-Up Debrief, the Introduction, Key Concepts, and Guided Practice examples, some of which have links to corresponding GeoGebra applets. Estimated times reflect presentation of these PowerPoint versions and applets.

Guided Practice, Scaffolded Practice, and Practice

For each Guided Practice in the Teacher Resource (TRB), corresponding worksheets are provided in the Student Workbook (SWB). Scaffolded Practice worksheets are also included to introduce the skills for each lesson. Each lesson also includes two sets of Practice problems. Guided Practice, Scaffolded Practice, and Practice problems may be assigned from the SWB worksheets for in-class work or homework assignments.

Please note: When a lesson is spread across more than one class period, the Guided Practice examples will also be spread across the same number of class periods. Assign Practice problems that correspond to the Guided Practice example(s) you select for each day, to ensure appropriate prior instruction.

Suggested Progression of One-, Two-, and Three-Day Lessons

One Day

- Warm-Up PowerPoint and Debrief (10 minutes)
- Introduction and Key Concepts (10 minutes)
- One or two Guided Practice examples, including PowerPoint applet presentation for selected examples as appropriate (15 minutes)
- Problem-Based Task and discussion (15 minutes)
- Homework: Practice problems from the SWB

Two Days

Day 1

- Warm-Up PowerPoint and Debrief (10 minutes)
- Introduction and Key Concepts (15 minutes)
- Selected Scaffolded Practice problems and two or three Guided Practice examples, including PowerPoint applet presentation for selected examples as appropriate (15 minutes)
- In-class practice: selected Scaffolded Practice and Practice problems, or a conceptual task or activity
- Homework: selected Practice problems from the SWB

Day 2

- Review Practice problems (10 minutes)
- Selected Scaffolded Practice problems and one or two remaining Guided Practice examples, including PowerPoint applet presentation for selected examples as appropriate (10 minutes)
- Problem-Based Task and discussion (30 minutes)
- In-class practice: selected Scaffolded Practice and Practice problems, or a conceptual task or activity
- Homework: the remainder of the Practice problems

Three Days

Day 1

- Warm-Up PowerPoint and Debrief (10 minutes)
- Introduction and Key Concepts (25 minutes)
- Selected Scaffolded Practice problems and one or two Guided Practice examples, including PowerPoint applet presentation for selected examples as appropriate (15 minutes)
- Homework: selected Practice problems

Day 2

- Review Practice problems (15 minutes)
- Two or three remaining Guided Practice examples, including PowerPoint applet presentation for selected examples as appropriate (15 minutes)
- Problem-Based Task and discussion (20 minutes)
- Homework: the remaining Practice problems

Day 3

- Review Practice problems (15 minutes)
- In-class practice: selected Scaffolded Practice and Practice problems, or a conceptual task or activity
- Homework: the remaining Practice problems

Assessments, Conceptual Tasks and Activities, and Station Activities

Each topic includes a Pre-Assessment and a Progress Assessment, and each unit concludes with a Unit Assessment. Units also feature sets of Station Activities, and selected topics include Conceptual Task along with links to interactive web-based Conceptual Activities, to complement instruction.

Pre-Assessments are short, multiple-choice assessments with five problems, designed to evaluate prior knowledge of the upcoming topic. These are brief and should not factor into a student's grade. Pre-Assessments are often given on the same day as the preceding topic's Progress Assessment, to prepare for the upcoming topic.

Progress Assessments include 10 multiple-choice problems and one extended-response problem. Progress Assessments may not take an entire class period. The additional time may be used to review before the assessment, work through the Conceptual Activities, or to begin the next topic after the assessment.

Unit Assessments include 12 multiple-choice problems and three extended-response problems, and generally require a full class period.

Conceptual Tasks provide opportunities for students to deepen their understanding and develop their conceptual knowledge of math concepts.

Conceptual Activities are digital math resources that allow students to explore mathematical ideas with engaging, real-world problems and interactive games.

Station Activities generally require a full class period for the students to rotate through each station and then to engage in a class discussion at the end.

Unit 1: Inferences and Conclusions from Data (42 Days)

Unit Overview: Students learn about using the normal curve, as well as about populations versus random samples and random sampling. Then, they learn about strategies for collecting data, including surveys, experiments, and observational studies. Students estimate sample proportions and sample means and develop tools for comparing treatments and reading reports. Finally, they look at making and analyzing decisions with data.

Topic A: Using the Normal Curve

Students explore the concept of probability distribution, and especially the normal distribution and uniform distribution. They are also introduced to standard deviation. Next, students look more closely at normal distributions and learn how to find the *z*-score. Finally, students learn three methods for determining whether a given situation follows a normal distribution.

Day	Area of study/content	Standard(s)
Day 1 Day 2	Topic A Pre-Assessment 1.1: Normal Distributions and the 68–95–99.7 Rule	S–ID.A.4★
Day 3 Day 4	1.2: Standard Normal Calculations	S–ID.A.4★
Day 5	1.3: Assessing Normality	S–ID.A.4★
Day 6	Station Activities Set 1: *z*-scores	S–ID.A.4★
Day 7	Topic A Progress Assessment/Conceptual Task Topic B Pre-Assessment	

Topic B: Populations Versus Random Samples and Random Sampling

This topic begins with an in-depth look at the differences between populations and samples, as well as how to determine the reliability and validity of a sample. Then, students explore simple random sampling and sampling bias. The topic ends with a look at several other methods of sampling, such as cluster sampling, systematic sampling, and stratified sampling.

Day	Area of study/content	Standard(s)
Day 8 Day 9	1.4: Differences Between Populations and Samples	S–IC.A.1★
Day 10 Day 11	1.5: Simple Random Sampling	S–IC.A.2★
Day 12 Day 13	1.6: Other Methods of Random Sampling	S–IC.A.2★
Day 14	Topic B Progress Assessment Topic C Pre-Assessment	

Topic C: Surveys, Experiments, and Observational Studies

This topic focuses on various ways of collecting data. First, students look at the diverse ways that data can be collected. Then, they investigate more closely the steps needed to design a study so that the outcomes are reliable and relevant to the subject being studied.

Day	Area of study/content	Standard(s)
Day 15 Day 16	1.7: Identifying Surveys, Experiments, and Observational Studies	S–IC.B.3★
Day 17 Day 18	1.8: Designing Surveys, Experiments, and Observational Studies	S–IC.B.3★
Day 19	Topic C Progress Assessment/Conceptual Task Topic D Pre-Assessment	

Topic D: Estimating Sample Proportions and Sample Means

Students learn about sample proportions and the standard error for the proportion. They explore binomial experiments and binomial distributions, as well as the addition rule for mutually exclusive events. Then, they study sample means and their relationship to population means, and take a look at the margin of error and the confidence level. Finally, students investigate how to calculate the probability that a given value falls within a specified interval.

Day	Area of study/content	Standard(s)
Day 20 Day 21	1.9: Estimating Sample Proportions	S–IC.B.4★
Day 22 Day 23	1.10: The Binomial Distribution	S–IC.B.4★
Day 24 Day 25	1.11: Estimating Sample Means	S–IC.B.4★
Day 26 Day 27	1.12: Estimating with Confidence	S–IC.B.4★
Day 28	Station Activities Set 2: Distributions and Estimating with Confidence	S–ID.A.4★ S–IC.B.4★
Day 29	Topic D Progress Assessment/Conceptual Task Topic E Pre-Assessment	

Topic E: Comparing Treatments and Reading Reports

Students learn what a treatment is and how to evaluate the confidence level of a given treatment. They explore how to use a *t*-test to establish the statistical significance of a set of data. Next, they investigate the concept of simulations and how to design an accurate simulation. Finally, students learn how to question the validity of the conclusions and assumptions made based on reported data.

Day	Area of study/content	Standard(s)
Day 30 Day 31	1.13: Evaluating Treatments	S–IC.B.5★
Day 32 Day 33	1.14: Designing and Simulating Treatments	S–IC.B.5★
Day 34 Day 35	1.15: Reading Reports	S–IC.B.6★
Day 36	Topic E Progress Assessment Topic F Pre-Assessment	

Topic F: Making and Analyzing Decisions

Students explore ways of making decisions based on the probability of certain events occurring, and they learn how to calculate the expected value. They then focus on analyzing the accuracy and fairness of situations and outcomes.

Day	Area of study/content	Standard(s)
Day 37 Day 38	1.16: Making Decisions	S–MD.B.6★ (+)
Day 39 Day 40	1.17: Analyzing Decisions	S–MD.B.7★ (+)
Day 41	Topic F Progress Assessment	
Day 42	**Unit Assessment**	

Unit 2: Polynomial Relationships (34 Days)

Unit Overview: Students begin by exploring polynomial structures and operating with polynomials. Then, they learn how to prove identities, graph polynomial functions, and solve systems of equations with polynomials. Finally, students learn about geometric series.

Topic A: Polynomial Structures and Operating with Polynomials

Students review the vocabulary used when discussing polynomials. They practice adding, subtracting, and multiplying polynomials and simplifying the results.

Day	Area of study/content	Standard(s)
Day 1	Topic A Pre-Assessment 2.1: Structures of Expressions	A–SSE.A.1a★
Day 2	2.2: Adding and Subtracting Polynomials	A–APR.A.1
Day 3 Day 4	2.3: Multiplying Polynomials	A–APR.A.1
Day 5	Topic A Progress Assessment/Conceptual Task Topic B Pre-Assessment	

Topic B: Proving Identities

Students begin this topic by exploring common polynomial identities, such as the difference of two squares and the square of sums. Then they expand their investigation to include complex numbers in the polynomial identities. They end the topic with an exploration of the Binomial Theorem and a look at its usefulness when finding binomial expansions.

Day	Area of study/content	Standard(s)
Day 6 Day 7	2.4: Polynomial Identities	A–SSE.A.1b★ A–SSE.A.2 A–APR.C.4
Day 8 Day 9	2.5: Complex Polynomial Identities	N–CN.C.8 (+) A–SSE.A.1b★ A–SSE.A.2 A–APR.C.4
Day 10 Day 11	2.6: The Binomial Theorem	A–SSE.A.1a★ A–SSE.A.1b★ A–SSE.A.2 A–APR.C.4 A–APR.C.5 (+)
Day 12	Topic B Progress Assessment Topic C Pre-Assessment	

Topic C: Graphing Polynomial Functions

Students begin this topic by analyzing the end behavior and number of turns of the graphs of polynomial functions. Then they learn synthetic division of polynomials, and apply this to the Remainder Theorem. Their study of polynomials continues with an exploration of how to find the zeros of a polynomial function. Lastly, students learn the Rational Root Theorem as a means of locating the zeros of polynomials that are difficult to factor.

Day	Area of study/content	Standard(s)
Day 13 Day 14	2.7: Describing End Behavior and Turns	F–IF.C.7c★
Day 15 Day 16	2.8: The Remainder Theorem	A–APR.B.2
Day 17 Day 18	2.9: Finding Zeros	A–APR.B.3 N–CN.C.9 (+) F–IF.C.7c★
Day 19 Day 20	2.10: The Rational Root Theorem	A–APR.B.3
Day 21	Station Activities Set 1: Polynomial Functions	N–CN.C.9 (+) A–SSE.A.2 A–APR.B.2 A–APR.B.3 F–IF.A.2 F–IF.C.7c★
Day 22	Topic C Progress Assessment/Conceptual Task Topic D Pre-Assessment	

Topic D: Solving Systems of Equations with Polynomials

This brief topic focuses on how to find the solution(s) to a system of equations graphically when at least one of the equations is a polynomial. The systems of equations in this topic also include absolute value equations and exponential equations.

Day	Area of study/content	Standard(s)
Day 23 Day 24	2.11: Solving Systems of Equations Graphically	A–REI.D.11★
Day 25	Topic D Progress Assessment Topic E Pre-Assessment	

Topic E: Geometric Series

This topic provides an in-depth look at geometric series, beginning with the associated vocabulary. Students learn the difference between recursive and explicit formulas. Next, students explore how to find the sum of a finite geometric series, followed by how to find the sum of an infinite geometric series.

Day	Area of study/content	Standard(s)
Day 26 Day 27	2.12: Geometric Sequences	A–SSE.B.4★
Day 28 Day 29	2.13: Sum of a Finite Geometric Series	A–SSE.B.4★
Day 30 Day 31	2.14: Sum of an Infinite Geometric Series	A–SSE.B.4★
Day 32	Station Activities Set 2: Sequences and Series	A–SSE.B.4★
Day 33	Topic E Progress Assessment	
Day 34	**Unit Assessment**	

Unit 3: Rational and Radical Relationships (18 Days)

Unit Overview: Students work with operations involving rational expressions. Then, they learn to solve rational and radical equations.

Topic A: Operating with Rational Expressions

Students learn the definition of a rational expression and how to simplify rational expressions. Then, they focus on adding and subtracting rational expressions, followed by multiplying and dividing them.

Day	Area of study/content	Standard(s)
Day 1	Topic A Pre-Assessment 3.1: Structures of Rational Expressions	A–SSE.A.1a★ A–SSE.A.1b★ A–SSE.A.2
Day 2 Day 3	3.2: Adding and Subtracting Rational Expressions	A–APR.D.7 (+) A–SSE.A.2
Day 4 Day 5	3.3: Multiplying Rational Expressions	A–APR.D.7 (+) A–SSE.A.2
Day 6 Day 7	3.4: Dividing Rational Expressions	A–APR.D.6 A–APR.D.7 (+) A–SSE.A.2
Day 8	Topic A Progress Assessment/Conceptual Task Topic B Pre-Assessment	

Topic B: Solving Rational and Radical Equations

Students first learn how to solve rational equations, and then how to solve radical equations. The topic ends with students solving systems of equations algebraically, including polynomial, radical, and rational equations.

Day	Area of study/content	Standard(s)
Day 9 Day 10	3.5: Solving Rational Equations	A–REI.A.2
Day 11	Station Activities Set 1: Rational Expressions and Equations	A–SSE.A.2 A–APR.D.6 A–APR.D.7 (+) A–REI.A.2
Day 12 Day 13	3.6: Solving Radical Equations	A–REI.A.2
Day 14 Day 15	3.7: Solving Systems of Equations	A–REI.D.11★
Day 16	Station Activities Set 2: Solving Systems of Equations	A–REI.D.11★
Day 17	Topic B Progress Assessment/Conceptual Task	
Day 18	**Unit Assessment**	

Unit 4: Trigonometry of General Triangles and Trigonometric Functions (24 Days)

Unit Overview: Students begin the unit with a study of radians and the unit circle. Then, they explore the trigonometry of general angles, including the Law of Sines and the Law of Cosines. Finally, students move on to modeling periodic phenomena with the graphs of the sine and cosine functions.

Topic A: Radians and the Unit Circle

Students explore the meaning of radian measure, as well as the relationship between radians, radius, and arc length measurements on a circle. They also learn to convert from radians to degrees and vice versa. Then, students learn about the unit circle, how to calculate distances using the three basic trigonometric functions, and how to find reference angles. This is followed by the exploration of the special angles on the unit circle. Finally, students learn how to evaluate trigonometric functions with the help of the unit circle.

Day	Area of study/content	Standard(s)
Day 1	Topic A Pre-Assessment 4.1: Radians	F–TF.A.1 F–TF.A.2
Day 2 Day 3	4.2: The Unit Circle	F–TF.A.2
Day 4 Day 5	4.3: Special Angles in the Unit Circle	F–TF.A.2
Day 6 Day 7	4.4: Evaluating Trigonometric Functions	F–TF.A.2
Day 8	Station Activities Set 1: Trigonometric Functions	F–TF.A.2
Day 9	Topic A Progress Assessment/Conceptual Task Topic B Pre-Assessment	

Topic B: Trigonometry of General Angles

Students discover how to find the measures of missing side lengths and angle measurements if the given triangle is *not* a right triangle, by using the Law of Sines and the Law of Cosines. After learning these two useful formulas, students use the Law of Sines and the Law of Cosines to solve problems.

Day	Area of study/content	Standard(s)
Day 10 Day 11	4.5: Proving the Law of Sines	G–SRT.D.9 (+) G–SRT.D.10 (+)
Day 12 Day 13	4.6: Proving the Law of Cosines	G–SRT.D.10 (+)
Day 14 Day 15	4.7: Applying the Laws of Sines and Cosines	G–SRT.D.11 (+)
Day 16	Station Activities Set 2: The Laws of Sines and Cosines	G–SRT.D.9 (+) G–SRT.D.10 (+) G–SRT.D.11 (+)
Day 17	Topic B Progress Assessment/Conceptual Task Topic C Pre-Assessment	

Topic C: Graphs of Trigonometric Functions

Students begin studying the periodic nature of trigonometric functions by looking at the graphs of the sine, cosine, and tangent functions. This includes analyzing the period, frequency, amplitude, and midline of these functions.

Day	Area of study/content	Standard(s)
Day 18 Day 19	4.8: Periodic Phenomena and Amplitude, Frequency, and Midline	F–TF.B.5★
Day 20 Day 21 Day 22	4.9: Using Trigonometric Functions to Model Periodic Phenomena	F–TF.B.5★
Day 23	Topic C Progress Assessment	
Day 24	**Unit Assessment**	

Unit 5: Mathematical Modeling of Inverse, Logarithmic, and Trigonometric Functions (23 Days)

Unit Overview: Students begin by learning about the inverses of quadratics and other functions. This leads to learning about graphing and interpreting logarithmic functions and models. Students continue exploring and applying the graphs of the sine and cosine functions learned previously.

Topic A: Inverses of Functions

Students learn how to find the inverse of a quadratic function, and then how to find the inverse of other functions, such as rational and radical functions.

Day	Area of study/content	Standard(s)
Day 1 Day 2	Topic A Pre-Assessment 5.1: Determining Inverses of Quadratic Functions	F–BF.B.4a
Day 3 Day 4	5.2: Determining Inverses of Other Functions	F–BF.B.4a
Day 5	Station Activities Set 1: Inverse Functions	F–BF.B.4a
Day 6	Topic A Progress Assessment Topic B Pre-Assessment	

Topic B: Modeling Logarithmic Functions

Students are introduced to logarithmic functions, and they explore logarithmic functions as inverses of exponential functions. Next, students investigate common logarithms and natural logarithms. This is followed by a look at the graphs of logarithmic functions. Finally, students learn how analyze and interpret situations that can be modeled with logarithmic functions.

Day	Area of study/content	Standard(s)
Day 7 Day 8	5.3: Logarithmic Functions as Inverses	F–BF.B.4a F–LE.A.4★
Day 9 Day 10	5.4: Common Logarithms	F–IF.C.8 F–LE.A.4★
Day 11 Day 12	5.5: Natural Logarithms	F–IF.C.8 F–LE.A.4★
Day 13 Day 14	5.6: Graphing Logarithmic Functions	F–IF.C.7e★
Day 15 Day 16	5.7: Interpreting Logarithmic Models	F–IF.B.4★ F–IF.B.5★ F–IF.B.6★
Day 17	Topic B Progress Assessment/Conceptual Task Topic C Pre-Assessment	

Topic C: Modeling Trigonometric Functions

Students return to the analysis of the sine and cosine functions they studied previously, this time with a closer investigation into the translation of trigonometric functions.

Day	Area of study/content	Standard(s)
Day 18 Day 19	5.8: Graphing the Sine Function	F–IF.C.7e★
Day 20 Day 21	5.9: Graphing the Cosine Function	F–IF.C.7e★
Day 22	Topic C Progress Assessment/Conceptual Task	
Day 23	**Unit Assessment**	

Unit 6: Mathematical Modeling and Choosing a Model (38 Days)

Unit Overview: Students revisit the process of creating equations in one variable and explore creating constraints and rearranging formulas. They then learn about transforming models and combining functions. Students review various kinds of functions, including linear, exponential, quadratic, piecewise, step, absolute value, square root, and cube root functions, all with an eye to choosing a model for a real-world situation. Finally, students consider geometric models, including two-dimensional cross sections of three-dimensional objects.

Topic A: Creating Equations

Students begin by reviewing vocabulary associated with equations and inequalities, and they practice writing equations to represent various scenarios. Next, they take a closer look at the constraints that exist for some situations and some equations, and learn how to find restricted domains. Finally, students apply their equation-solving skills to formulas, when they solve formulas for a given variable.

Day	Area of study/content	Standard(s)
Day 1 Day 2	Topic A Pre-Assessment 6.1: Creating Equations in One Variable	A–CED.A.1★
Day 3 Day 4	6.2: Representing and Interpreting Constraints	A–CED.A.3★
Day 5 Day 6	6.3: Rearranging Formulas	A–CED.A.4★
Day 7	Topic A Progress Assessment Topic B Pre-Assessment	

Topic B: Transforming a Model and Combining Functions

Students take an in-depth look at transformations of functions. Next, they analyze functions both graphically and algebraically to determine whether the functions are even, odd, or neither. Finally, students learn how to combine two functions by adding, subtracting, multiplying, and dividing them, as well as by substituting one function into the other (composition of functions).

Day	Area of study/content	Standard(s)
Day 8 Day 9	6.4: Transformations of Parent Graphs	F–BF.B.3
Day 10 Day 11	6.5: Recognizing Odd and Even Functions	F–BF.B.3
Day 12 Day 13	6.6: Combining Functions	F–BF.A.1b★
Day 14	Topic B Progress Assessment/Conceptual Task Topic C Pre-Assessment	

Topic C: Comparing Properties Within and Between Functions

Students compare the features and the limitations of a real-world mathematical situation and the "pure" mathematical model that can represent it, with a goal of finding the most accurate mathematical model possible. Next, students learn to calculate the average rate of change of nonlinear functions. The topic ends with an analysis of how to compare two functions that represent a real-world scenario.

Day	Area of study/content	Standard(s)
Day 15 Day 16	6.7: Reading and Identifying Key Features of Real-World Situation Graphs	F–IF.B.4★ F–IF.B.5★ F–IF.B.6★
Day 17 Day 18	6.8: Calculating Average Rates of Change	F–IF.B.6★
Day 19 Day 20	6.9: Comparing Functions	F–IF.B.6★ F–IF.C.9
Day 21	Topic C Progress Assessment/Conceptual Task Topic D Pre-Assessment	

Topic D: Choosing a Model

Students learn how to choose the best model (linear, exponential, or quadratic) for a given situation by analyzing the given data. Then, they analyze more models, including piecewise functions, step functions, and absolute value functions. Students discover which of these functions apply to different types of situations. Finally, they look at situations that are best modeled with square root and cube root functions.

Day	Area of study/content	Standard(s)
Day 22 Day 23	6.10: Linear, Exponential, and Quadratic Functions	A–CED.A.2★ F–IF.B.4★ F–IF.B.5★ F–BF.B.3
Day 24 Day 25	6.11: Piecewise, Step, and Absolute Value Functions	F–IF.B.4★ F–IF.B.5★ F–IF.C.7b★ F–BF.B.3
Day 26 Day 27	6.12: Square Root and Cube Root Functions	F–IF.B.4★ F–IF.B.5★ F–IF.C.7b★ F–BF.B.3
Day 28	Station Activities Set 1: Choosing a Model	F–IF.B.4★ F–IF.B.5★ F–IF.C.7b★ F–BF.B.3
Day 29	Topic D Progress Assessment/Conceptual Task Topic E Pre-Assessment	

Topic E: Geometric Modeling

Students find ways to model geometric situations, first by looking at cross sections of three-dimensional objects, and how these cross-sectional areas can be used to model a given problem. The modeling continues as students explore the concept of density, and how the density of a quantity can be compared to a basic unit of area or volume to solve real-world problems. Finally, students explore the role of mathematics in the design of products and services.

Day	Area of study/content	Standard(s)
Day 30 Day 31	6.13: Two-Dimensional Cross Sections of Three-Dimensional Objects	G–GMD.B.4 G–MG.A.1★
Day 32 Day 33	6.14: Density	G–MG.A.2★
Day 34 Day 35	6.15: Design	G–MG.A.3★
Day 36	Station Activities Set 2: Geometric Modeling	G–GMD.B.4 G–MG.A.1★ G–MG.A.2★ G–MG.A.3★
Day 37	Topic E Progress Assessment	
Day 38	**Unit Assessment**	

End-of-Course Assessment	
Day 39	This Learnosity-based interactive assessment is available through Canvas or Walch's Curriculum Engine. For more information, contact us at success@bwwalch.com.

Graphic Organizers

Overview

Graphic organizers can be a versatile tool in your classroom. Organizers offer an easy, straightforward way to visually present a wide range of material. Research suggests that graphic organizers support learning in the classroom for all levels of learners. Gifted students, students on grade level, and students with learning difficulties all benefit from the use of graphic organizers. They reduce the cognitive demand on students by helping them access information quickly and easily. Using graphic organizers, learners can understand content more clearly and can take concise notes. Ultimately, learners find it easier to retain and apply what they've learned.

Graphic organizers help foster higher-level thinking skills. They help students identify main ideas and details in their reading. They make it easier for students to see patterns such as cause and effect, comparing and contrasting, and chronological order. Organizers also help students master critical-thinking skills by asking them to recall, evaluate, synthesize, analyze, and apply what they've learned. Research suggests that graphic organizers contribute to better test scores because they help students understand relationships between key ideas, and enable them to be more focused as they study.

Types of Graphic Organizers

There are four main purposes for using graphic organizers in mathematics and a variety of tools within each category:

Purpose 1: Organizing, Categorizing, and Classifying	**Purpose 2: Problem Solving**	**Purpose 3: Understanding Mathematical Information**	**Purpose 4: Communicating Mathematical Information**
Tables Flowcharts Webs Venn Diagrams	Number Lines Geometric Drawings Factor Trees Attribute Tables Cause and Effect Maps Coordinate Plane Probability Trees	Frayer Model Semantic Map/ Concept Map Compare-and-Contrast Diagram	Line Graphs Bar Charts

Tables

A table is simply a grid with rows and columns. Tables are useful because information stored in a table is easy to find—much easier than the same information embedded in text.

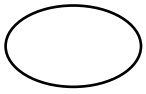
Usually, a table has a row (horizontal) for each item being listed. The columns (vertical) provide places for details about the listed items—the things they have in common. The places where the rows and columns meet are called cells. In each cell, we write information that fits both the topic of the row (the thing being listed) and the topic of the column (the aspect being examined). To create a table, we make rows and columns to fit the number of items and attributes.

Flowcharts

Flowcharts are graphic organizers that show the steps in a process. Flowcharts can be very simple—just a series of boxes with one step in each box. However, there is also a more formal type of flowchart. These flowcharts use special symbols to show different things, such as starting and stopping points, or points where decisions must be made. These symbols make flowcharts especially useful for showing complicated processes.

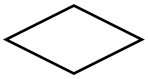
Each step in a flowchart is written in a box. The boxes are connected by arrows to show the sequence of steps. The boxes aren't all rectangular; different shapes are used to indicate different actions. The shapes and symbols are a kind of visual shorthand. Whenever a certain symbol is used, it always has the same meaning.

- Circles and ovals show starting and stopping points. They often contain the words start or stop. The "start" circle or oval has no arrows in and one arrow out. The "stop" circle or oval has one arrow in and no arrows out.

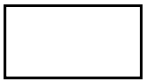


- Arrows show the direction in which the process is moving.

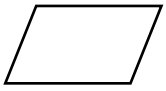
- Diamonds show points where a decision must be made or a question must be answered. The question can usually be answered either "yes" or "no."



- Rectangles and squares show steps where a process or an operation takes place.



- Parallelograms show input or output, such as writing or printing a result or solution.



Webs

Webs are graphic organizers that help take notes, identify important ideas, and show relationships between and among pieces of information. In a web, the main idea is written in the center circle. Details are recorded in other circles with lines to connect related topics. Circles or lines can be added or deleted as necessary.

Number Lines

In its simplest form, a number line is any line that uses equally spaced marks to show numbers. Number lines are used to visualize equalities and inequalities, positive and negative numbers, and measurements of all kinds. They can "map" math problems, especially ones that involve negative numbers or distances.

Geometric Drawings

A geometric drawing is a representation on paper (or some other surface) of a geometric figure. The geometric drawings we make can never be as perfect as the geometric figures they represent, but as long as they are reasonably accurate, they can help us visualize the figures. In fact, it's often impossible to solve a geometry problem without making a drawing.

Factor Trees

There are several ways to find factors. One that helps to visually keep track of all the factors is called a factor tree. This is a diagram with a tree-like shape. It uses "branches" to show the factors of a number.

All whole numbers other than 1 can be written as the product of factors. A prime number is a number that has only two factors, itself and 1. An example of a prime number is 13. Its only factors are 13 and 1. A composite number is a number that has more than two factors. An example of a composite number is 6. Its factors include 6, 3, 2, and 1. Prime factors are factors that are also prime numbers. The greatest common factor (GCF) of two numbers is the largest number that is a factor of both numbers.

Coordinate Plane

This is the plane determined by a horizontal number line, called the x-axis, and a vertical number line, called the y-axis, intersecting at a point called the origin. A coordinate plane can be used to illustrate locations and relationships using ordered pairs of numbers.

Venn Diagrams

A set is a list of objects in no particular order. Items in a set can be numbers, but they can also be letters or words. Venn diagrams are a visual way of showing how sets of things can include one another, overlap, or be distinct from one another.

Venn diagrams are often used to compare and contrast things. But they are also a useful tool to sort and classify information. You can use Venn diagrams to take notes on material that shows relationships between things or ideas. You can also use them to solve certain types of word problems. When a word problem names two or three different categories and asks you how many items fall into each category, a Venn diagram can be a useful problem-solving tool.

A Venn diagram begins with a rectangle representing the universal set. Then each set in the problem is represented by a circle. Circles can be separate, overlapping, or one within another. When two circles overlap, it means that the two sets intersect. Some members of one set are also members of the other set.

Venn Diagrams AND **Compare-and-Contrast Diagrams**

The Venn diagram is an organizing device for planning comparisons and contrasts. A completed Venn diagram helps students categorize and organize similarities and differences, and provides a blueprint for a comparison-and-contrast exercise. The compare-and-contrast diagram provides a structure to identify or list similarities and differences between two objects.

Attribute Tables

To solve logic problems, you need a way to keep track of the subjects and which attributes they have or don't have. An attribute table can help. This is a table with a row for each subject in the problem, and a column for each attribute. The rows and columns meet to form cells. Because the attributes in logic problems are usually exclusive, you can use Xs or check marks (✔) to show which attribute belongs to which subject.

Cause and Effect Maps

Cause and effect maps help you work through information to make sense of it. Write each cause in the oval. Write all its effects in the boxes. Add or delete ovals and boxes as needed.

Frayer Model

The Frayer Model is a word categorization activity that helps learners to develop their understanding of concepts. Using this model, students provide a definition, list characteristics, and provide examples and non-examples of the concept.

Semantic Map

A semantic word map allows students to conceptually explore their knowledge of a new term or concept by mapping it with other related words, concepts, or phrases that are similar in meaning. Semantic maps portray the schematic relations that compose a concept. It assumes that there are multiple relations between a concept and the knowledge that is associated with the concept.

Line Graphs

Line graphs are often used to show how things change over time. They clearly show trends in data and can let you make predictions about future trends, too. Line graphs use two number lines, one horizontal and one vertical. The horizontal number line is called the *x*-axis. The vertical line is called the *y*-axis. The *x*-axis often shows the passage of time. The *y*-axis often shows a quantity of some kind, such as height, speed, cost, and so forth.

Bar Charts

Bar charts are useful when you want to compare things or to show how one thing changes over time. They are a good way to show overall trends. Bar charts use horizontal or vertical bars to represent data. Longer bars represent higher values. Different colors can be used to show different variables. When you look at a bar chart, it's easy to see which element has the greatest value—the one with the longest bar.

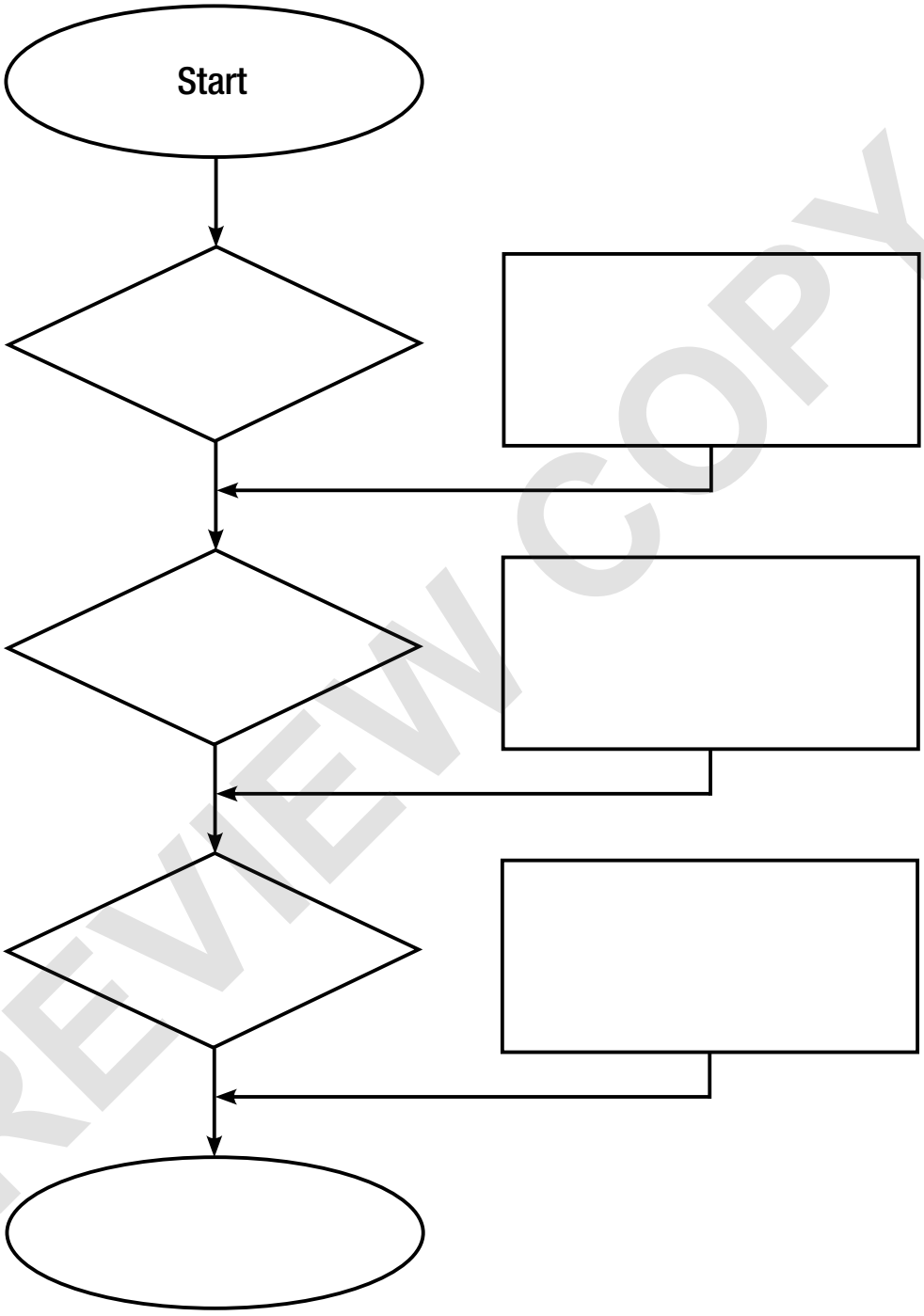
Bar charts have an *x*-axis (horizontal) and a *y*-axis (vertical). If the graph is being used to show how something changes over time, the *x*-axis has numbers for the time period. If the graph is being used to compare things, the *x*-axis shows which things are being compared. The *y*-axis has numbers that show how much of each thing there is.

Probability Trees

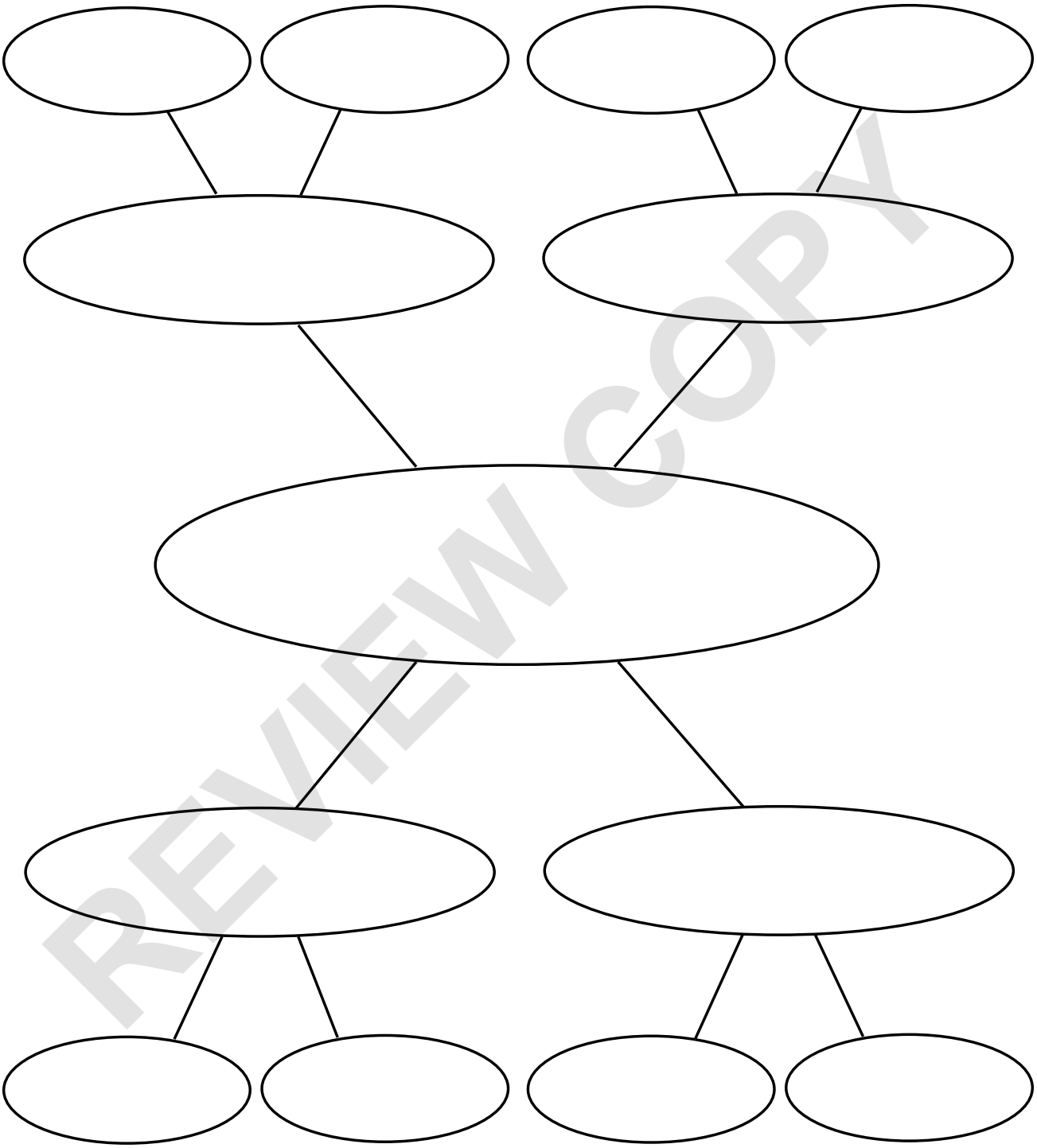
When we have probability problems with many possible outcomes, or events that depend on one another, probability trees can help. Probability trees show all the possible outcomes of an event. Whenever a problem calls for figuring out how many possible outcomes there are, and the probability that any one of them will happen, a probability tree can be useful.

Table

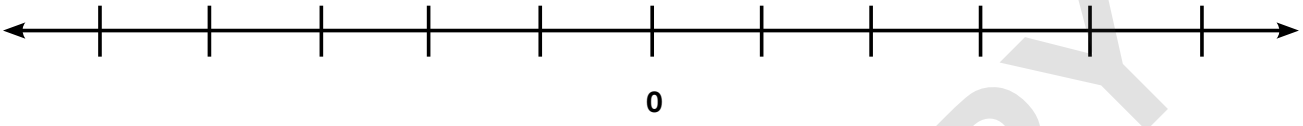
Flowchart



Web



Number Line



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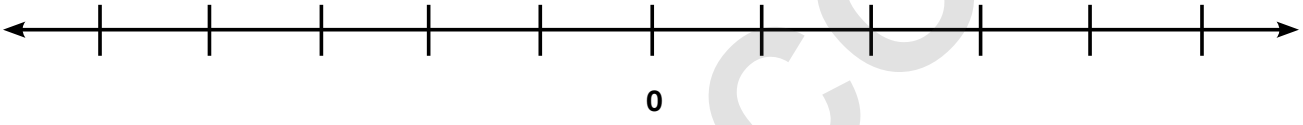

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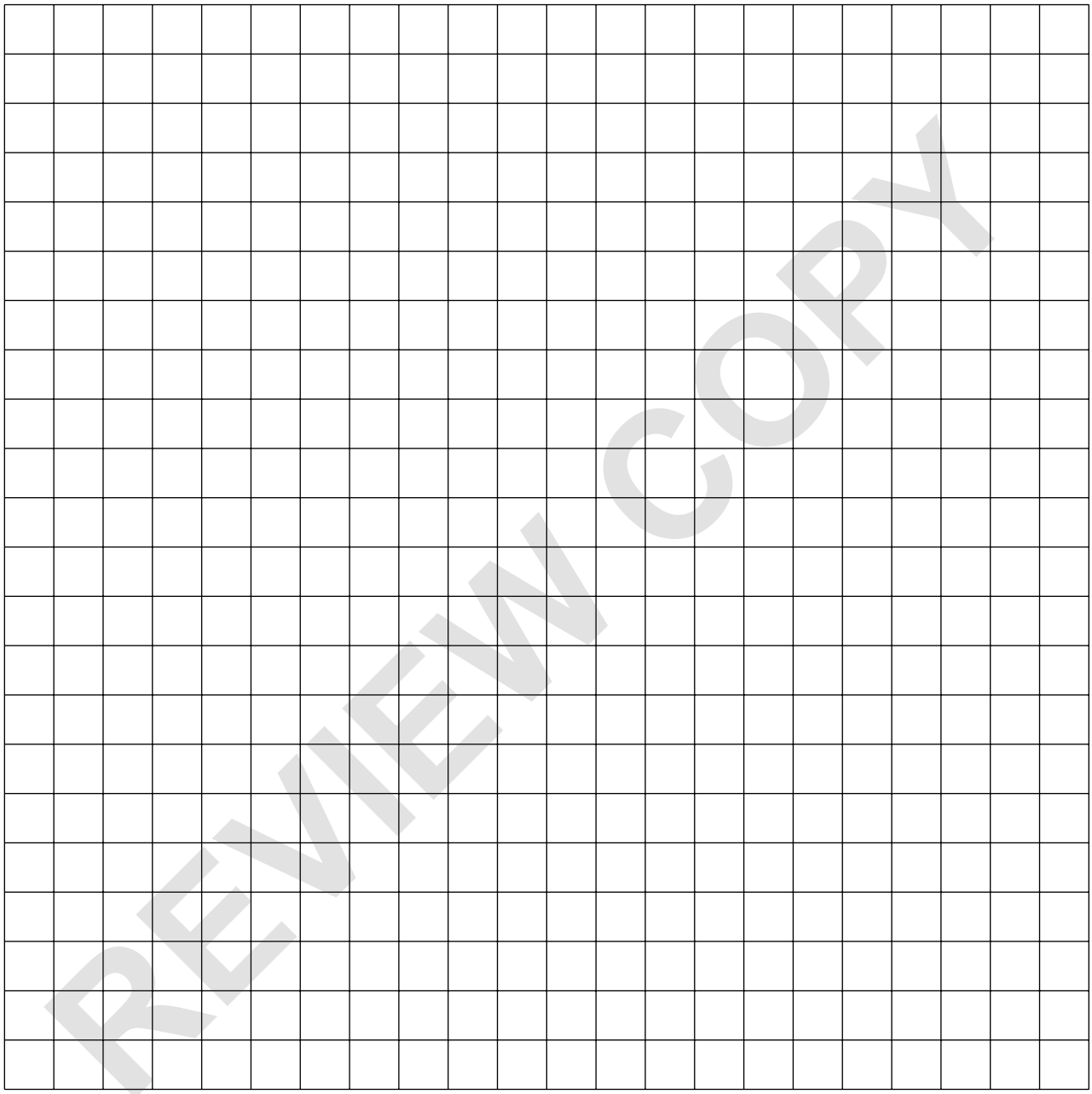
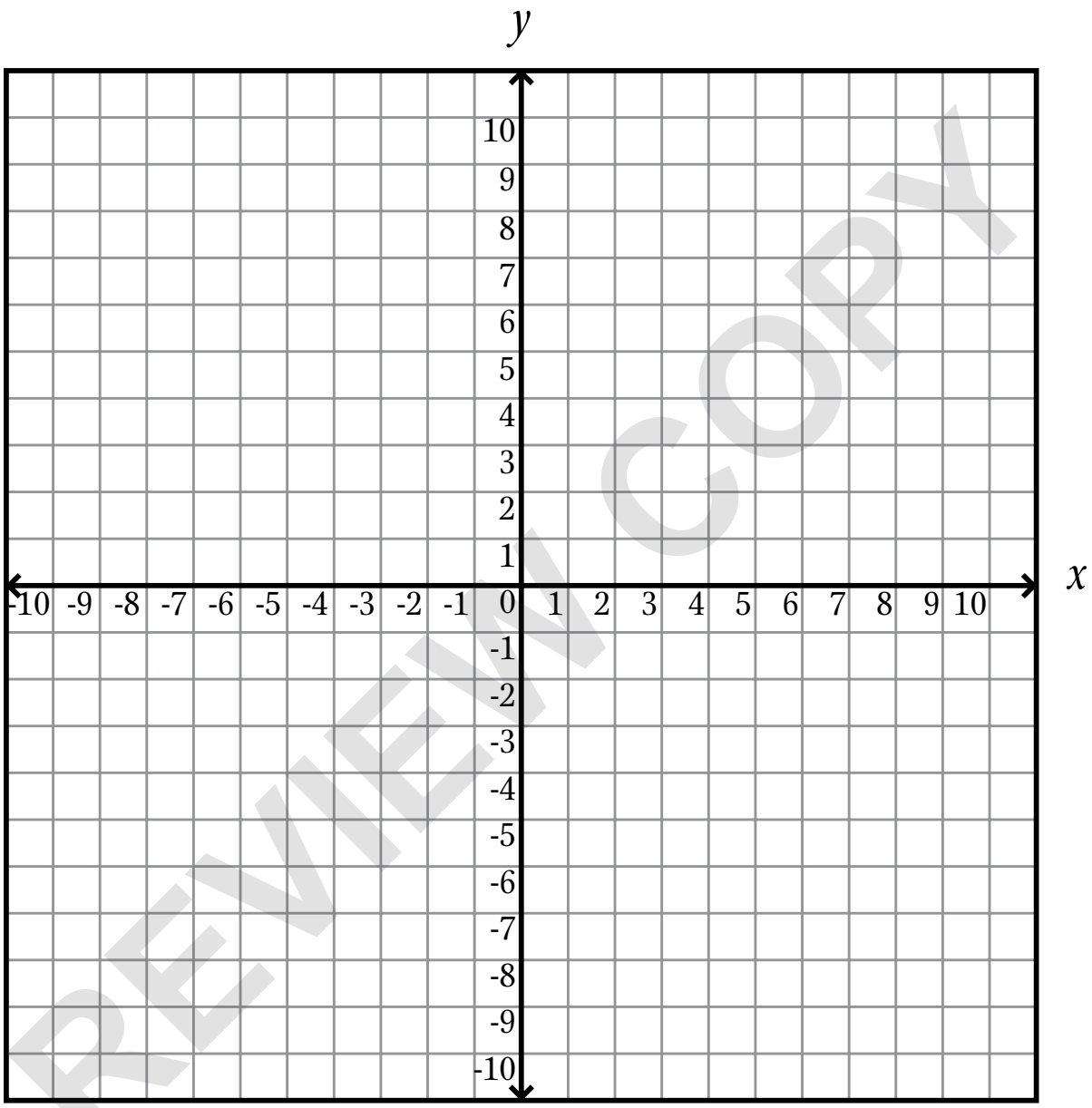
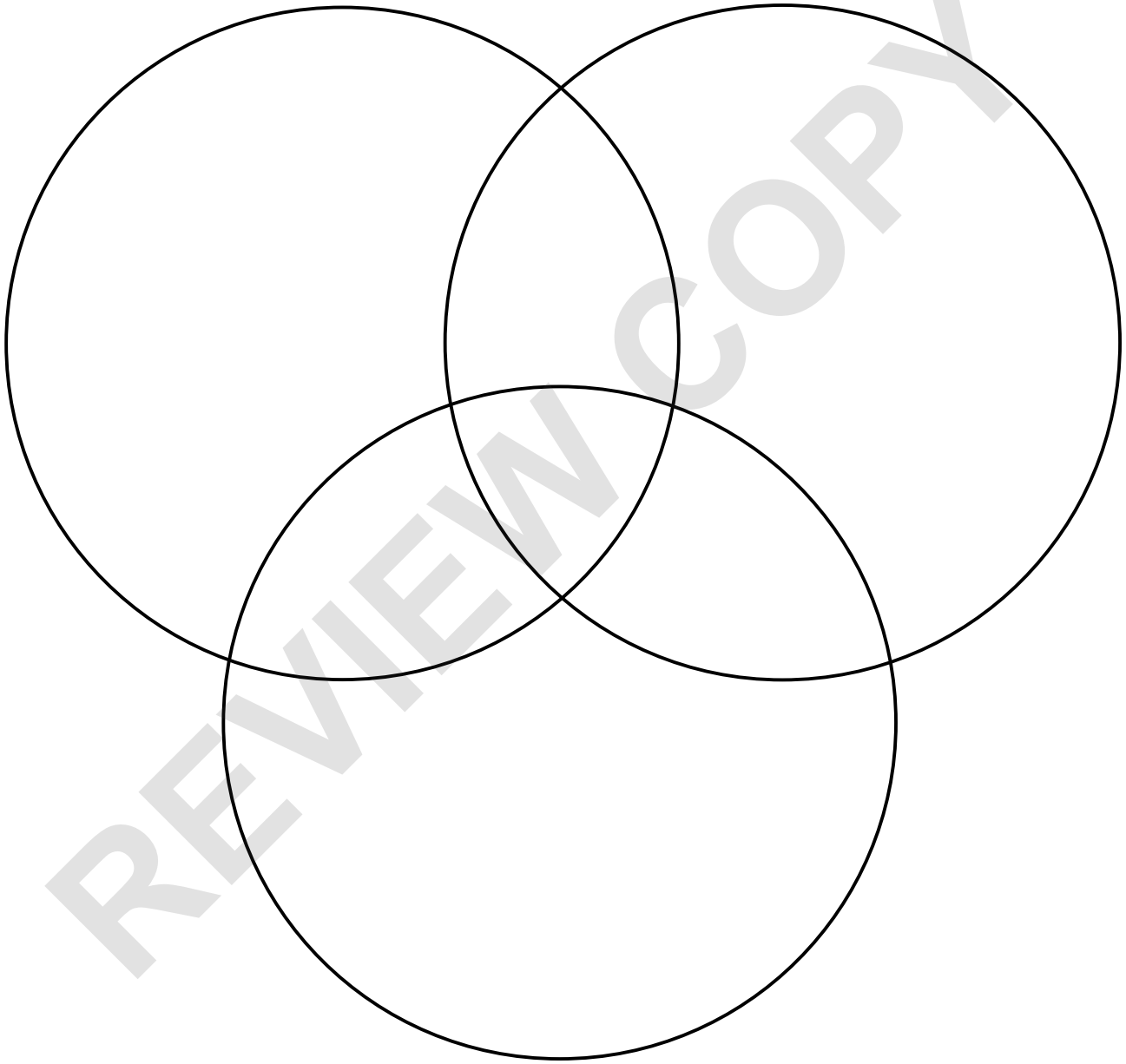
Geometric Drawing



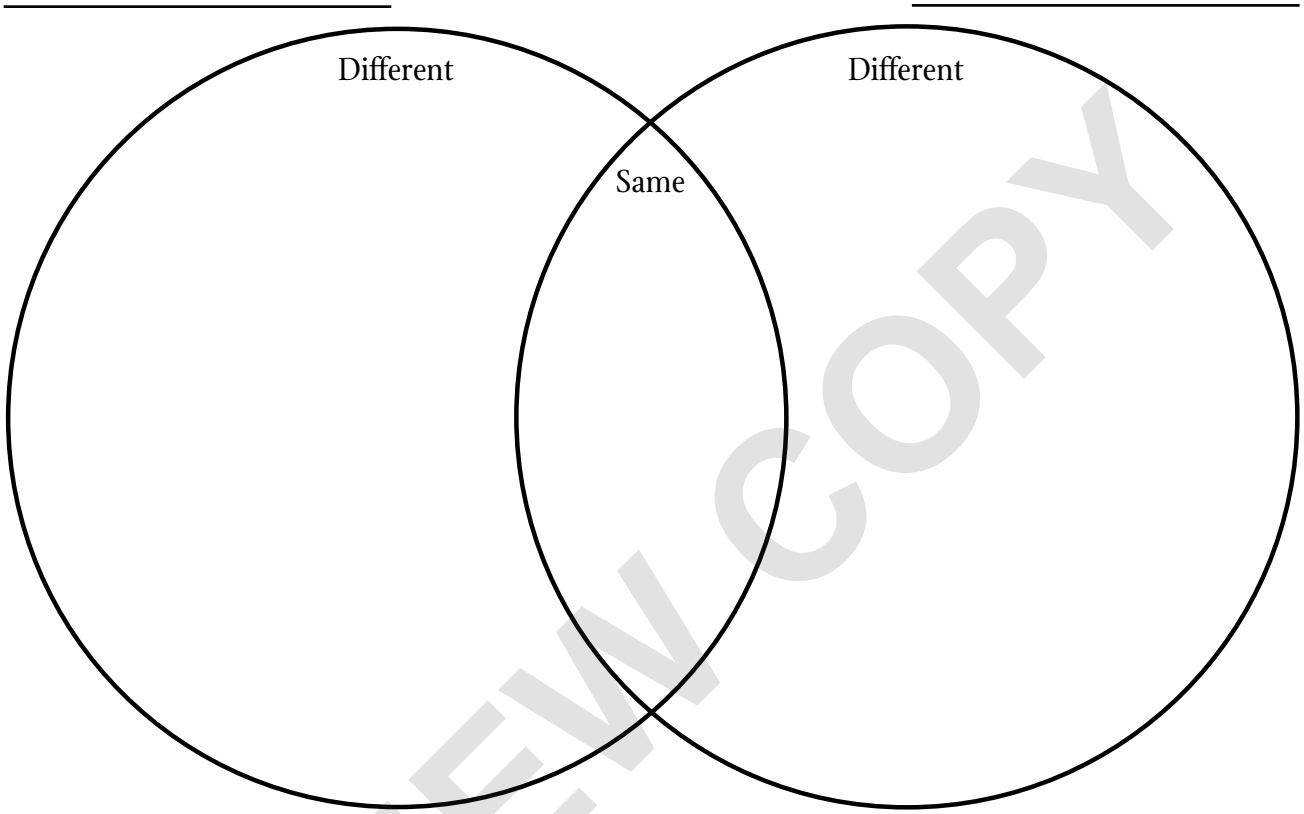
Coordinate Plane



Venn Diagram



Venn Diagram



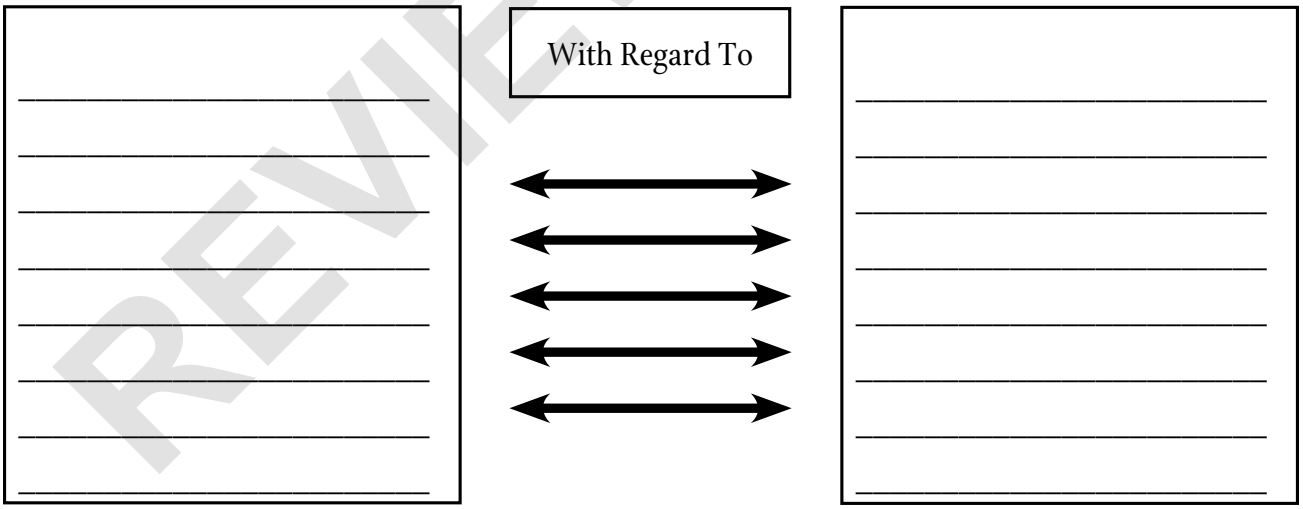
Compare-and-Contrast Diagram

Item 1 _____ Item 2 _____

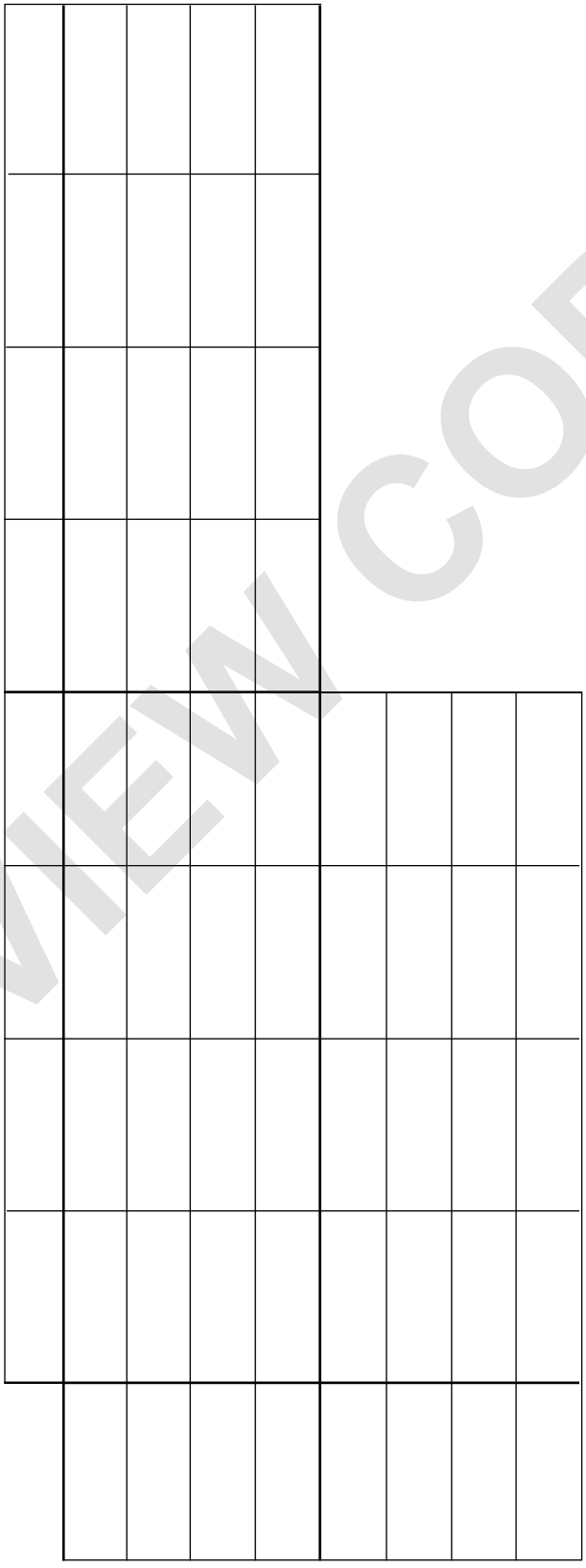
How Alike?

How Different?

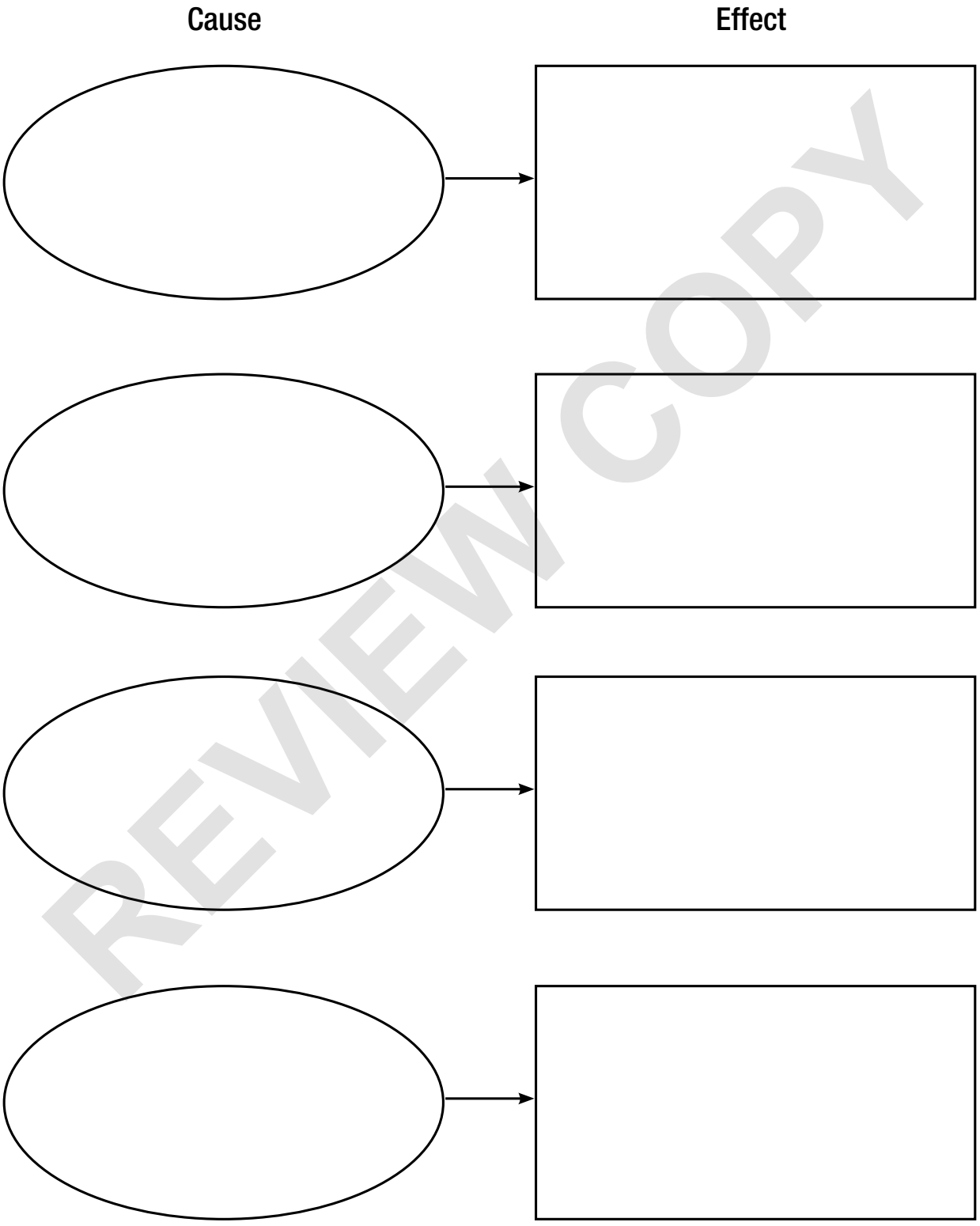
With Regard To



Attribute Table



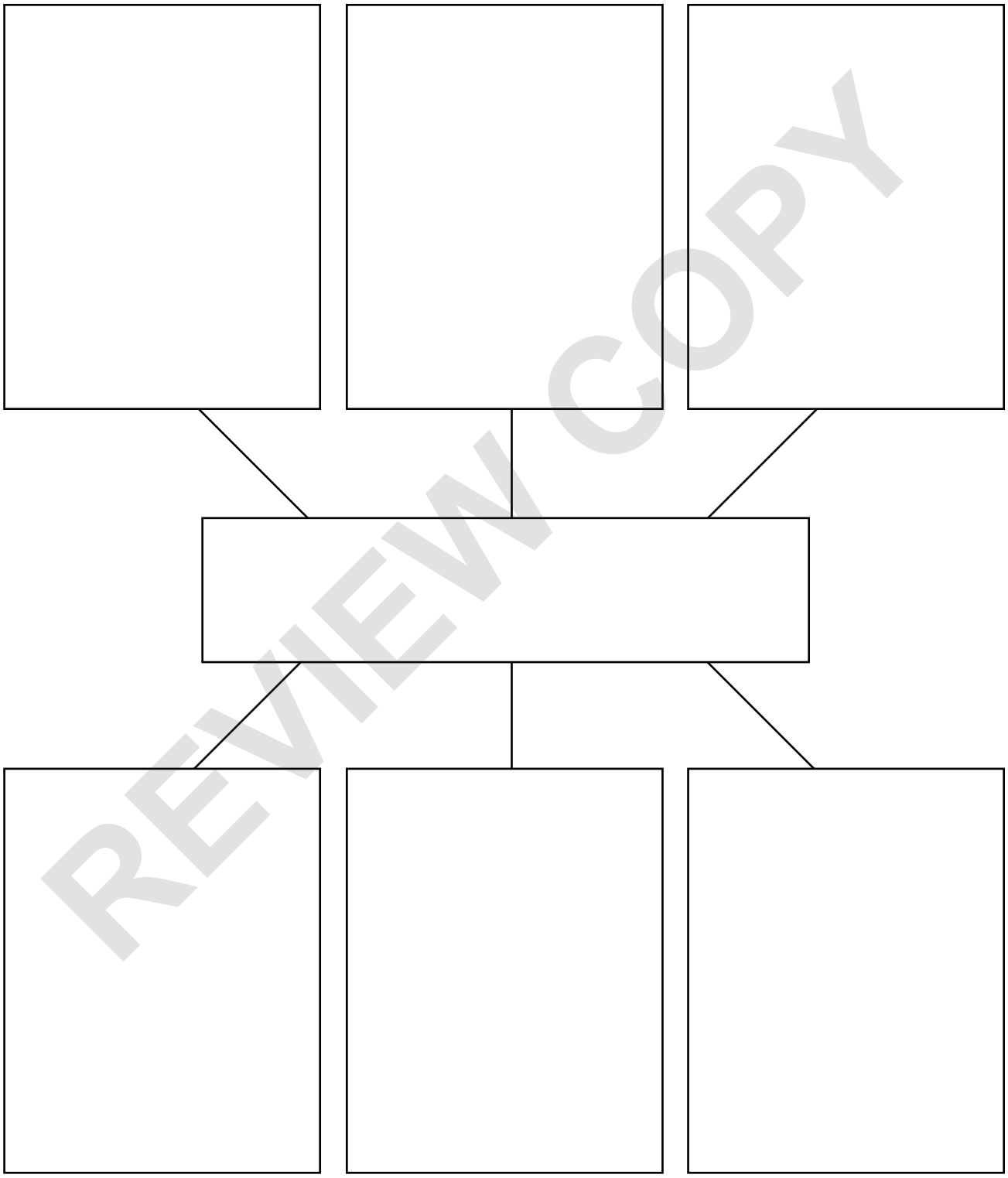
Cause and Effect Map



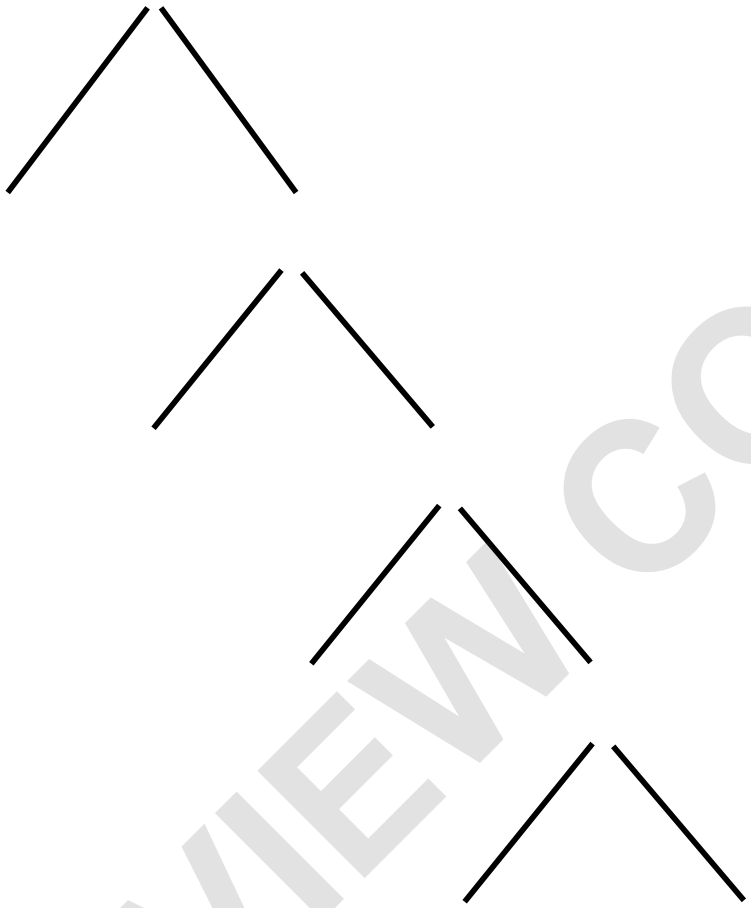
Frayer Model

Definition	Characteristics
WORD	
Examples from Life	Non-Examples

Semantic Map/Concept Map

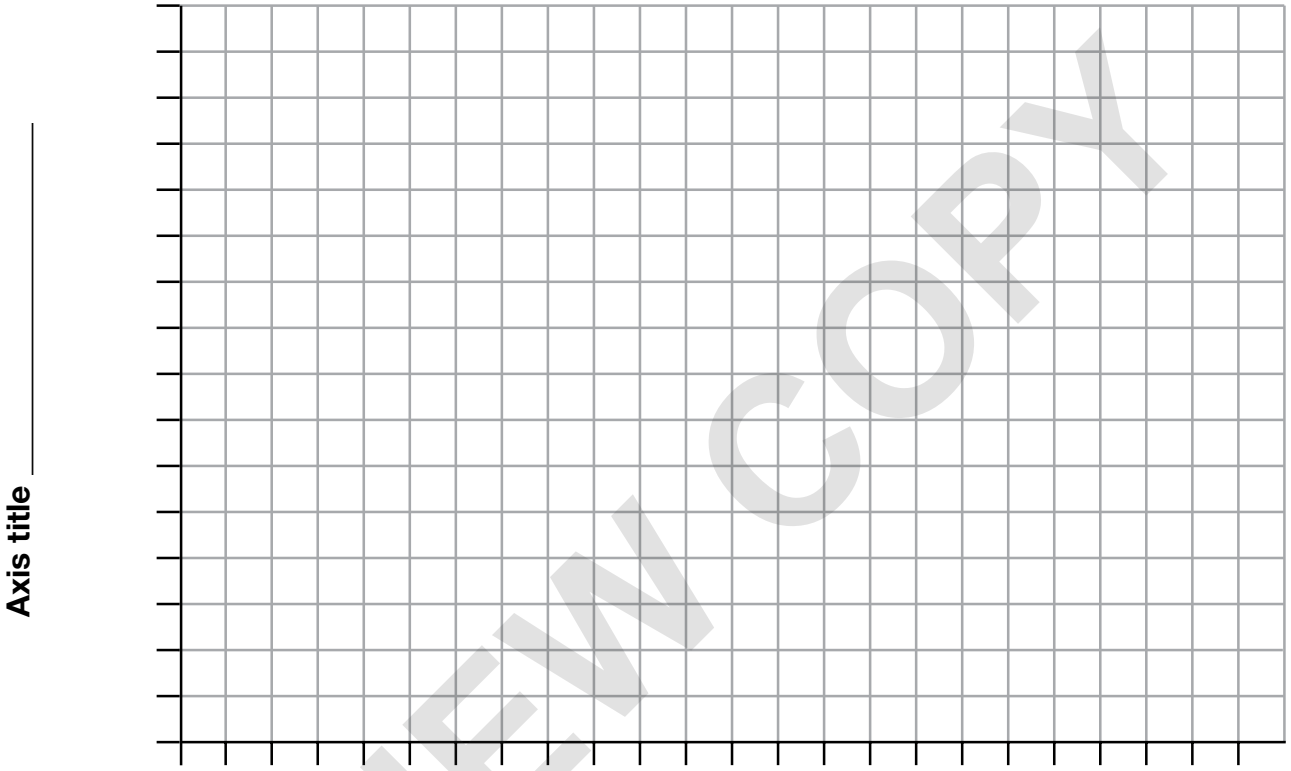


Factor Tree



Line Graph

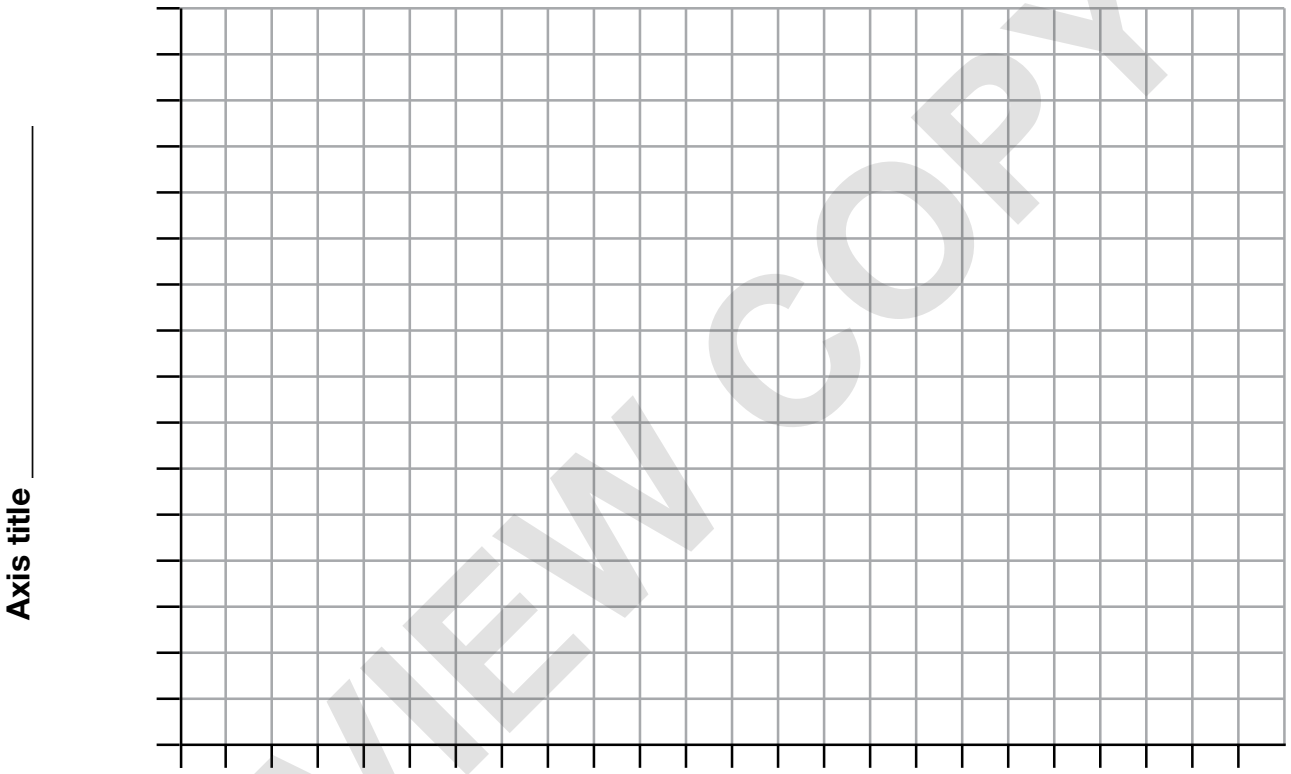
Graph title _____



Axis title _____

Bar Chart/Histogram

Graph title _____



KEY

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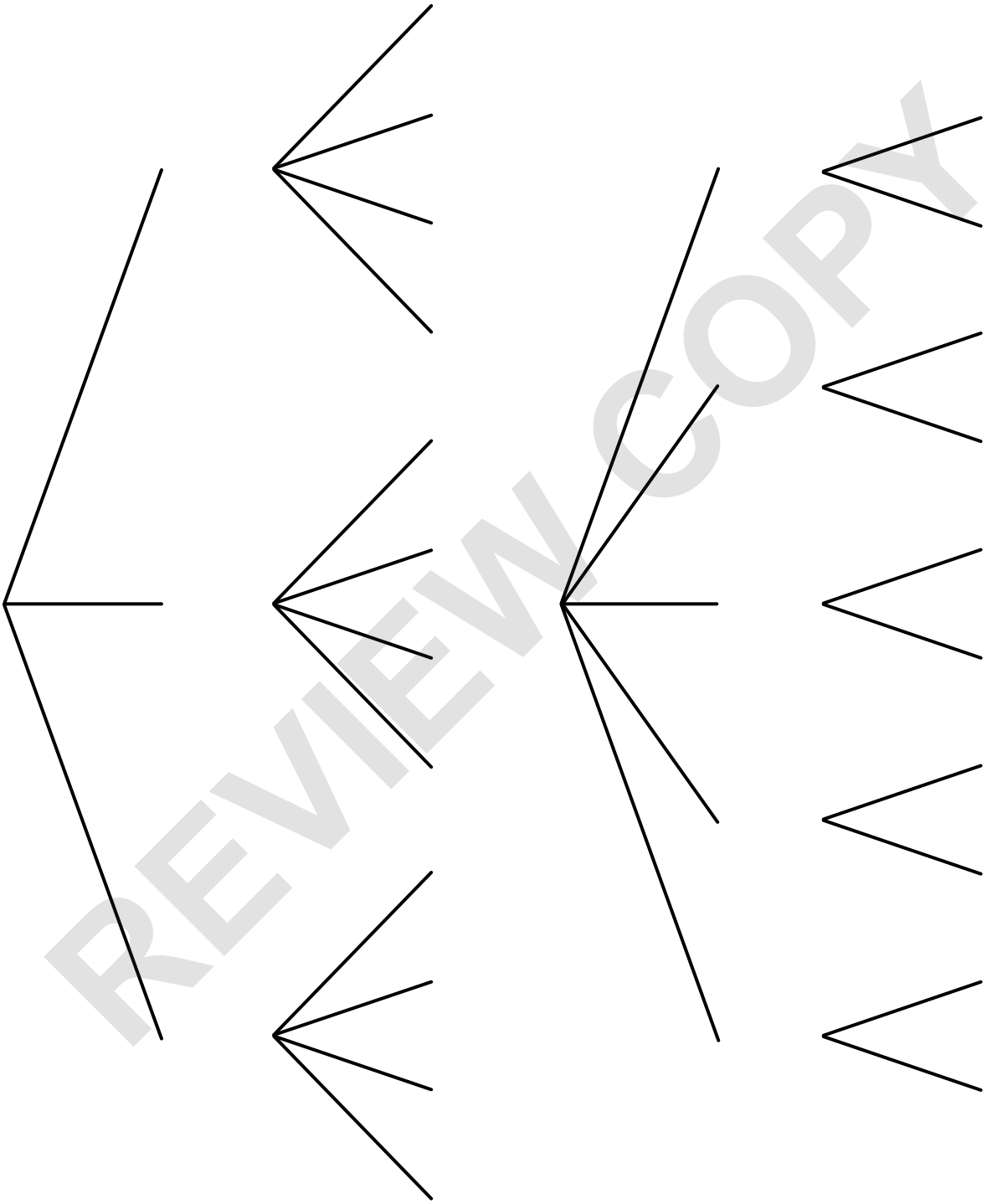
Axis title _____

Probability Trees



ALGEBRA

Symbols	
$\approx$	Approximately equal to
$\neq$	Is not equal to
$\lvert a \rvert$	Absolute value of a
$\sqrt{a}$	Square root of a
∞	Infinity
[	Inclusive on the lower bound
]	Inclusive on the upper bound
(	Non-inclusive on the lower bound
)	Non-inclusive on the upper bound
Σ	Sigma
Δ	Delta

General	
(x, y)	Ordered pair
$(x, 0)$	x-intercept
$(0, y)$	y-intercept

Linear Equations	
$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$y = mx + b$	Slope-intercept form
$ax + by = c$	General form
$y - y_1 = m(x - x_1)$	Point-slope form

Exponential Equations	
$A = P\left(1 + \frac{r}{n}\right)^{nt}$	Compounded interest formula
Compounded...	n (number of times per year)
Yearly/annually	1
Semiannually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

Exponential Functions	
$1 + r$	Growth factor
$1 - r$	Decay factor
$f(t) = a(1+r)^t$	Exponential growth function
$f(t) = a(1-r)^t$	Exponential decay function
$f(x) = ab^x$	Exponential function in general form
$y = ab^{\frac{x}{t}}$	Exponential equation

Binomial Theorem
$\sum_{k=0}^{n} \frac{n!}{(n-k)!k!} \bullet a^{n-k}b^k = 1a^nb^0 + \frac{n}{1}a^{n-1}b^1 + \frac{n(n-1)}{1 \bullet 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1 \bullet 2 \bullet 3}a^{n-3}b^3 + \cdots + 1a^0b^n$

Functions	
$f(x)$	Function notation, "f of x"
$f^{-1}(x)$	Inverse function notation
$f(x) = mx + b$	Linear function
$f(x) = b^x + k$	Exponential function
$f(x) = ax^2 + b^x + c$	Quadratic function
$(f + g)(x) = f(x) + g(x)$	Addition
$(f - g)(x) = f(x) - g(x)$	Subtraction
$(f \bullet g)(x) = f(x) \bullet g(x)$	Multiplication
$(f \circ g)(x) = f(g(x))$	Composition
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	Division
$\frac{f(b) - f(a)}{b - a}$	Average rate of change
$r = \frac{\Delta f(x)}{\Delta g(x)}$	Concise rate of change
$f(-x) = -f(x)$	Odd function
$f(-x) = f(x)$	Even function
$f(x) = \lfloor x \rfloor$	Floor/greatest integer function
$f(x) = \lceil x \rceil$	Ceiling/least integer function
$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$	Polynomial function
$f(x) = a\sqrt[3]{(x - h)} + k$	Cube root function
$f(x) = a\sqrt[n]{(x - h)} + k$	Radical function
$f(x) = a\lvert x - h\rvert + k$	Absolute value function
$f(x) = \frac{p(x)}{q(x)};\ \ q(x) \neq 0$	Rational function
$\log_a bx = c$	Logarithmic function

Quadratic Functions and Equations	
$x=\frac{-b}{2a}$	Axis of symmetry
$x=\frac{p+q}{2}$	Axis of symmetry using the midpoint of the x-intercepts
$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$	Vertex
$f(x) = ax^2 + bx + c$	General form
$f(x) = a(x - h)^2 + k$	Vertex form
$f(x) = a(x - p)(x - q)$	Factored/intercept form
$b^2 - 4ac$	Discriminant
$x^2+bx+\left(\frac{b}{2}\right)^2$	Perfect square trinomial
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	Quadratic formula

Properties of Exponents	
Property	General rule
Zero Exponent	$a^0 = 1$
Negative Exponent	$b^{-\frac{m}{n}}=\frac{1}{b^{\frac{m}{n}}}$
Product of Powers	$a^m \bullet a^n = a^{m+n}$
Quotient of Powers	$\frac{a^m}{a^n}=a^{m-n}$
Power of a Power	$\left(b^m\right)^n = b^{mn}$
Power of a Product	$(bc)^n = b^n c^n$
Power of a Quotient	$\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}$

Multiplication of Complex Conjugates
$(a + bi)(a - bi) = a^2 + b^2$

Common Polynomial Identities	
$(a + b)^2 = a^2 + 2ab + b^2$	Square of Sums
$(a - b)^2 = a^2 - 2ab + b^2$	Square of Differences
$a^2 - b^2 = (a + b)(a - b)$	Difference of Two Squares
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	Sum of Two Cubes
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	Difference of Two Cubes

Radicals to Rational Exponents
$\sqrt[n]{a}=a^{\frac{1}{n}}$
$\sqrt[n]{x^m}=x^{\frac{m}{n}}$

Imaginary Numbers
$i=\sqrt{-1}$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

Logarithmic Functions	
e	Base of a natural logarithm
$\log_a b=\frac{\log b}{\log a}$	Change of base formula
$\frac{2\pi}{b}$	Period
$-\frac{b}{a}$	Phase shift

Properties of Radicals
$\sqrt{ab}=\sqrt{a}\bullet\sqrt{b}$
$\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$

Series and Sequences	
$r = \frac{a_n}{a_{n-1}}$	Common ratio
$a_n = a_1 \bullet r^{n-1}$	Explicit formula for a geometric sequence
$\sum_{k=1}^{n} a_1 r^{k-1}$	Finite geometric series
$\sum_{k=1}^{\infty} a_1 r^{k-1}$	Infinite geometric series
$a_n = a_{n-1} \bullet r$	Recursive formula for a geometric sequence
$S_n = \frac{a_1(1-r^n)}{1-r}$	Sum formula for a finite geometric series
$S_n = \frac{a_1}{1-r}$	Sum formula for an infinite geometric series
$P = \sum_{k=1}^{n} A\left(\frac{1}{1+i}\right)^{k-1}$	Amortization loan formula

Properties of Logarithms	
Product property	$\log_a (x \bullet y) = \log_a x + \log_b y$
Quotient property	$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
Power property	$\log_a x^y = y \bullet \log_a x$

STATISTICS AND DATA ANALYSIS

Symbols	
$\varnothing$	Empty/null set
$\cap$	Intersection, "and"
$\cup$	Union, "or"
$\subset$	Subset
$\overline{A}$	Complement of Set A
!	Factorial
${}_nC_r$	Combination
${}_nP_r$	Permutation
μ	Population mean
$\overline{x}$	Sample mean
σ	Standard deviation of a population
s	Standard deviation of a sample
$\hat{p}$	Sample proportion
SEM	Standard error of the mean
SEP	Standard error of the proportion
MOE	Margin of error
CI	Confidence interval
df	Degrees of freedom

Empirical Rule/68–95–99.7 Rule
$\mu \pm 1\sigma \approx 68\%$
$\mu \pm 2\sigma \approx 95\%$
$\mu \pm 3\sigma \approx 99.7\%$

Common Critical Values

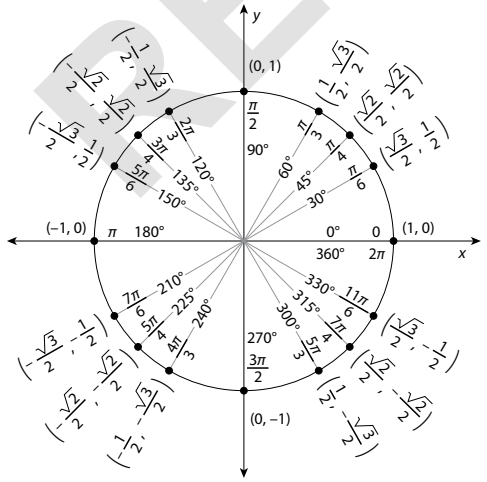
Confidence level	99%	98%	96%	95%	90%	80%	50%
Critical value (z_c)	2.58	2.33	2.05	1.96	1.645	1.28	0.6745

Formulas	
$\mu = \frac{x_1 + x_2 + \cdots + x_n}{n}$	Mean of a population
$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$	Mean of a sample
$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}}$	Standard deviation of a population
$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n}}$	Standard deviation of a sample
$z = \frac{x - \mu}{\sigma}$	*z*-score
$\hat{p} = \frac{p}{n}$	Sample proportion
$\text{SEM} = \frac{s}{\sqrt{n}}$	Standard error of the mean
$\text{SEP} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	Standard error of the proportion
$\text{MOE} = \pm z_c \frac{s}{\sqrt{n}}$	Margin of error of a sample mean
$\text{MOE} = \pm z_c \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	Margin of error for a sample proportion
$\text{CI} = \hat{p} \pm z_c \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	Confidence interval for a sample population with proportion $\hat{p}$
$\text{CI} = \overline{x} \pm z_c \frac{s}{\sqrt{n}}$	Confidence interval for a sample population with mean $\overline{x}$
$t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	*t*-value for two sets of sample data
$t = \frac{\overline{x} - \mu_0}{\frac{s}{\sqrt{n}}}$	*t*-value for sample data and population
$df = \frac{n_1 - 1 + n_2 - 1}{2}$	Degrees of freedom

Rules and Equations	
$P(E)=\dfrac{\#\text{ of outcomes in } E}{\#\text{ of outcomes in sample space}}$	Probability of event E
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$	Addition rule
$P(\overline{A})=1-P(A)$	Complement rule
$P(B\mid A)=\dfrac{P(A\cap B)}{P(A)}$	Conditional probability
$E(X)=p_1P(X_1)+p_2P(X_2)+p_3P(X_3)$	Expected value
$P(A\cap B)=P(A)\bullet P(B\mid A)$	Multiplication rule
$P(A\cap B)=P(A)\bullet P(B)$	Multiplication rule if A and B are independent
${}_nC_r=\dfrac{n!}{(n-r)!r!}$	Combination
${}_nP_r=\dfrac{n!}{(n-r)!}$	Permutation
$n!=n\bullet(n-1)\bullet(n-2)\bullet\cdots\bullet 1$	Factorial
$P=\binom{n}{x}p^xq^{n-x}$	Binomial probability distribution

GEOMETRY

Unit Circle



Symbols	
$\overset{\frown}{ABC}$	Major arc length
$\overset{\frown}{AB}$	Minor arc length
$\angle$	Angle
$\odot$	Circle
$\cong$	Congruent
$\overleftrightarrow{PQ}$	Line
$\overline{PQ}$	Line segment
$\overrightarrow{PQ}$	Ray
$\parallel$	Parallel
$\perp$	Perpendicular
$\bullet$	Point
$\triangle$	Triangle
$\square$	Parallelogram
A'	Prime
$^\circ$	Degrees
θ	Theta
ϕ	Phi
π	Pi
ρ	Rho

Trigonometric Ratios		
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$	$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$	$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

Volume	
$V = lwh$	Rectangular prism
$V = Bh$	Prism
$V = \frac{1}{3}\pi r^2 h$	Cone
$V = \frac{1}{3}Bh$	Pyramid
$V = \pi r^2 h$	Cylinder
$V = \frac{4}{3}\pi r^3$	Sphere

Pythagorean Theorem
$a^2 + b^2 = c^2$

Trigonometric Identities
$\sin\theta = \cos(90° - \theta)$
$\cos\theta = \sin(90° - \theta)$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\csc\theta = \frac{1}{\sin\theta}$
$\sec\theta = \frac{1}{\cos\theta}$
$\cot\theta = \frac{1}{\tan\theta}$
$\cot\theta = \frac{\cos\theta}{\sin\theta}$
$\sin^2\theta + \cos^2\theta = 1$

Area	
$A = lw$	Rectangle
$A = \frac{1}{2}bh$	Triangle
$A = \frac{1}{2}ab\sin C$	Triangle with unknown height
$A = \pi r^2$	Circle
$A = \frac{1}{2}(b_1 + b_2)h$	Trapezoid

Distance Formula
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Pi Defined
$\pi = \frac{\text{circumference}}{\text{diameter}} = \frac{\text{circumference}}{2 \bullet \text{radius}}$

Circumference of a Circle	
$C = 2\pi r$	Circumference given the radius
$C = \pi d$	Circumference given the diameter

Converting Between Degrees and Radians
$\frac{\text{radian measure}}{\pi} = \frac{\text{degree measure}}{180}$

Laws of Sines and Cosines	
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Law of Sines
$c^2 = a^2 + b^2 - 2ab \cos C$	Law of Cosines

Inverse Trigonometric Functions
Arcsin $\theta = \sin^{-1}\theta$
Arccos $\theta = \cos^{-1}\theta$
Arctan $\theta = \tan^{-1}\theta$

Arc Length	
$s = \theta r$	Arc length (θ in radians)

Density		
$\text{Density}_{\text{Area}} = \frac{\text{mass or quantity}}{\text{number of } \textit{square} \text{ units}}$	or $\rho_A = \frac{m}{A}$	Area density
$\text{Density}_{\text{Volume}} = \frac{\text{mass or quantity}}{\text{number of } \textit{cubic} \text{ units}}$	or $\rho = \frac{m}{V}$	Volume density

MEASUREMENTS

Length
Metric
1 kilometer (km) = 1000 meters (m)
1 meter (m) = 100 centimeters (cm)
1 centimeter (cm) = 10 millimeters (mm)
Customary
1 mile (mi) = 1760 yards (yd)
1 mile (mi) = 5280 feet (ft)
1 yard (yd) = 3 feet (ft)
1 foot (ft) = 12 inches (in)

Volume and Capacity
Metric
1 liter (L) = 1000 milliliters (mL)
Customary
1 gallon (gal) = 4 quarts (qt)
1 quart (qt) = 2 pints (pt)
1 pint (pt) = 2 cups (c)
1 cup (c) = 8 fluid ounces (fl oz)

Weight and Mass
Metric
1 kilogram (kg) = 1000 grams (g)
1 gram (g) = 1000 milligrams (mg)
1 metric ton (MT) = 1000 kilograms
Customary
1 ton (T) = 2000 pounds (lb)
1 pound (lb) = 16 ounces (oz)

English	Unit/Topic	Español
	#	
68–95–99.7 rule a rule that states percentages of data under the normal curve are as follows: $\mu \pm 1\sigma \approx 68\%$, $\mu \pm 2\sigma \approx 95\%$, and $\mu \pm 3\sigma \approx 99.7\%$; also known as the *Empirical Rule*	1/A	**regla 68–95–99,7** regla que establece los siguientes porcentajes de datos bajo la curva normal: $\mu \pm 1\sigma \approx 68\%$, $\mu \pm 2\sigma \approx 95\%$ y $\mu \pm 3\sigma \approx 99,7\%$; también se la conoce como *Regla Empírica*
	A	
absolute value equation an equation of the form $y = \lvert ax + b \rvert + c$, where x is the independent variable, y is the dependent variable, and a, b, and c are real numbers	2/D	**ecuación de valor absoluto** ecuación de la forma $y = \lvert ax + b \rvert + c$, donde x es la variable independiente, y es la variable dependiente y a, b y c son números reales
absolute value function a function of the form $f(x) = \lvert ax + b \rvert + c$, where x is the independent variable and a, b, and c are real numbers	6/D	**función de valor absoluto** función de la forma $f(x) = \lvert ax + b \rvert + c$, donde x es la variable independiente a, b y c son números reales
addition rule for mutually exclusive events If events A and B are mutually exclusive, then the probability that A or B will occur is the sum of the probability of each event; $P(A \text{ or } B) = P(A) + P(B)$.	1/D	**regla de adición para eventos mutuamente excluyentes** si los eventos A y B son mutuamente excluyentes, la probabilidad de que A o B suceda es la suma de la probabilidad de cada evento; $P(A \text{ or } B) = P(A) + P(B)$.
alternative hypothesis any hypothesis that differs from the null hypothesis; that is, a statement that indicates there is a difference in the data from two treatments; represented by H_a	1/E	**hipótesis alternativa** toda hipótesis que difiera de la hipótesis nula; es decir, una afirmación que indica que existe una diferencia en los datos de dos tratamientos; representada por H_a
ambiguous case a situation wherein the Law of Sines produces two possible answers. This only occurs when the lengths of two sides and the measure of the non-included angle are given (SSA).	4/B	**caso ambiguo** situación en la cual la Ley de Senos produce dos respuestas posibles. Esto solo ocurre cuando están dadas las longitudes de los dos lados y la medida del ángulo no incluido (SSA).

English	Unit/Topic	Español
amplitude the coefficient a or c of the sine or cosine term in a function of the form $f(x) = a \sin bx$ or $g(x) = c \cos dx$; on a graph of the cosine or sine function, the vertical distance from the y-coordinate of the maximum point on the graph to the midline of the cosine or sine curve	4/C 5/C	**amplitud** el coeficiente a o c del término de seno o coseno en una función de la forma $f(x) = a \sin bx$ o $g(x) = c \cos dx$; en un gráfico de la función seno o coseno, la distancia vertical desde la coordenada y del punto máximo en la gráfica hasta la línea media de la curva de seno o coseno
arccosine the inverse of the cosine function, written $\cos^{-1}\theta$ or $\arccos\theta$	4/B	**arcocoseno** inversa de la función coseno; se expresa $\cos^{-1}\theta$ o $\arccos\theta$
arc length the distance between the endpoints of an arc; written as $d(\overset{\frown}{ABC})$ or $m\overset{\frown}{AC}$	4/A	**longitud de arco** distancia entre los puntos extremos de un arco; se expresa como $d(\overset{\frown}{ABC})$ o $m\overset{\frown}{AC}$
arcsine the inverse of the sine function, written $\sin^{-1}\theta$ or $\arcsin\theta$	4/B	**arcoseno** inversa de la función seno; se expresa $\text{sen}^{-1}\theta$ o $\text{arcsen}\,\theta$
argument the result of raising the base of a logarithm to the power of the logarithm, so that b is the argument of the logarithm $\log_a b = c$; the term $cx + d$ in a cosine or sine function of the form $f(x) = a + b \sin(cx + d)$ or $g(x) = a + b \cos(cx + d)$	5/B 5/C	**argumento** el resultado de elevar la base de un logaritmo a la potencia del logaritmo, de manera que b es el argumento del logaritmo $\log_a b = c$; el término $cx + d$ en una función coseno o seno de la forma $f(x) = a + b \sin(cx + d)$ o $g(x) = a + b \cos(cx + d)$
asymptote a line that a function gets closer and closer to as one of the variables increases or decreases without bound	6/C	**asíntota** una línea que una función se acerca cada vez más cerca de una de las variables aumenta o disminuye sin límite
average rate of change the ratio of the difference of output values to the difference of the corresponding input values: $\frac{f(b)-f(a)}{b-a}$; a measure of how a quantity changes over some interval	6/C	**tasa de cambio promedio** proporción de la diferencia de valores de salida a la diferencia de valores correspondientes de entrada: $\frac{f(b)-f(a)}{b-a}$; medida de cuánto cambia una cantidad en cierto intervalo
axis of rotation a line about which a plane figure can be rotated in three-dimensional space to create a solid figure, such as a diameter or a symmetry line	6/E	**eje de rotación** línea alrededor de la cual puede girar la figura de un plano en un espacio de tres dimensiones para crear una figura sólida, como un diámetro o una línea de simetría

English	Unit/Topic	Español
	B	
base the quantity that is being raised to an exponent in an exponential expression; in a^x, a is the base; or, the quantity that is raised to an exponent which is the value of the logarithm, such as 2 in the equation $\log_2 g(x) = 3 - x$	5/B	**base** cantidad que es elevada a un exponente en una expresión exponencial; en a^x, a es la base; o, la cantidad que se eleva a un exponente que es el valor del logaritmo, tal que 2 en la ecuación $\log_2 g(x) = 3 - x$
bias leaning toward one result over another; having a lack of neutrality	1/B 1/C 1/E	**sesgo** inclinación por un resultado sobre otro; carecer de neutralidad
biased sample a sample in which some members of the population have a better chance of inclusion in the sample than others	1/B	**muestra sesgada** muestra en la cual algunos miembros de la población tienen una mayor posibilidad de ser incluidos en la muestra que otros
binomial experiment an experiment in which there are a fixed number of trials, each trial is independent of the others, there are only two possible outcomes (success or failure), and the probability of each outcome is constant from trial to trial	1/D	**experimento binomial** experimento en el que existe un número fijo de pruebas, cada prueba es independiente de las demás, existen dos resultados posibles (éxito o fracaso) y la probabilidad de cada resultado es constante de prueba a prueba
binomial probability distribution formula the distribution of the probability, P, of exactly x successes out of n trials, if the probability of success is p and the probability of failure is q; given by the formula $P = \binom{n}{x} p^x q^{n-x}$	1/D	**fórmula de distribución binomial de probabilidad** la distribución de la probabilidad, P, de exactamente x éxitos entre n pruebas, si la probabilidad de éxito es p y la probabilidad de fracaso es q; dada por la fórmula $P = \binom{n}{x} p^x q^{n-x}$

English	Unit/Topic	Español
Binomial Theorem a theorem stating that a binomial $(a + b)^n$ can be expanded using the formula $\sum_{k=0}^{n} \frac{n!}{(n-k)!k!} \bullet a^{n-k}b^k = 1a^n b^0 + \frac{n}{1} \bullet a^{n-1}b^1 + \frac{n(n-1)}{1 \bullet 2} \bullet a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1 \bullet 2 \bullet 3} \bullet a^{n-3}b^3 + \cdots + 1a^0 b^n$	2/B	**Teorema del Binomio** teorema que expresa que un binomio $(a + b)^n$ puede ampliarse utilizando la fórmula $\sum_{k=0}^{n} \frac{n!}{(n-k)!k!} \bullet a^{n-k}b^k = 1a^n b^0 + \frac{n}{1} \bullet a^{n-1}b^1 + \frac{n(n-1)}{1 \bullet 2} \bullet a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1 \bullet 2 \bullet 3} \bullet a^{n-3}b^3 + \cdots + 1a^0 b^n$
boundary condition a constraint or limit on a function or domain value based on real-world conditions or restraints in the problem or its solution	6/C	**condición de contorno** restricción o límite en un valor de dominio o función basado en condiciones del mundo real o restricciones en el problema o su solución
C		
central angle an angle with its vertex at the center of a circle	4/A	**ángulo central** ángulo con su vértice en el centro de un círculo
chance variation a measure showing how precisely a sample reflects the population, with smaller sampling errors resulting from large samples and/or when the data clusters closely around the mean; also called *sampling error*	1/B	**variación aleatoria** medida que muestra cómo una muestra refleja con precisión la población, con errores de muestreo más pequeños que resultan de muestras grandes y/o cuando los datos se agrupan estrechamente alrededor de la media; también llamada *error de muestreo*
closure a system is closed, or shows closure, under an operation if the result of the operation is within the system	2/A	**cierre** un sistema es cerrado, o tiene cierre, en una operación si el resultado de la misma está dentro del sistema
cluster sample a sample in which naturally occurring groups of population members are chosen for a sample	1/B	**muestreo en grupos** muestra en la cual se eligen para una muestra grupos naturalmente ya formados de miembros de la población
coefficient the number multiplied by a variable in an algebraic expression	2/A	**coeficiente** número multiplicado por una variable en una expresión algebraica

English	Unit/Topic	Español
combination a subset of a group of objects taken from a larger group of objects; the order of the objects does not matter, and objects may be repeated. A combination of size r from a group of n objects can be represented using the notation ${}_nC_r$, where ${}_nC_r = \frac{n!}{(n-r)!r!}$.	1/B	**combinación** subconjunto de un grupo de objetos tomado de un grupo más grande de objetos; el orden de los objetos no importa y los objetos pueden repetirse. Se puede representar una combinación de tamaño r de un grupo de n objetos con la notación ${}_nC_r$, donde ${}_nC_r = \frac{n!}{(n-r)!r!}$.
combination of functions the process of combining two or more functions using the operations of addition, subtraction, multiplication, or division to create a new function	6/B	**combinación de funciones** proceso de combinar dos o más funciones utilizando las operaciones de adición, sustracción, multiplicación o división para crear una nueva función
common denominator a quantity that is a shared multiple of the denominators of two or more fractions	3/A 3/B	**denominador común** cantidad que es un múltiplo compartido de los denominadores de dos o más fracciones
common logarithm a base-10 logarithm which is usually written without the number 10, such as $\log x = \log_{10} x$	5/B	**logaritmo común** logaritmo de base 10 que se escribe normalmente sin el número 10, como $\log x = \log_{10} x$
common ratio the ratio of a term in a geometric sequence to the previous term in that sequence; indicated by the variable r and given by the formula $r = \frac{a_n}{a_{n-1}}$	2/E	**relación común** la relación de un término en una secuencia geométrica con el término anterior en esa secuencia; indicada por la variable r y dada por la fórmula $r = \frac{a_n}{a_{n-1}}$
complex conjugate the complex number that when multiplied by another complex number produces a value that is wholly real; the complex conjugate of $a + bi$ is $a - bi$	2/B 2/C	**conjugado de número complejo** número complejo que cuando se multiplica por otro número complejo produce un valor totalmente real; el conjugado complejo de $a + bi$ es $a - bi$

English	Unit/Topic	Español
Complex Conjugate Theorem Let $p(x)$ be a polynomial with real coefficients. If $a + bi$ is a root of the equation $p(x) = 0$, where a and b are real and $b \neq 0$, then $a - bi$ is also a root of the equation.	2/C	**Teorema de Complejo Conjugado** suponiendo que $p(x)$ es un polinomio con coeficientes reales, si $a + bi$ es una raíz de la ecuación $p(x) = 0$, donde a y b son reales y $b \neq 0$, entonces $a - bi$ es también una raíz de la ecuación.
complex number a number of the form $a + bi$, where a and b are real numbers and i is the imaginary unit	2/B	**número complejo** número en la forma $a + bi$, donde a y b son números reales e i es la unidad imaginaria
composition of functions the process of substituting one function for the independent variable of another function to create a new function	6/B	**composición de funciones** proceso de sustituir una función por la variable independiente de otra función para crear una función nueva
conditional probability of *B* given *A* the probability that event B occurs, given that event A has already occurred. If A and B are two events from a sample space with $P(A) \neq 0$, then the conditional probability of B given A, denoted $P(B \mid A)$, has two equivalent expressions: $P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{\text{number of outcomes of } (A \text{ and } B)}{\text{all outcomes in } A}$	1/F	**probabilidad condicional de *B* dado *A*** probabilidad de que el evento B se produzca, dado que el evento A ya se ha producido. Si A y B son dos eventos de un espacio de muestra con $P(A) \neq 0$, entonces la probabilidad condicional de B dado A, indicado $P(B \mid A)$, tiene dos expresiones equivalentes: $P(B \mid A) = \frac{P(A \text{ y } B)}{P(A)} = \frac{\text{cantidad de resultados de (A y B)}}{\text{todos los resultados en A}}$
confidence interval an interval of numbers within which it can be claimed that repeated samples will result in the calculated parameter; generally calculated using the estimate plus or minus the margin of error	1/D	**intervalo de confianza** intervalo de números dentro del cual se puede afirmar que las muestras repetidas tendrán como resultado el parámetro calculado; generalmente se calcula usando la estimación más o menos el margen de error
confidence level the probability that a parameter's value can be found in a specified interval; also called *level of confidence*	1/D 1/E	**nivel de confianza** probabilidad de que se pueda encontrar el valor de un parámetro en un intervalo específico; también llamado *grado de confianza*

English	Unit/Topic	Español
confounding variable an ignored or unknown variable that influences the result of an experiment, survey, or study	1/C 1/E	**variable de confusión** una variable ignorada o desconocida que influye sobre el resultado de un experimento, encuesta o estudio
congruent having the same shape, size, lines, and angles; the symbol for representing congruency between figures is ≅	6/E	**congruente** tener la misma forma, tamaño, rectas y ángulos; el símbolo para representar la congruencia entre números es ≅
constant term a term whose value does not change	2/A	**término constante** término cuyo valor no cambia
constraint a limit or restriction on the domain, range, and/or solutions of a mathematical or real-world problem	6/A	**restricción** límite en el dominio, rango y/o soluciones de un problema matemático o del mundo real
continuous data a set of values for which there is at least one value between any two given values	1/A	**datos continuos** conjunto de valores para el que existe al menos un valor entre dos valores dados
continuous distribution the graphed set of values, a curve, in a continuous data set	1/A	**distribución continua** conjunto de valores representado gráficamente, una curva, en un conjunto de datos continuos
continuous function a function that does not have a break in its graph across a specified domain	6/D	**función continua** función que no tiene una interrupción en su curva a lo largo de un dominio específico
control group the group of participants in a study who are not subjected to the treatment, action, or process being studied in the experiment, in order to form a comparison with participants who are subjected to it	1/C	**grupo de control** grupo de participantes en un estudio que no están sujetos al tratamiento, acción o proceso que está en estudio en el experimento con el fin de establecer una comparación con participantes que sí lo están.
convenience sample a sample in which members are chosen to minimize the time, effort, or expense involved in sampling	1/B	**muestreo de conveniencia** muestreo en el cual se eligen los miembros para minimizar el tiempo, esfuerzo o gasto involucrado en este proceso

English	Unit/Topic	Español
converge to approach a finite limit. If the sequence of the partial sums of a series approaches the value of a given number (the limit), then the entire series converges to that limit; that is, the series has a sum. An infinite series converges when the absolute value of the common ratio r is less than 1 $(\lvert r\rvert<1)$.	2/E	**converger** acercarse a un límite finito. Si la secuencia de las sumas parciales de una serie se acerca al valor de un número dado (el límite), entonces la serie completa converge hacia ese límite; es decir, la serie tiene una suma. Una serie infinita converge cuando el valor absoluto de la relación común r es menor que 1 $(\lvert r\rvert<1)$.
correlation a measure of the power of the association between exactly two quantifiable variables	1/E	**correlación** una medida de la potencia de la asociación entre exactamente dos variables cuantificables
cosecant the reciprocal of sine, $\csc\theta=\frac{1}{\sin\theta}$; the cosecant of θ = csc θ = $\frac{\text{length of hypotenuse}}{\text{length of opposite side}}$	4/A	**cosecante** la recíproca del seno, $\csc\theta=\frac{1}{\sin\theta}$; la cosecante de θ = csc θ = $\frac{\text{largo de la hipotenusa}}{\text{largo del lado opuesto}}$
cosine a trigonometric function of an acute angle in a right triangle that is the ratio of the length of the side adjacent to the length of the hypotenuse; the cosine of θ = cos θ = $\frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$	4/A 4/B 5/C	**coseno** función trigonométrica de un ángulo agudo en un triángulo rectángulo que es la proporción de la longitud de lado adyacente a la longitud de la hipotenusa; el coseno de θ = cos θ = $\frac{\text{longitud del lado adyacente}}{\text{longitud de la hipotenusa}}$
cosine function a trigonometric function of the form $f(x) = a\cos[b(x-c)]+d$, in which a, b, c, and d are constants and x is a variable defined in radians over the domain $(-\infty,\infty)$	4/C 5/C	**función del coseno** función trigonométrica de la forma $f(x) = a\cos[b(x-c)]+d$, donde a, b, c y d son constantes y x es una variable definida en radianes a lo largo del dominio $(-\infty,\infty)$

English	Unit/Topic	Español
cotangent the reciprocal of tangent, $\cot\theta = \frac{1}{\tan\theta}$; the cotangent of θ = cot $\theta = \frac{\text{length of adjacent side}}{\text{length of opposite side}}$	4/A	**cotangente** recíproco de la tangente, $\cot\theta = \frac{1}{\tan\theta}$; la cotangente de θ = cot $\theta = \frac{\text{longitud del lado adyacente}}{\text{longitud del lado opuesto}}$
coterminal angles angles that, when drawn in standard position, share the same terminal side	4/A	**ángulos coterminales** ángulos que, cuando están trazados en una posición estándar, comparten el mismo lado terminal
critical value a measure of the number of standards of error to be added to or subtracted from the mean in order to achieve the desired confidence level; also known as z_c-*value*	1/D	**valor crítico** medida de la cantidad de estándares de error que se suma o se resta de la media para lograr el nivel de confianza deseado; también conocido como *valor* z_c
cross section the plane figure formed by the intersection of a plane with a solid figure, where the plane is at a right angle to the surface of the solid figure	6/E	**sección transversal** figura del plano formada por la intersección de un plano con una figura sólida, donde el plano está en un ángulo recto a la superficie de la figura sólida
cube root function a function that contains the cube root of a variable. The general form is $f(x) = a\sqrt[3]{(x-h)-k}$, where a, h, and k are real numbers.	6/D	**función raíz cúbica** función que contiene la raíz cúbica de una variable. La forma general es $y = a\sqrt[3]{(x-h)}+k$, donde a, h, y k son números reales.
cycle the smallest representation of a cosine or sine function graph as defined over a restricted domain; equal to one repetition of the period of a function	4/C 5/C	**ciclo** la representación más pequeña de una gráfica de la función coseno o seno definida a través de un dominio restringido; igual a una repetición del período de una función
D		
data numbers in context	1/C	**datos** números en contexto
data fitting the process of assigning a rule, usually an equation or formula, to a collection of data points as a method of predicting the values of new dependent variables that result from new independent-variable values	6/A	**ajuste de datos** proceso de asignar una regla, normalmente una ecuación o fórmula, a una colección de puntos de datos como un método de predecir los valores de nuevas variables dependientes que vienen de nuevos valores de variables independientes

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data point a point (x, y) on a two-dimensional coordinate plane that represents the value of an independent variable (x) that results in a specific dependent variable value (y). The term also refers to solutions for an equation or inequality in one variable that originate from a real-world situation. A data point is also called an *ordered pair*.	6/A	**punto de datos** un punto (x, y) en un plano de coordenadas de dos dimensiones que representa el valor de una variable independiente (x) que da como resultado un valor específico de variable dependiente (y). El término también se refiere a las soluciones para una ecuación o desigualdad en una variable que proviene de una situación del mundo real. Un punto de datos se denomina también un *par ordenado*.
degree a unit used to measure angles. One degree represents $\frac{1}{360}$ of a full rotation.	4/A 5/C	**grado** unidad usada para medir ángulos. Un grado representa $\frac{1}{360}$ de una rotación completa.
degree of a one-variable polynomial the greatest exponent of the variable in a polynomial	2/A	**grado de un polinomio de una variable** el mayor exponente de la variable en un polinomio
degrees of freedom (*df*) the number of data values that are free to vary in the final calculation of a statistic; that is, values that can change or move without violating the constraints on the data	1/E	**grados de libertad (*df*)** la cantidad de valores de datos que varían libremente en el cálculo final de una estadística; es decir, los valores que pueden cambiar o moverse sin violar las restricciones en los datos
***delta* (Δ)** a Greek letter commonly used to represent the change in a value	6/C	***delta* (Δ)** letra griega utilizada comúnmente para representar el cambio en un valor
denominator the value located below the line of a rational expression or fraction; the divisor	3/A	**denominador** el valor ubicado debajo de la línea de una expresión racional o fracción; el divisor
density the amount, number, or other quantity per unit of area or volume of some substance or population being studied	6/E	**densidad** la cantidad, el número u otra cantidad por unidad de área o volumen de alguna sustancia o población que está siendo estudiada
dependent system a system of equations that intersect at every point	2/D 3/B	**sistema dependiente** sistema de ecuaciones que se cruzan en cada punto

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dependent variable labeled on the *y*-axis; the quantity that is based on the input values of the independent variable; the output variable of a function	6/A	**variable dependiente** designada en el eje de *y*; cantidad que se basa en los valores de entrada de la variable independiente; variable de salida de una función
depressed polynomial the result of dividing a polynomial by one of its binomial factors	2/C	**polinomio deprimido** el resultado de dividir un polinomio por uno de sus factores binómicos
descending order polynomials ordered by the power of the variables, with the largest power listed first and the constant last	2/A	**orden descendente** polinomios ordenados según la potencia de las variables, con la potencia más grande indicada primero y la constante, última
desirable outcome the data sought or hoped for, represented by *p*; also known as *favorable outcome* or *success*	1/D	**resultado deseado** datos buscados o esperados, representado por *p*; también conocido como *resultado favorable* o *éxito*
discontinuous function a function that has a break, hole, or jump in the graph	6/D	**función discontinua** una función que tiene un descanso, agujero, o salto en el gráfico
discrete data a set of values with gaps between successive values	1/A	**datos discretos** conjunto de valores con interrupciones entre valores sucesivos
discrete function a function in which every element of the domain is individually separate and distinct	2/E 6/D	**función discreta** función en la cual cada elemento del dominio está individualmente separado y distinguible
diverge to *not* approach a finite limit. If a series does not have a sum (that is, the sequence of its partial sums does not approach a given number), then the series diverges. An infinite series diverges when the absolute value of the common ratio *r* is greater than 1 $(\lvert r\rvert > 1)$.	2/E	**divergir** *no* acercarse a un límite finito. Si una serie no tiene una suma (es decir, la secuencia de sus sumas parciales no se acerca a un número dado), entonces la serie diverge. Una serie infinita diverge cuando el valor absoluto de la relación común *r* es mayor que 1 $(\lvert r\rvert > 1)$.
domain the set of all input values (*x*-values) that satisfy the given function without restriction	5/A 6/A	**dominio** conjunto de todos los valores de entrada (valores de *x*) que satisfacen la función dada sin restricciones
double-blind study a study in which neither the researcher nor the participants know who has been subjected to the treatment, action, or process being studied, and who is in a control group	1/C	**estudio doble-ciego** estudio en el cual ni el investigador ni los participantes saben quién se sometió al tratamiento, acción o proceso que está siendo estudiado y quién está en un grupo de control

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	E	
e an irrational number with an approximate value of 2.71828; *e* is the base of the natural logarithm ($\ln x$ or $\log_e x$)	5/B	***e*** número irracional con un valor aproximado de 2,71828; *e* es la base del logaritmo natural (In x o $\log_e x$)
empirical probability the number of times an event actually occurs divided by the total number of trials, given by the formula $P(E)=\frac{\text{number of occurrences of the event}}{\text{total number of trials}}$; also called *experimental probability*	1/F	**probabilidad empírica** cantidad de veces que se produce un evento dividido por la cantidad total de pruebas, dada por la fórmula $P(E)=\frac{\text{cantidad de ocurrencias del evento}}{\text{cantidad total de pruebas}}$; también se denomina *probabilidad experimental*
Empirical Rule a rule that states percentages of data under the normal curve are as follows: $\mu\pm1\sigma\approx68\%$, $\mu\pm2\sigma\approx95\%$, and $\mu\pm3\sigma\approx99.7\%$; also known as the *68–95–99.7 rule*	1/A	**Regla Empírica** regla que establece los siguientes porcentajes de datos bajo la curva normal: $\mu\pm1\sigma\approx68\%$, $\mu\pm2\sigma\approx95\%$ y $\mu\pm3\sigma\approx99{,}7\%$; también se conoce como *la regla 68–95–99,7*
empty set a set that has no elements, denoted by $\varnothing$; the solution to a system of equations with no intersection points, denoted by { }	2/D 3/B	**conjunto vacío** un conjunto que no tiene elementos, indicado por $\varnothing$; la solución a un sistema de ecuaciones sin puntos de intersección, indicado por { }
end behavior the behavior of the graph as x approaches positive or negative infinity	2/C	**comportamiento final** el comportamiento del gráfico a medida que x se acerca al infinito positivo o negativo
even-degree polynomial function a polynomial function in which the highest exponent is an even number. Both ends of the graph of an even-degree polynomial function will extend in the same direction, either upward or downward.	2/C	**función polinómica de grado par** función polinómica en la cual el exponente mayor es un número par. Ambos extremos del gráfico de una función polinómica de grado par se extenderán en la misma dirección, hacia arriba o hacia abajo.
even function a function that, when evaluated for $-x$, results in a function that is the same as the original function; $f(-x) = f(x)$	6/B	**función par** función que, cuando se la evalúa para $-x$, tiene como resultado una función que es igual a la original; $f(-x) = f(x)$

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expected value an estimate of value that is determined by finding the product of a total value and a probability of a given event; symbolized by $E(X)$	1/F	**valor esperado** estimación de valor que se determina al encontrar el producto de un valor total y una probabilidad de un evento dado; simbolizado por $E(X)$
experimental probability the number of times an event actually occurs divided by the total number of trials, given by the formula $P(E) = \frac{\text{number of occurrences of the event}}{\text{total number of trials}}$; also called *empirical probability*	1/F	**probabilidad experimental** cantidad de veces que se produce un evento dividido por la cantidad total de pruebas, dada por la fórmula $P(E) = \frac{\text{cantidad de ocurrencias del evento}}{\text{cantidad total de pruebas}}$; también se denomina *probabilidad empírica*
experiment a process or action that has observable results	1/C	**experimento** proceso o acción con resultados observables
explicit formula for a geometric sequence a formula used to find any term in a sequence. The formula is $a_n = a_1 \bullet r^{n-1}$, where n is a positive integer that represents the number of terms in the sequence, r is the common ratio, a_1 is the value of the first term in the sequence, and a_n is the value of the nth term of the sequence.	2/E	**fórmula explícita para una secuencia geométrica** fórmula usada para encontrar algún término en una secuencia. La fórmula es $a_n = a_1 \bullet r^{n-1}$, donde n es un número entero positivo que representa la cantidad de términos en la secuencia, r es la relación común, a_1 es el valor del primer término de la secuencia y a_n es el valor del término n de la secuencia.
exponential equation an equation of the form $y = ab^x$, where x is the independent variable, y is the dependent variable, and a and b are real numbers	2/D 6/A	**ecuación exponencial** ecuación de la forma $y = ab^x$, en la que x es la variable independiente, y es la variable dependiente, y a y b son números reales
exponential expression an expressionthat contains a base raised to a power/ exponent	2/A	**expresión exponencial** expresión que incluye una base elevada a una potencia o exponente
exponential function a function of the form $f(x) = ab^{cx}$, in which a, b, and c are constants; a function that has a variable in the exponent, such as $f(x) = 0.4 \bullet 6^{-0.2x}$	5/B 6/D	**función exponencial** función de la fórmula $f(x) = ab^{cx}$ en la cual a, b y c son constantes; una función que tiene una variable en el exponente, tal como $f(x) = 0{,}4 \bullet 6^{-0{,}2x}$

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extraneous solution a solution of an equation that arises during the solving process, but which is not a solution of the original equation	3/B	**solución extraña** solución de una ecuación que surge durante el proceso de resolución pero que no es una solución de la ecuación original
extrema the minima or maxima of a function	6/C	**extremos** los mínimos o máximos de una función
	F	
factor one of two or more numbers or expressions that when multiplied produce a given product	2/A	**factor** uno de dos o más números o expresiones que cuando se multiplican generan un producto determinado
factorial the product of an integer and all preceding positive integers, represented using a ! symbol; $n! = n \bullet (n-1) \bullet (n-2) \bullet \cdots \bullet 1$. For example, 5! = 5 • 4 • 3 • 2 • 1. By definition, 0! = 1.	1/B 1/D 2/B	**factorial** producto de un entero y todos los enteros positivos anteriores, que se representa con el símbolo !; $n! = n \bullet (n-1) \bullet (n-2) \bullet \ldots \bullet 1$. Por ejemplo, 5! = 5 • 4 • 3 • 2 • 1. Por definición, 0! = 1.
factor of a polynomial any polynomial that divides evenly into the function $p(x)$	2/C	**factor de un polinomio** todo polinomio que divide de manera uniforme en la función $p(x)$
Factor Theorem The binomial $x - a$ is a factor of the polynomial $p(x)$ if and only if $p(a) = 0$.	2/C	**Teorema del Factor** el binomio $x - a$ es un factor del polinomio $p(x)$ si y solo si $p(a) = 0$.
failure the occurrence of an event that was not sought out or wanted, represented by q; also known as *undesirable outcome* or *unfavorable outcome*	1/D	**fracaso** ocurrencia de un evento que no fue buscado ni deseado, representado por q; también conocido como *resultado no deseado* o *resultado desfavorable*
fair describes a situation or game in which all of the possible outcomes have an equal chance of occurring	1/F	**equitativo** describe una situación o juego en el cual todos los resultados posibles tienen igual probabilidad de producirse
false negative result a determination that an experiment has produced an incorrect negative result	1/F	**resultado falso negativo** determinación de que un experimento ha producido un resultado negativo incorrecto
false positive result a determination that an experiment has produced an incorrect positive result	1/F	**resultado falso positivo** determinación de que un experimento ha producido un resultado positivo incorrecto

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family of functions a set of functions whose graphs have the same general shape as their parent function. The parent function is the function with a simple algebraic rule that represents the family of functions.	6/B	**familia de funciones** conjunto de funciones cuyas gráficas tienen la misma forma general que su función raíz. La función raíz es la función con una regla algebraica simple que representa la familia de funciones.
favorable outcome the data sought or hoped for, represented by *p*; also known as *desirable outcome* or *success*	1/D	**resultado favorable** datos buscados o esperados, representados por *p*; también conocido como *resultado deseado* o *éxito*
finite limited in number	2/E	**finito** limitado en número
finite geometric series a series that has a limited or definite number of terms; can be written as $\sum_{k=1}^{n} a_1 r^{k-1}$, where *n* is a positive integer that represents the number of terms in the series, a_1 is the first term, *r* is the common ratio, and *k* is the number of the term	2/E	**serie geométrica finita** serie que tiene una cantidad limitada o definida de términos; puede ser escrita como $\sum_{k=1}^{n} a_1 r^{k-1}$, donde *n* es un número entero positivo que representa la cantidad de términos en la serie, a_1 es el primer término, *r* es la relación común y *k* es el número del término
formula a mathematical statement of the relationship between two or more variables	6/A	**fórmula** afirmación matemática de la relación entre dos o más variables
fraction a ratio of two expressions or quantities	3/A	**fracción** relación de dos expresiones o cantidades
frequency of a periodic function the reciprocal of the period for a periodic function; indicates how often the function repeats	4/C	**frecuencia de una función periódica** recíproca del período para una función periódica; indica con que frecuencia se repite la función
function a relation in which every element of the domain is paired with exactly one element of the range; that is, for every value of *x*, there is exactly one value of *y*	5/A	**función** relación en la que cada elemento del dominio se empareja con un único elemento del rango; es decir, para cada valor de *x*, existe exactamente un valor de *y*

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Fundamental Theorem of Algebra If $p(x)$ is a polynomial function of degree $n \geq 1$ with complex coefficients, then the related equation $p(x) = 0$ has at least one complex solution (root).	2/C	**Teorema fundamental del álgebra** Si $P(x)$ es una función polinómica de grado $n \geq 1$ con coeficientes complejos, entonces la ecuación relacionada $P(x) = 0$ tiene al menos una solución compleja (raíz).
	G	
geometric sequence an ordered list of terms in which each new term is the product of the preceding term and a common ratio, r. For a sequence to be geometric, r cannot be equal to 1 ($r \neq 1$).	2/E	**secuencia geométrica** lista ordenada de términos en la cual cada término nuevo es el producto del término anterior y una relación común, r. Para que una secuencia sea geométrica, r no puede ser igual a 1 ($r \neq 1$).
geometric series the sum of a specified number of terms from a geometric sequence	2/E	**serie geométrica** suma de una cantidad específica de términos de una secuencia geométrica
	H	
half plane a region containing all points on one side of a boundary, which is a line or curve that continues in both directions infinitely. The line or curve may or may not be included in the region. A half plane can be used to represent a solution to an inequality statement.	6/A	**semiplano** región que contiene todos los puntos en un lado de un límite, que es una línea recta o curva que continúa en ambas direcciones de manera infinita. La línea recta o curva puede o no estar incluida en la región. Un semiplano puede utilizarse para representar una solución a una afirmación de desigualdad.
hypothesis a statement that you are trying to prove or disprove	1/E	**hipótesis** afirmación que usted intenta probar o desaprobar
hypothesis testing assessing data in order to determine whether the data supports (or fails to support) the hypothesis as it relates to a parameter of the population	1/E	**prueba de hipótesis** evaluación de datos para determinar si los datos respaldan (o no respaldan) la hipótesis mientras se relaciona con un parámetro de la población

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	I	
imaginary number any number of the form *bi*, where *b* is a real number, $i=\sqrt{-1}$, and $b \neq 0$	2/B	**número imaginario** cualquier número de la forma *bi*, en el que *b* es un número real, $i=\sqrt{-1}$, y $b \neq 0$
imaginary unit, *i* the letter *i*, used to represent the non-real value $i=\sqrt{-1}$	2/B	**unidad imaginaria, *i*** la letra *i*, utilizada para representar el valor no real $i=\sqrt{-1}$
included angle the angle between two sides	4/B	**ángulo incluido** ángulo entre dos lados
independent system a system of equations with a finite number of points of intersection	2/D	**sistema independiente** sistema de ecuaciones con una cantidad finita de puntos de intersección
independent variable labeled on the *x*-axis; the quantity that changes based on values chosen; the input variable of a function	6/A	**variable independiente** designada en el eje *x*; cantidad que cambia según los valores seleccionados; variable de entrada de una función
inequality a mathematical statement that compares the value of an expression in one independent variable to the value of a dependent variable using the comparison symbols >, <, ≥, and ≤	6/A	**desigualdad** afirmación matemática que compara el valor de una expresión en una variable independiente con el valor de una variable dependiente usando los símbolos de comparación >, <, ≥ y ≤
inference a conclusion reached upon the basis of evidence and reasoning	1/B	**inferencia** conclusión alcanzada sobre la base de evidencia y razonamiento
infinite geometric series a series that has an unlimited number of terms $(n=\infty)$; can be written as $\sum_{k=1}^{\infty} a_1 r^{k-1}$, where a_1 is the first term, *r* is the common ratio, and *k* is the number of the term	2/E	**serie geométrica infinita** serie que tiene una cantidad ilimitada de términos $(n=\infty)$; se puede escribir como $\sum_{k=1}^{\infty} a_1 r^{k-1}$, donde a_1 es el primer término, *r* es la relación común y *k* es el número del término
infinite having no limit; represented by the infinity symbol, ∞	2/E	**infinito** no tiene límite; representado por el símbolo de infinitud ∞
initial condition a constraint or limit on a function or domain value that exists in the form of a *y*-intercept or other starting-point mathematical restraint in a real-world problem or solution	6/C	**condición inicial** restricción o límite en el valor de una función o de un dominio que existe en la forma de un intercepto de *y* u otra restricción matemática del punto de inicio en un problema o solución del mundo real

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initial side the stationary ray of an angle from which the measurement of the angle starts	4/A	**lado inicial** rayo fijo de un ángulo desde el cual comienza la medición del ángulo
integer a number that is not a fraction or decimal	2/C	**entero** un número que no es una fracción ni un decimal
Integral Zero Theorem If the coefficients of a polynomial function are integers such that $a_n = 1$ and $a_0 \neq 0$, then any rational zeros of the function must be factors of a_0.	2/C	**Teorema Integral Cero** si los coeficientes de una función polinómica son números enteros tales como $a_n = 1$ y $a_0 \neq 0$, entonces todos los ceros racionales de la función deben ser factores de a_0.
interval a set of values between a lower bound and an upper bound	1/A	**intervalo** conjunto de valores entre un límite inferior y un límite superior
inverse function the function that may result from switching the x- and y-variables in a given function; the inverse of $f(x)$ is written as $f^{-1}(x)$	5/A	**función inversa** función que se produce como resultado de cambiar las variables x y y en una función determinada; la inversa de $f(x)$ se expresa como $f^{-1}(x)$
inverse relation a relation $g(x)$ such that $g(f(x)) = x$ and $f(g(y)) = y$ where $f(x)$ is a function	5/A	**relación inversa** una relación $g(x)$ tal que $g(f(x)) = x$ y $f(g(y)) = y$ donde $f(x)$ es una función
Irrational Root Theorem If a polynomial $p(x)$ has rational coefficients and $a + b\sqrt{c}$ is a root of the polynomial equation $p(x) = 0$, where a and b are rational and $\sqrt{c}$ is irrational, then $a - b\sqrt{c}$ is also a root of $p(x) = 0$.	2/C	**Teorema de la Raíz Irracional** Si un polinomio $p(x)$ tiene coeficientes racionales y $a + b\sqrt{c}$ es una raíz de la ecuación de polinomios $p(x) = 0$, donde a y b son racionales y $\sqrt{c}$ es irracional, entonces $a - b\sqrt{c}$ es también una raíz de $p(x) = 0$.
L		
Law of Cosines a formula for any triangle which states $c^2 = a^2 + b^2 - 2ab\cos C$, where C is the included angle in between sides a and b, and c is the nonadjacent side across from $\angle C$	4/B	**Ley de Cosenos** fórmula para todo triángulo que establece $c^2 = a^2 + b^2 - 2ab\cos C$, donde C es el ángulo incluido entre los lados a y b, y c es el lado no adyacente u opuesto $\angle C$

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Law of Sines a formula for any triangle which states $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$, where a represents the measure of the side opposite $\angle A$, b represents the measure of the side opposite $\angle B$, and c represents the measure of the side opposite $\angle C$	4/B	**Ley de Senos** fórmula para todo triángulo que establece $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$, donde a representa la medida del lado opuesto $\angle A$, b representa la medida del lado opuesto $\angle B$ y c representa la medida del lado opuesto $\angle C$
leading coefficient the coefficient of the term with the highest power	2/A	**coeficiente líder** coeficiente del término con la mayor potencia
least common denominator (LCD) the least common multiple of the denominators of two or more fractions	3/A 3/B	**denominador común mínimo (DCM)** múltiplo común mínimo de los denominadores de dos o más fracciones
level of confidence the probability that a parameter's value can be found in a specified interval; also called *confidence level*	1/D 1/E	**grado de confianza** probabilidad de que se pueda encontrar el valor de un parámetro en un intervalo específico; también llamado *nivel de confianza*
like terms terms that contain the same variables raised to the same power	2/A	**términos semejantes** términos que contienen las mismas variables elevadas a la misma potencia
linear equation an equation that can be written in the form $ax + by = c$, where a, b, and c are constants; can also be written as $y = mx + b$, in which m is the slope and b is the y-intercept. The graph of a linear equation is a straight line; its solutions are the infinite set of points on the line.	2/D 6/A	**ecuación lineal** ecuación que puede expresarse en la forma $ax + by = c$, donde a, b y c son constantes; también puede escribirse como $y = mx + b$, en donde m es la pendiente y b es el intercepto de y. El gráfico de una ecuación lineal es una línea recta; sus soluciones son el conjunto infinito de puntos en la línea.
linear function a function that can be written in the form $ax + by = c$, where a, b, and c are constants; can also be written as $f(x) = mx + b$, in which m is the slope and b is the y-intercept. The graph of a linear function is a straight line; its solutions are the infinite set of points on the line.	6/D	**función lineal** función que puede expresarse en la forma $ax + by = c$, donde a, b y c son constantes; también puede escribirse como $f(x) = mx + b$, donde m es la pendiente y b es el intercepto de y. El gráfico de una función lineal es una línea recta; sus soluciones son el conjunto infinito de puntos en la línea.

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local maximum the greatest value of a function for a particular interval of the function; also known as a *relative maximum*	2/C 6/C	**máximo local** el mayor valor de una función para un intervalo específico de la función; también conocido como *máximo relativo*
local minimum the least value of a function for a particular interval of the function; also known as a *relative minimum*	2/C 6/C	**mínimo local** el menor valor de una función para un intervalo específico de la función; también conocido como *mínimo relativo*
logarithmic equation an equation of the form $y = \log_a x$, which is an inverse of the exponential equation $y = a^x$ if $x > 0$	6/A	**ecuación logarítmica** ecuación de la forma $y = \log_a x$, que es una inversa de la ecuación exponencial $y = a^x$ si $x > 0$
logarithmic function the inverse of an exponential function; for the exponential function $g(x) = 5^x$, the inverse logarithmic function is $x = \log_5 g(x)$	5/B	**función logarítmica** la inversa de una función exponencial; para la función exponencial $g(x) = 5^x$, la función logarítmica inversa es $x = \log_5 g(x)$
M		
margin of error the quantity that represents the level of confidence in a calculated parameter, abbreviated MOE. The margin of error can be calculated by multiplying the critical value by the standard deviation, if known, or by the SEM.	1/D	**margen de error** cantidad que representa el nivel de confianza en un parámetro calculado, abreviado MOE. El margen de error puede calcularse multiplicando el valor crítico por la desviación estándar, si se conoce, o por el SEM.
maximum the greatest value or highest point of a function	4/C	**máximo** el mayor valor o punto más alto de una función
mean a measure of center in a set of numerical data, computed by adding the values in a data set and then dividing the sum by the number of values in the data set; denoted as the Greek lowercase letter *mu*, μ; given by the formula $\mu = \frac{x_1 + x_2 + \ldots + x_n}{n}$, where each *x*-value is a data point and *n* is the total number of data points in the set	1/A	**media** medida del centro en un conjunto de datos numéricos, calculada al sumar los valores en un conjunto de datos y luego al dividir la suma por el número de valores en el conjunto de datos; indicada con la letra griega minúscula *mu*, μ; dada por la fórmula $\mu = \frac{x_1 + x_2 + \ldots + x_n}{n}$, donde cada valor de x es un punto de datos y *n* es la cantidad total de puntos de datos en el conjunto

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measurement bias bias that occurs when the tool used to measure the data is not accurate, current, or consistent	1/E	**sesgo de medición** sesgo que se produce cuando la herramienta utilizada para medir los datos no es exacta, actual o constante
median the middle-most value of a data set; 50% of the data is less than this value, and 50% is greater than it	1/A	**mediana** valor máximo-medio de un conjunto de datos; el 50 % de los datos es menor que este valor y el otro 50 % es mayor que él
midline in a cosine function or sine function of the form $f(x) = a + \sin x$ or $g(x) = a + \cos x$, a horizontal line of the form $y = a$ that bisects the vertical distance on a graph between the minimum and maximum function values	4/C 5/C	**línea media** en una función del coseno o en una función del seno de la forma $f(x) = a + \sin x$ o $g(x) = a + \cos x$, una línea horizontal de la forma $y = a$ que divide en dos la distancia vertical en un gráfico entre los valores de funciones mínimos y máximos
minimum the least value or lowest point of a function	4/C	**mínimo** el menor valor o el punto más bajo de una función
mu, μ a Greek letter used to represent mean	1/A	***mu,*** μ letra griega usada para representar la media
multiplicity (of a zero) the number of times a zero of a polynomial function occurs	2/C	**multiplicidad (de un cero)** la cantidad de veces que sucede cero en una función polinómica
mutually exclusive events events that have no outcomes in common. If A and B are mutually exclusive events, then they cannot both occur.	1/D	**eventos mutuamente excluyent es** eventos que no tienen resultados en común. Si A y B son eventos mutuamente excluyentes, entonces no pueden producirse ambos.
N		
natural logarithm a logarithm whose base is the irrational number e; usually written in the form "ln," which means "$\log_e$."	5/B	**logaritmo natural** logaritmo cuya base es el número irracional e; escrito normalmente en la forma "ln", que significa "$\log_e$".
negatively skewed a distribution in which there is a "tail" of isolated, spread-out data points to the left of the median. "Tail" describes the visual appearance of the data points in a histogram. Data that is negatively skewed is also called *skewed to the left*.	1/A	**sesgado negativamente** distribución en la cual existe una "cola" de puntos de datos aislados y esparcidos a la izquierda de la mediana. La "cola" describe la apariencia visual de los puntos de datos en un histograma. Los datos que están sesgados negativamente también se denominan *sesgados a la izquierda*.

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neutral not biased or skewed toward one side or another; regarding surveys, neutral refers to phrasing questions in a way that does not lead the response toward one particular answer or side of an issue	1/C	**neutral** no sesgado hacia un lado u otro; respecto de las encuestas, neutral se refiere a la formulación de preguntas de una manera que no conduzca la respuesta hacia una respuesta o lado específico de un tema
nonresponse bias bias that occurs when the respondents to a survey have different characteristics than nonrespondents, causing the population that does not respond to be underrepresented in the survey's results	1/E	**sesgo sin respuesta** sesgo que se produce cuando los encuestados de una encuesta tienen características diferentes de los no encuestados, dando pie a que la población que no responde sea subrepresentada en los resultados de la encuesta
normal curve a symmetrical curve representing the normal distribution	1/A	**curva normal** curva simétrica que representa la distribución normal
normal distribution a set of values that are continuous, are symmetric to a mean, and have higher frequencies in intervals close to the mean than equal-sized intervals away from the mean	1/A	**distribución normal** conjunto de valores que son continuos, simétricos a una media y tienen frecuencias más altas en intervalos cercanos a la media que los intervalos de igual tamaño lejos de la media
null hypothesis the statement or idea that will be tested, represented by H_0; generally characterized by the concept that there is no relationship between the data sets, or that the treatment has no effect on the data	1/E	**hipótesis nula** afirmación o idea que será probada, representada por H_0; caracterizada generalmente por el concepto de que no hay relación entre los conjuntos de datos o que el tratamiento no tiene efecto en los datos
numerator the value located above the line of a rational expression or fraction; the dividend	3/A	**numerador** el valor ubicado por encima de la línea de una expresión racional o fracción; el dividendo
	O	
oblique triangle a triangle that does not contain a right angle	4/B	**triángulo oblicuo** triángulo que no contiene un ángulo recto
observational study a study in which all data, including observations and measurements, are recorded in a way that does not change the subject that is being measured or studied	1/C	**estudio observacional** estudio en el cual todos los datos, incluyendo las observaciones y las mediciones, están registrados de tal manera que no cambian el objeto que está siendo medido o estudiado

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odd-degree polynomial function a polynomial function in which the highest exponent is an odd number. One end of the graph of an odd-degree polynomial function will extend upward and the other end will extend downward.	2/C	**función polinómica de grado impar** función polinómica en la cual el exponente mayor es un número impar. Un extremo del gráfico de una función polinómica de grado impar se extenderá hacia arriba y el otro extremo se extenderá hacia abajo.
odd function a function that, when evaluated for $-x$, results in a function that is the opposite of the original function; $f(-x) = -f(x)$	6/B	**función impar** función que, cuando se evalúa para $-x$, tiene como resultado una función que es lo opuesto a la función original; $f(-x) = -f(x)$
one-tailed test a *t*-test performed on a set of data to determine if the data could belong in one of the tails of the bell-shaped distribution curve; with this test, the area under only one tail of the distribution is considered	1/E	**prueba de una cola o unilateral** una prueba *t* realizada en un conjunto de datos para determinar si estos podrían pertenecer a una de las colas de la curva de distribución con forma de campana; con esta prueba, solo se considera el área debajo de una cola de la distribución
one-to-one correspondence the feature of a function whereby each value in the domain corresponds to a unique function value; that is, if $x = a$ and $x = b$, the two points would be $(a, f(a))$ and $(b, f(b))$, and if $a \neq b$, then $f(a) \neq f(b)$ for a function to exhibit one-to-one correspondence	5/A	**correspondencia uno a uno** la característica de una función por la cual cada valor del dominio corresponde a un único valor de función; es decir, si $x = a$ y $x = b$, los dos puntos serían $(a, f(a))$ y $(b, f(b))$, y si $a \neq b$, entonces $f(a) \neq f(b)$ para que una función exhiba una correspondencia uno a uno
ordered pair a point (x, y) on a two-dimensional coordinate plane that represents the value of an independent variable (x) that results in a specific dependent variable value (y). The term also refers to solutions for an equation or inequality in one variable that originate from a real-world situation. An ordered pair is also called a *data point*.	6/A	**par ordenado** un punto (x, y) en un plano de coordenadas de dos dimensiones que representa el valor de una variable independiente (x) que da como resultado un valor específico de variable dependiente (y). El término también se refiere a las soluciones para una ecuación o desigualdad en una variable que proviene de una situación del mundo real. Un par ordenado también se denomina un *punto de datos*.
outcome the observable result of an experiment	1/C	**resultado** producto observable de un experimento
outlier a value far above or below other values of a distribution	1/A	**valor atípico** valor muy por encima o muy por debajo de otros valores de una distribución

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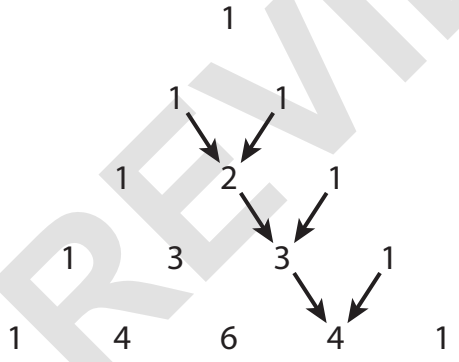
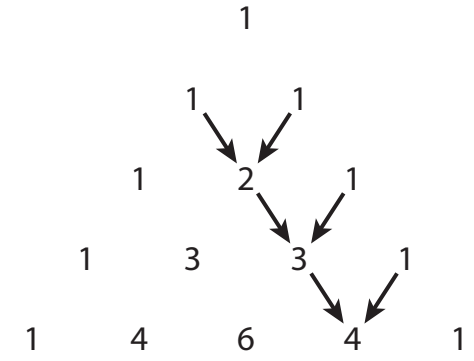
	P	
parameter numerical value(s) representing the data in a set, including proportion, mean, and variance	1/B 1/D 1/E	**parámetro** valores numéricos que representan los datos en un conjunto, incluyendo la proporción, la media y la varianza
parent function a function with a simple algebraic rule that represents a family of functions. The graphs of the functions in the family have the same general shape as the parent function.	5/C 6/A 6/B	**función principal** función con una regla algebraica simple que representa una familia de funciones. Los gráficos de las funciones en la familia tienen la misma forma general que la función principal.
partial sum the sum of part of a series	2/E	**suma parcial** la suma de la parte de una serie
Pascal's Triangle a triangle displaying a pattern of numbers in which the terms in additional rows are found by adding pairs of terms in previous rows, so that any given term is the sum of the two terms directly above it. The number 1 is the top number of the triangle, and is also the first and last number of each row.  	2/B	**triángulo de Pascal** triángulo que muestra un patrón de números en el cual los términos se encuentran en filas adicionales al agregar pares de términos en filas anteriores, de manera que cualquier término dado es la suma de los dos términos directamente por encima de este. El número 1 es el número superior del triángulo y también es el primero y el último de cada fila. 
period in a cosine or sine function graph, the horizontal distance from a maximum to a maximum or from a minimum to a minimum; one repetition of the period of a function is called a *cycle*	4/C 5/C	**período** en una curva de la función del seno o coseno, distancia horizontal desde un máximo a un máximo o desde un mínimo a un mínimo; una repetición del período de una función se llama *ciclo*

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periodic function a function whose values repeat at regular intervals	4/C 6/C	**función periódica** función cuyos valores se repiten a intervalos regulares
periodic phenomena real-life situations that repeat at regular intervals and can be represented by a periodic function	4/C	**fenómenos periódicos** situaciones de la vida real que se repiten a intervalos regulares y se pueden representar mediante una función periódica
phase shift on a cosine or sine function graph, the horizontal distance by which the curve of a parent function is shifted by the addition of a constant or other expression in the argument of the function	5/C	**cambio de fase** en una gráfica de la función del seno o del coseno, distancia horizontal por la cual la curva de una función raíz se cambia por la adición de una constante u otra expresión en el argumento de la función
piecewise function a function that is defined by two or more expressions on separate portions of the domain	6/D	**función por partes** función definida por dos o más expresiones en porciones separadas del dominio
placebo a substance that is used as a control in testing new medications; the substance has no medicinal effect on the subject	1/C	**placebo** sustancia que se utiliza como control en las pruebas de medicamentos nuevos; la sustancia no tiene efecto medicinal sobre el sujeto
plane a flat, two-dimensional figure without depth that has at least three non-collinear points and extends infinitely in all directions	6/E	**plano** figura plana, bidimensional, sin profundidad, que tiene al menos tres puntos no colineales y se extiende infinitamente en todas direcciones
plane figure a two-dimensional shape on a plane	6/E	**figura plana** forma bidimensional sobre un plano
point(s) of intersection in a graphed system of equations, the ordered pair(s) where graphed functions intersect on a coordinate plane	2/D 3/B	**punto(s) de intersección** en un sistema de ecuaciones graficado, par(es) ordenado(s) donde las funciones de la gráfica se cruzan sobre un plano de coordenadas
polyhedron a three-dimensional object that has faces made of polygons	6/E	**poliedro** objeto tridimensional que tiene caras compuestas por polígonos
polynomial an expression that contains variables, numeric quantities, or both, where variables are raised to integer powers greater than or equal to 0	2/A	**polinomio** expresión que contiene variables, cantidades numéricas o ambas en donde las variables se elevan a potencias de números enteros mayores o iguales a 0

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polynomial equation an equation of the general form $y = a_n x^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$, where a_1 is a rational number, $a_n \neq 0$, and n is a nonnegative integer and the highest degree of the polynomial	2/D	**ecuación polinómica** ecuación de la forma general $y = a_n x^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$, donde a_1 es un número racional, $a_n \neq 0$ y n es un número entero no negativo y el grado más alto del polinomio
polynomial function a function of the general form $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$, where a_1 is a rational number, $a_n \neq 0$, and n is a nonnegative integer and the highest degree of the polynomial	2/A 2/C	**función polinómica** función de la forma general $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$, donde a_1 es un número racional, $a_n \neq 0$ y n es un número entero no negativo y el grado más alto del polinomio
polynomial identity a true equation that is often generalized so it can apply to more than one example	2/B	**identidad del polinomio** ecuación verdadera que suele generalizarse para que pueda aplicarse a más de un ejemplo
population all of the people, objects, or phenomena of interest in an investigation; the entire data set	1/D	**población** todas las personas, los objetos o fenómenos de interés en una investigación; el conjunto completo de datos
population all of the people, objects, or phenomena of interest in an investigation	1/A 1/B 1/D	**población** todas las personas, los objetos o fenómenos de interés en una investigación
population average the sum of all quantities in a population, divided by the total number of quantities in the population; typically represented by μ; also known as *population mean*	1/D	**promedio de la población** suma de todas las cantidades de una población, dividida por el número total de cantidades de la población; representada normalmente por μ; también se conoce como *media poblacional*
population mean the sum of all quantities in a population, divided by the total number of quantities in the population; typically represented by μ; also known as *population average*	1/D	**media poblacional** suma de todas las cantidades de una población, dividida por el número total de cantidades de la población; representada normalmente μ; también se conoce como *promedio de la población*

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positively skewed a distribution in which there is a "tail" of isolated, spread-out data points to the right of the median. "Tail" describes the visual appearance of the data points in a histogram. Data that is positively skewed is also called *skewed to the right*.	1/A	**sesgado positivamente** distribución en la cual existe una "cola" de puntos de datos aislados esparcidos hacia la derecha de la mediana. La "cola" describe la apariencia de los puntos de datos en un histograma. Los datos que están positivamente sesgados también se denominan *sesgados a la derecha*.
power the quantity that shows the number of times the base is being multiplied by itself in an exponential expression, such as x in the logarithmic function $x = \log_5 g(x)$ and its exponential function, $g(x) = 5^x$	5/B	**potencia** cantidad que muestra el número de veces que se multiplica la base por sí misma en una expresión exponencial, tal que x en la función logarítmica $x = \log_5 g(x)$ y su función exponencial $g(x) = 5^x$
probability distribution the values of a random variable with associated probabilities	1/A	**distribución de probabilidad** los valores de una variable aleatoria con probabilidades asociadas
proportion a statement of equality between two ratios	3/B	**proporción** afirmación de igualdad entre dos relaciones
***p*-value** a number between 0 and 1 that determines whether to accept or reject the null hypothesis	1/E	**valor *p*** número entre 0 y 1 que determina si se acepta o se rechaza la hipótesis nula
Q		
quadratic equation an equation that can be written in the form $ax^2 + bx + c = 0$, where x is the variable, a, b, and c are constants, and $a \neq 0$	2/D 3/A 6/A	**ecuación cuadrática** ecuación que se puede expresar en la forma $ax^2 + bx + c = 0$, donde x es la variable, a, b, y c son constantes, y $a \neq 0$
quadratic expression an algebraic expression that can be written in the form $ax^2 + bx + c$, where x is the variable, a, b, and c are constants, and $a \neq 0$	3/A	**expresión cuadrática** expresión algebraica que se puede expresar en la forma $ax^2 + bx + c$, donde x es la variable, a, b, y c son constantes, y $a \neq 0$

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quadratic formula a formula that states the solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. A quadratic equation in this form can have no real solutions, one real solution, or two real solutions.	3/A	**fórmula cuadrática** fórmula que establece que las soluciones de una ecuación cuadrática de la forma $ax^2 + bx + c = 0$ están dadas por $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Una ecuación cuadrática en esta forma tener ningún solución real, o tener una solución real, o dos soluciones reales.
quadratic function a function defined by a second-degree expression of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$ and a, b, and c are constants. The graph of any quadratic function is a parabola.	5/A 6/D	**función cuadrática** función definida por una expresión de segundo grado de la forma $f(x) = ax^2 + bx + c$, donde $a \neq 0$ y a, b y c son constantes. La representación gráfica de toda función cuadrática es una parábola.
	R	
radian the measure of the central angle that intercepts an arc equal in length to the radius of the circle; π radians = 180°	4/A 5/C	**radián** medida del ángulo central que intercepta un arco de longitud igual al radio del círculo; π radianes = 180°
radical equation an algebraic equation in which at least one term includes a radical expression	3/B	**ecuación radical** ecuación algebraica en la cual al menos un término incluye una expresión radical
radical expression an expression containing a root	3/B	**expresión radical** expresión que contiene una raíz
random the designation of a group or sample that has been formed without following any kind of pattern and without bias. Each group member has been selected without having more of a chance than any other group member of being chosen.	1/C	**aleatorio** designación de un grupo o muestra que se formó sin seguir ninguna clase de patrón y sin sesgo. Cada miembro del grupo se seleccionó sin tener más probabilidades de ser elegido que cualquier otro miembro del grupo.

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randomization the selection of a group, subgroup, or sample without following a pattern, so that the probability of any item in the set being generated is equal; the process used to ensure that a sample best represents the population	1/C	**aleatorización** selección de un grupo, subgrupo o muestra sin seguir un patrón, de manera que la probabilidad de cualquier elemento en el conjunto que está siendo generado sea igual; proceso utilizado para asegurar que la muestra sea la que mejor represente a la población
random number generator a tool used to select a number without following a pattern, where the probability of generating any number in the set is equal	1/B 1/F	**generador de números aleatorios** herramienta utilizada para seleccionar un número sin seguir un patrón, donde la probabilidad de generar cualquier número en el conjunto es igual
random sample a subset or portion of a population or set that has been selected without bias, with each item in the population or set having the same chance of being found in the sample	1/B 1/D	**muestra aleatoria** subconjunto o porción de población o conjunto que ha sido seleccionado sin sesgo, con cada elemento de la población o conjunto con la misma probabilidad de encontrarse en la muestra
random variable a variable whose numerical value changes depending on each outcome in a sample space; the values of a random variable are associated with chance variation	1/A	**variable aleatoria** variable cuyo valor numérico cambia según cada resultado en un espacio de muestra; los valores de una variable aleatoria están asociados con una variación al azar
range the set of all outputs of a function; the set of *y*-values that are valid for the function	5/A 6/A	**rango** conjunto de todas las salidas de una función; conjunto de valores de *y* que son válidos para la función
rate of change a ratio that describes how much one quantity changes with respect to the change in another quantity; also known as the slope of a line	6/C	**tasa de cambio** proporción que describe cuánto cambia una cantidad con respecto al cambio de otra cantidad; también se la conoce como pendiente de una recta
ratio the relation between two quantities; can be expressed in words, fractions, decimals, or as a percentage	3/B	**proporción** relación entre dos cantidades; puede expresarse en palabras, fracciones, decimales o como porcentaje
rational equation an algebraic equation that contains at least one rational expression	3/B	**ecuación racional** una ecuación algebraica que contiene al menos una expresión racional

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rational expression an expression made of the ratio of two polynomials, in which a variable appears in the denominator	3/A	**expresión racional** expresión que resulta de la relación de dos polinomios, en la cual una variable aparece en el denominador
rational number any number that can be written as $\frac{m}{n}$, where m and n are integers and $n \neq 0$; any number that can be written as a decimal that ends or repeats	3/B	**números racionales** números que pueden expresarse como $\frac{m}{n}$, en los que m y n son enteros y $n \neq 0$; cualquier número que puede escribirse como decimal finito o periódico
Rational Root Theorem If the polynomial $p(x)$ has integer coefficients, then every rational root of the polynomial equation $p(x) = 0$ can be written in the form $\frac{p}{q}$, where p is a factor of the constant term $p(x)$ and q is a factor of the leading coefficient of $p(x)$.	2/C	**Teorema de la Raíz Racional** Si el polinomio $p(x)$ tiene coeficientes enteros, entonces toda raíz racional de la ecuación polinómica $p(x) = 0$ puede escribirse en la forma $\frac{p}{q}$, donde p es un factor del término constante $p(x)$ y q es un factor del coeficiente principal de $p(x)$.
reciprocal a number that, when multiplied by the original number, has a product of 1	3/A	**recíproco** número que multiplicado por el número original tiene producto 1
recursive formula for a geometric sequence a formula used to find the next term in a sequence. The formula is $a_n = a_{n-1} \bullet r$, where n is a positive integer that represents the number of terms in the sequence and r is the common ratio.	2/E	**fórmula recursiva para una secuencia geométrica** fórmula utilizada para encontrar el término siguiente en una secuencia. La fórmula es $a_n = a_{n-1} \bullet r$, donde n es un número entero positivo que representa la cantidad de términos en la secuencia y r es la relación común.
reference angle the acute angle that the terminal side makes with the x-axis. The sine, cosine, and tangent of the reference angle are the same as that of the original angle (except for the sign, which is based on the quadrant in which the terminal side is located).	4/A	**ángulo de referencia** el ángulo agudo que forma el lado terminal con el eje x. El seno, el coseno y la tangente del ángulo de referencia son iguales a los del ángulo original (con excepción del signo, que se basa en el cuadrante en el que se ubica el lado terminal).

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regular polyhedron a polyhedron with faces that are all congruent regular polygons; the angles created by the intersecting faces are congruent, and the cross sections are similar figures	6/E	**poliedro regular** poliedro cuyas caras son todas polígonos regulares congruentes; los ángulos creados por las caras que se cruzan son congruentes y las secciones transversales son figuras similares
relation a relationship between two variables in which at least one value of the domain or independent variable, x, is matched with one or more values of the dependent or range variable, y	5/A	**relación** relación entre dos variables en la que al menos un valor del dominio o variable independiente, x, concuerda con uno o más valores de la variable de rango o dependiente, y
relative maximum the greatest value of a function for a particular interval of the function; also known as a *local maximum*	2/C 6/C	**máximo relativo** el mayor valor de una función para un intervalo en particular de la función; también conocido como *máximo local*
relative minimum the least value of a function for a particular interval of the function; also known as a *local minimum*	2/C 6/C	**mínimo relativo** el menor valor de una función para un intervalo en particular de la función; también conocido como *mínimo local*
reliability the degree to which a study or experiment performed many times would have similar results	1/B	**confiabilidad** grado en el cual un estudio o experimento realizado varias veces tendría resultados similares
Remainder Theorem For a polynomial $p(x)$ and a number a, dividing $p(x)$ by $x - a$ results in a remainder of $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	2/C	**Teorema del Resto** para un polinomio $p(x)$ y un número a, dividiendo $p(x)$ por $x - a$ resulta un resto de $p(a)$, entonces $p(a) = 0$ si y solo si $(x - a)$ es un factor de $p(x)$.
repeated root a polynomial function with a root that occurs more than once	2/C	**raíz repetida** función polinómica con una raíz que aparece más de una vez
representative sample a sample in which the characteristics of the people, objects, or items in the sample are similar to the characteristics of the population	1/B	**muestra representativa** muestra en la cual las características de las personas, los objetos o elementos en ella son similares a las características de la población
response bias bias that occurs when responses by those surveyed have been influenced in some manner	1/E	**sesgo de respuesta** sesgo que se produce cuando las respuestas de los encuestados fueron influenciadas de alguna manera
restricted domain a subset of a function's defined domain	6/A	**dominio restringido** subconjunto del dominio definido de una función
***rho* (ρ)** a lowercase Greek letter commonly used to represent density	6/E	***rho* (ρ)** letra griega minúscula comúnmente utilizada para representar densidad

English	Unit/Topic	Español
root the *x*-intercept of a function; also known as *zero*	2/C	**raíz** intercepto de *x* de una función; también conocida como *cero*
rotation in three dimensions, a transformation in which a plane figure is moved about one of its sides, a fixed point, or a line that is not located in the plane of the figure, such that a solid figure is produced	6/E	**rotación** en tres dimensiones, transformación en la cual una figura plana se mueve sobre uno de sus lados, un punto fijo o una línea que no está ubicada en el plano de la figura, de manera que se produce una figura sólida
S		
sample a subset of the population	1/A 1/B	**muestra** subconjunto de la población
sample average the sum of all quantities in a sample divided by the total number of quantities in the sample, typically represented by $\overline{x}$; also known as *sample mean*	1/D	**promedio de la muestra** suma de todas las cantidades en una muestra dividida por el número total de cantidades en la muestra, normalmente representada por $\overline{x}$; también se conoce como *media de la muestra*
sample mean the sum of all quantities in a sample divided by the total number of quantities in the sample, typically represented by $\overline{x}$; also known as *sample average*	1/D	**media de la muestra** suma de todas las cantidades en una muestra dividida por el número total de cantidades en la muestra, normalmente representada por $\overline{x}$; también se conoce como *promedio de la muestra*
sample population a portion of the population; the number of elements or observations in a sample population is represented by *n*	1/D	**población de la muestra** porción de la población; la cantidad de elementos u observaciones en una población de muestra se representa por *n*
sample proportion the fraction of favorable results *p* from a sample population *n*; conventionally represented by $\hat{p}$, which is pronounced "p hat." The formula for the sample proportion is $\hat{p}=\frac{p}{n}$, where *p* is the number of favorable outcomes and *n* is the number of elements or observations in the sample population.	1/D	**proporción de la muestra** fracción de los resultados favorables *p* de una población de muestra *n*; convencionalmente representada por $\hat{p}$, que se pronuncia "p hat". La fórmula para la proporción de la muestra es $\hat{p}=\frac{p}{n}$, donde *p* es la cantidad de resultados favorables y *n* es la cantidad de elementos u observaciones en la población de la muestra.

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sample survey a survey carried out using a sampling method so that only a portion of the population is surveyed rather than the whole population	1/C	**encuesta de muestra** encuesta realizada utilizando un método de muestreo para encuestar solo una porción de la población en lugar de toda la población.
sampling bias errors in estimation caused by flawed (non-representative) sample selection	1/B	**sesgo de muestreo** errores de cálculo ocasionados por una selección defectuosa (no representativa) de la muestra
sampling error a measure showing how precisely a sample reflects the population, with smaller sampling errors resulting from large samples and/or when the data clusters closely around the mean; also called *chance variation*	1/B	**error de muestreo** medición que demuestra qué tan precisamente refleja una muestra a una población, con pequeños errores de muestreo ocasionados por muestras grandes y/o cuando los datos se agrupan estrechamente alrededor de la media; también se llama *variación aleatoria*
scale the numbers representing the interval of a variable and the increments into which it is subdivided; usually includes the interval endpoints and the increments of the basic unit of the variable	6/D	**escala** números que representan el intervalo de una variable y los incrementos en los cuales esta se subdivide; generalmente incluye los extremos del intervalo y los incrementos de la unidad básica de la variable
secant the reciprocal of cosine, $\sec\theta = \frac{1}{\cos\theta}$; the secant of θ = sec θ = $\frac{\text{length of hypotenuse}}{\text{length of adjacent side}}$	4/A	**secante** recíproca del coseno, $\sec\theta = \frac{1}{\cos\theta}$; la secante de θ = sec θ = $\frac{\text{largo de la hipotenusa}}{\text{largo del lado adyacente}}$
sequence an ordered list of numbers or elements	2/E	**secuencia** lista ordenada de números o elementos
series the sum of the terms of a sequence	2/E	**serie** suma de los términos de una secuencia
***sigma* (lowercase), σ** a Greek letter used to represent standard deviation	1/A	***sigma* (minúscula) o σ** letra griega utilizada para representar la desviación estándar
***sigma* (uppercase), Σ** a Greek letter used to represent the summation of values	1/A 2/B 2/E	***sigma* (mayúscula) o Σ** letra griega utilizada para representar la sumatoria de valores

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similar figures two figures that are the same shape but not necessarily the same size; the symbol for representing similarity between figures is $\sim$.	6/E	**figuras similares** dos figuras que tienen la misma forma pero no necesariamente el mismo tamaño; el símbolo que representa la similitud entre figuras es $\sim$.
simple random sample a sample in which any combination of a given number of individuals in the population has an equal chance of selection	1/B	**muestra aleatoria simple** muestra en la cual cualquier combinación de una cantidad dada de individuos de la población tiene iguales posibilidades de selección
simulation a set of data that models an event that could happen in real life	1/E	**simulación** conjunto de datos que imita un evento que podría suceder en la vida real
sine a trigonometric function of an acute angle in a right triangle that is the ratio of the length of the opposite side to the length of the hypotenuse; the sine of $\theta = \sin\theta = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$	4/A 4/B 5/C	**seno** función trigonométrica de un ángulo agudo en un triángulo rectángulo que es la proporción de la longitud del lado opuesto a la longitud de la hipotenusa; sen de $\theta = \text{sen}\,\theta = \frac{\text{longitud del lado opuesto}}{\text{longitud de la hipotenusa}}$
sine curve a curve with a constant amplitude and period, which are given by a sine or cosine function; also called a *sine wave* or *sinusoid*	4/C	**curva del seno** curva con amplitud y período constantes que están dados por una función seno o coseno; también se denomina *onda de seno* o *sinusoide*
sine function a trigonometric function of the form $f(x) = a \sin [b(x - c)] + d$, in which a, b, c, and d are constants and x is a variable defined in radians over the domain $(-\infty,\infty)$	4/C 5/C	**función seno** función trigonométrica de la forma $f(x) = a \sin [b(x - c)] + d$, en la cual a, b, c y d son constantes y x es una variable expresada en radianes sobre el dominio $(-\infty,\infty)$
sine wave a curve with a constant amplitude and period given by a sine or cosine function; also called a *sine curve* or *sinusoid*	4/C	**onda senoidal** curva con amplitud y período constantes dados por una función seno o coseno; también se denomina *curva del seno* o *sinusoide*
sinusoid a curve with a constant amplitude and period given by a sine or cosine function; also called a *sine curve* or *sine wave*	4/C	**sinusoide** curva con amplitud o período constantes dados por una función seno o coseno; también se denomina *curva del seno* u *onda senoidal*

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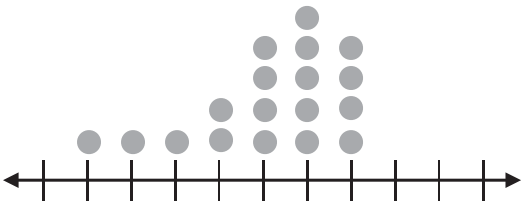
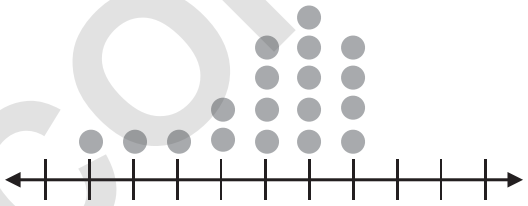
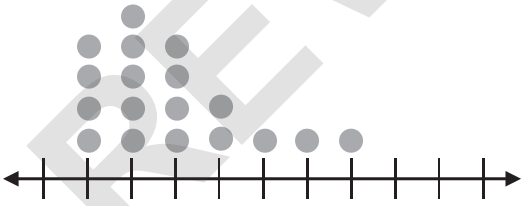
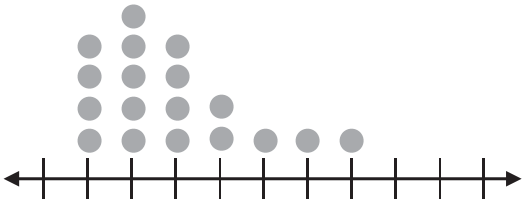
skewed to the left a distribution in which there is a "tail" of isolated, spread-out data points to the left of the median. "Tail" describes the visual appearance of the data points in a histogram. Data that is skewed to the left is also called *negatively skewed*. Example: 	1/A	**sesgado a la izquierda** distribución en la cual existe una "cola" de puntos de datos aislados extendidos hacia la izquierda de la mediana. La "cola" describe la apariencia de los puntos de datos en un histograma. Los datos sesgados a la izquierda también se denominan *negativamente sesgados*. Ejemplo: 
skew to distort or bias, as in data	1/C	**sesgar** distorsionar o afectar, como en el caso de los datos
skewed to the right a distribution in which there is a "tail" of isolated, spread-out data points to the right of the median. "Tail" describes the visual appearance of the data points in a histogram. Data that is skewed to the right is also called *positively skewed*. Example: 	1/A	**sesgado a la derecha** distribución en la cual existe una "cola" de puntos de datos aislados extendidos hacia la derecha de la mediana. La "cola" describe la apariencia de los puntos de datos en un histograma. Los datos sesgados a la derecha también se denominan *positivamente sesgados*. Ejemplo: 
solid figure a three-dimensional object that has length, width, and height (depth)	6/E	**figura sólida** objeto tridimensional que tiene largo, ancho y altura (profundidad)
solution set the set of ordered pairs that represent all of the solutions to an equation or a system of equations	3/B 6/A	**conjunto de soluciones** conjunto de pares ordenados que representa todas las soluciones para una ecuación o sistema de ecuaciones
spread refers to how data is spread out with respect to the mean; sometimes called *variability*	1/D	**dispersión** forma en que los datos se esparcen con respecto a la media; algunas veces se denomina *variabilidad*

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square root function a function that contains a square root of a variable. The general form is $f(x)=\sqrt{ax^2+bx+c}$, where a, b, and c are real numbers.	6/D	**función raíz cuadrada** función que contiene la raíz cuadrada de una variable. La forma general es $f(x)=\sqrt{ax^2+bx+c}$, donde a, b y c son números reales.
standard deviation how much the data in a given set is spread out, represented by s or σ. The standard deviation of a sample can be found using the following formula: $s=\sqrt{\frac{\sum\left(x_i-\overline{x}\right)^2}{n-1}}$. The standard deviation of a population can be found using the following formula: $\sigma=\sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}}$.	1/A 1/D	**desviación estándar** cuánto se extienden los datos en un conjunto dado, representada por s o σ. Se puede calcular la desviación estándar de una muestra utilizando la siguiente fórmula: $s=\sqrt{\frac{\sum\left(x_i-\overline{x}\right)^2}{n-1}}$. Se puede calcular la desviación estándar de una población utilizando la siguiente fórmula: $\sigma=\sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}}$.
standard error of the mean the variability of the mean of a sample; given by $\text{SEM}=\frac{s}{\sqrt{n}}$, where s represents the standard deviation and n is the number of elements or observations in the sample population	1/D	**error estándar de la media** variabilidad de la media de una muestra; dado por $\text{SEM}=\frac{s}{\sqrt{n}}$, donde s representa la desviación estándar y n la cantidad de elementos u observaciones en la población de la muestra

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standard error of the proportion the variability of the measure of the proportion of a sample, abbreviated SEP. The standard error (*SEP*) of a sample proportion $\hat{p}$ is given by the formula $SEP = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, where $\hat{p}$ is the sample proportion determined by the sample and n is the number of elements or observations in the sample population.	1/D	**error estándar de la proporción** variabilidad de la medida de la proporción de una muestra, abreviada SEP. El error estándar (*SEP*) de una proporción de la muestra $\hat{p}$ está dado por la fórmula $SEP = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, donde $\hat{p}$ es la proporción de la muestra determinada por la muestra y n representa la cantidad de elementos u observaciones en la población de la muestra.
standard normal distribution a normal distribution that has a mean of 0 and a standard deviation of 1; data following a standard normal distribution forms a normal curve when graphed	1/A	**distribución normal estándar** distribución normal que tiene una media de 0 y una desviación estándar de 1; los datos que siguen una distribución normal estándar forman una curva normal al graficarse
standard position (of an angle) a position in which the vertex of the angle is at the origin of the coordinate plane and is the center of the unit circle. The angle's initial side is located along the positive x-axis and the terminal side may be in any location.	4/A	**posición estándar (de un ángulo)** posición en la cual el vértice del ángulo está en el origen del plano de coordenadas y es el centro del círculo unitario. El lado inicial del ángulo está ubicado a lo largo del eje positivo x y el lado terminal puede estar en cualquier ubicación.
statistical significance a measure used to determine whether the outcome of an experiment is a result of the treatment being applied, as opposed to random chance	1/E	**relevancia estadística** medida utilizada para determinar si el resultado de un experimento es el resultado del tratamiento aplicado, en oposición al resultado producto del azar

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statistics a branch of mathematics focusing on how to collect, organize, analyze, and interpret information from data gathered; numbers used to summarize, describe, or represent sets of data	1/B 1/C	**estadística** rama de la matemática enfocada en la manera de recabar, organizar, analizar e interpretar la información proveniente de los datos reunidos; números utilizados para resumir, describir o representar conjuntos de datos
step function a function that is a series of disconnected constant functions	6/D	**función escalonada** función que es una serie de funciones constantes desconectadas
stratified sample a sample chosen by first dividing a population into subgroups of people or objects that share relevant characteristics, then randomly selecting members of each subgroup for the sample	1/B	**muestra estratificada** muestra escogida dividiendo primero una población en subgrupos de personas u objetos que comparten características relevantes, luego seleccionando al azar miembros de cada subgrupo para la muestra
subtended arc the section of an arc formed by a central angle that passes through the circle, thus creating the endpoints of the arc	4/A	**arco subtendido** sección de un arco formada por un ángulo central que pasa por el círculo, creando así los puntos extremos del arco
success the data sought or hoped for, represented by *p*; also known as *desirable outcome* or *favorable outcome*	1/D	**éxito** datos buscados o esperados, representados por *p*; también conocido como *resultado deseado* o *resultado favorable*
sum formula for a finite geometric series $S_n = \frac{a_1(1-r^n)}{1-r}$, where S_n is the sum, a_1 is the first term, *r* is the common ratio, and *n* is the number of terms	2/E	**fórmula de suma para una serie geométrica finita** $S_n = \frac{a_1(1-r^n)}{1-r}$ donde S_n es la suma, a_1 es el primer término, *r* es la relación común y *n* es la cantidad de términos
sum formula for an infinite geometric series $S_n = \frac{a_1}{1-r}$, where S_n is the sum, a_1 is the first term, and *r* is the common ratio	2/E	**fórmula de suma para una serie geométrica infinita** $S_n = \frac{a_1}{1-r}$, donde S_n es la suma, a_1 es el primer término y *r* es la relación común

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summation notation a symbolic way to represent a series (the sum of a sequence) using the uppercase Greek letter *sigma*, Σ	1/A 2/E	**notación sumatoria** forma simbólica de representar una serie (la suma de una secuencia) utilizando la letra griega mayúscula *sigma*, Σ
survey a study of particular qualities or attributes of items or people of interest to a researcher	1/C	**encuesta** estudio de las cualidades o atributos particulares de elementos o personas de interés para un investigador
symmetric distribution a data distribution in which a line can be drawn so that the left and right sides are mirror images of each other	1/A	**distribución simétrica** distribución de datos en la cual se puede trazar una línea de manera que los lados derecho e izquierdo sean imágenes especulares entre sí
symmetry of a function the property whereby a function exhibits the same behavior (e.g., graph shape, function values, etc.) for specific domain values and their opposites	6/C	**simetría de una función** propiedad por la cual una función exhibe el mismo comportamiento (por ej., forma de la gráfica, valores de la función, etc.) para valores específicos del dominio y sus opuestos
synthetic division a shorthand way of dividing a polynomial by a linear binomial	2/C	**división sintética** forma abreviada de dividir un polinomio por un binomio lineal
synthetic substitution the process of using synthetic division to evaluate a function by using only the coefficients	2/C	**sustitución sintética** proceso que utiliza la división sintética para evaluar una función utilizando solo los coeficientes
systematic sample a sample drawn by selecting people or objects from a list, chart, or grouping at a uniform interval; for example, selecting every fourth person	1/B	**muestra sistemática** la muestra se obtiene mediante la selección de personas u objetos a partir de una lista, una tabla o mediante la agrupación a intervalos regulares; por ej., eligiendo una de cada cuatro personas
system of equations a set of equations with the same unknowns	2/D 3/B	**sistema de ecuaciones** conjunto de ecuaciones con las mismas incógnitas

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	T	
tangent a trigonometric function of an acute angle in a right triangle that is the ratio of the length of the opposite side to the length of the adjacent side; the tangent of θ = tan θ = $\frac{\text{length of opposite side}}{\text{length of adjacent side}}$	4/A 4/B	**tangente** función trigonométrica de un ángulo agudo en un triángulo rectángulo que es la proporción de la longitud del lado opuesto a la longitud del lado adyacente; tangente de θ = tan θ = $\frac{\text{longitud del lado opuesto}}{\text{longitud del lado adyacente}}$
term an element in a sequence. In the sequence $\{a_1, a_2, a_3, ..., a_n\}$, a_1 is the first term, a_2 is the second term, a_3 is the third term, and a_n is the nth term; a number, a variable, or the product of a number and variable(s)	2/A 2/E	**término** elemento de una secuencia. En la secuencia $\{a_1, a_2, a_3, ..., a_n\}$, a_1 es el primer término, a_2 es el segundo término, a_3 es el tercer término y a_n es el término n; un número, una variable o el producto de un número y variable(s)
terminal side for an angle in standard position, the movable ray of an angle that can be in any location and which determines the measure of the angle	4/A	**lado terminal** para un ángulo en posición estándar, el rayo móvil de un ángulo que puede estar en cualquier ubicación y que determina la medida del ángulo
theoretical probability the probability that an outcome will occur as determined through reasoning or calculation, given by the formula $P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$	1/F	**probabilidad teórica** probabilidad de que un resultado se produzca como se determinó mediante razonamiento o cálculo, dado por la fórmula $P(E) = \frac{\text{cantidad de resultados en } E}{\text{cantidad de resultados en el espacio de muestreo}}$
***theta* (θ)** a Greek letter commonly used to refer to unknown angle measures	4/A	***theta* (θ)** letra griega generalmente utilizada para referirse a las medidas de un ángulo desconocido
translation in three dimensions, the horizontal or vertical movement of a plane figure in a direction that is not in the plane of the figure, such that a solid figure is produced	6/E	**traslación** en tres dimensiones, movimiento horizontal o vertical de una figura plana en una dirección que no está en el plano de la figura, de manera que se produzca una figura sólida

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treatment the process or intervention provided to the population being observed	1/E	**tratamiento** proceso o intervención efectuada sobre la población que está siendo observada
trial each individual event or selection in an experiment or treatment	1/D 1/E 1/F	**ensayo** cada evento o selección individual en un experimento o tratamiento
true negative result a determination that an experiment has produced a correct negative result	1/F	**resultado negativo verdadero** determinación de que un experimento ha producido un resultado negativo correcto
true positive result a determination that an experiment has produced a correct positive result	1/F	**resultado positivo verdadero** determinación de que un experimento ha producido un resultado positivo correcto
***t*-test** a procedure to establish the statistical significance of a set of data using the mean, standard deviation, and degrees of freedom for the sample or population	1/E	**prueba *t*** procedimiento para establecer la relevancia estadística de un conjunto de datos utilizando la media, la desviación estándar y los grados de libertad para la muestra o población
turning point a point where the graph of the function changes direction, from sloping upward to sloping downward or vice versa	2/C	**punto de inflexión** punto en el cual la gráfica de función cambia de dirección, de una inclinación o pendiente ascendente a una descendente o viceversa
***t*-value** the result of a *t*-test	1/E	**valor *t*** resultado de una prueba *t*
two-tailed test a *t*-test performed on a set of data to determine if the data could belong in either of the tails of the bell-shaped distribution curve; with this test, the area under both tails of the distribution is considered	1/E	**prueba de dos colas o prueba bilateral** prueba *t* realizada sobre un conjunto de datos para determinar si esos datos podrían pertenecer a alguna de las colas de una curva de distribución en forma de campana; con esta prueba, se tiene en cuenta el área bajo ambas colas de la distribución
U		
undesirable outcome the data not sought or hoped for, represented by *q*; also known as *unfavorable outcome* or *failure*	1/D	**resultado no deseado** datos no buscados o esperados, representados por *q*; también conocido como *resultado desfavorable* o *fracaso*
unfavorable outcome the data not sought or hoped for, represented by *q*; also known as *undesirable outcome* or *failure*	1/D	**resultado desfavorable** datos no buscados o esperados, representados por *q*; también conocido como *resultado no deseado* o *fracaso*

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uniform distribution a set of values that are continuous, are symmetric to a mean, and have equal frequencies corresponding to any two equally sized intervals. In other words, the values are spread out uniformly throughout the distribution.	1/A	**distribución uniforme** conjunto de valores que son continuos, simétricos respecto de la media y tienen frecuencias iguales que corresponden a cualquiera de dos intervalos del mismo tamaño. En otras palabras, los valores se extienden uniformemente en la distribución.
unit circle a circle with a radius of 1 unit. The center of the circle is located at the origin of the coordinate plane.	4/A 5/C	**círculo unitario** círculo con un radio de una unidad. El centro del círculo está ubicado en el origen del plano de coordenadas.
	V	
validity the degree to which the results obtained from a sample measure what they are intended to measure	1/B	**validez** el grado en el cual los resultados obtenidos de una muestra miden lo que se pretende que midan
variability refers to how data is spread out with respect to the mean; sometimes called *spread*	1/D	**variabilidad** hace referencia al modo en que se distribuyen los datos respecto de la media; algunas veces se denomina *dispersión*
voluntary response bias bias that occurs when the sample is not representative of the population due to the sample having the option of responding to the survey	1/E	**sesgo de respuesta voluntaria** sesgo que se produce cuando la muestra no es representativa de la población debido a que en la muestra existe la opción de responder a la encuesta (la respuesta es optativa)
	Z	
z_c-value a measure of the number of standards of error to be added or subtracted from the mean in order to achieve the desired confidence level; also known as *critical value*	1/D	**valor-z_c** medida de la cantidad de estándares de error a sumar o restar de la media para alcanzar el nivel de confianza deseado; conocido también como *valor crítico*
zero the *x*-intercept of a function; also known as *root*	2/C	**cero** el intercepto de *x* de una función; también se conoce como *raíz*
***z*-score** the number of standard deviations that a score lies above or below the mean; given by the formula $z=\frac{x-\mu}{\sigma}$	1/A	**puntuación *z*** cantidad de desviaciones estándar por encima o por debajo de la media que presenta la muestra; dada por la fórmula $z=\frac{x-\mu}{\sigma}$