Integrated Pathway

Mathematics I

Program Overview

TEACHER RESOURCE

BW WALCH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

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Introduction to the Program

Introduction

The *New Mexico Integrated Pathway Math I* program is a complete set of materials developed around the Common Core State Standards (CCSS), the overview of the Integrated Pathway for the Common Core State Mathematics Standards, and the Mathematics I content map found in Appendix A of the Common Core State Standards. Topics are built around accessible core curricula, ensuring that the *New Mexico Integrated Pathway Math I* program is useful for striving students and diverse classrooms.

This program realizes the benefits of exploratory and investigative learning and employs a variety of instructional models to meet the learning needs of students with a range of abilities.

The *New Mexico Integrated Pathway Math I* program includes components that support problem-based learning, instruct and coach as needed, provide practice, and assess students' skills. Instructional tools and strategies are embedded throughout.

The program includes:

- More than 150 hours of lessons, addressing the six units of CCSS IP: Mathematics I
- Essential Questions for each instructional topic
- Vocabulary
- Instruction and Guided Practice
- Problem-based Tasks and Coaching questions
- Step-by-step graphing calculator instructions for the TI-Nspire and the TI-83/84
- Station activities to promote collaborative learning and problem-solving skills

Purpose of Materials

The *New Mexico Integrated Pathway Math I* program has been organized to coordinate with the CCSS Integrated Pathway: Mathematics I content map and specifications from Appendix A of the Common Core State Standards.

Each topic includes activities that offer opportunities for exploration and investigation. These activities incorporate concept and skill development and guided practice, then move on to the application of new skills and concepts in problem-solving situations. Throughout the lessons and activities, problems are contextualized to enhance rigor and relevance.

This program includes all the topics addressed in the CCSS Integrated Pathway: Mathematics I content map. These include:

- Relationships Between Quantities
- Linear and Exponential Relationships
- Reasoning with Equations
- Descriptive Statistics
- Congruence, Proof, and Constructions
- Connecting Algebra and Geometry Through Coordinates

The eight Mathematical Practices described in the Common Core are infused throughout:

- CCSS.MP.1: Make sense of problems and persevere in solving them.
- CCSS.MP.2: Reason abstractly and quantitatively.
- CCSS.MP.3: Construct viable arguments and critique the reasoning of others.
- CCSS.MP.4: Model with mathematics.
- CCSS.MP.5: Use appropriate tools strategically.
- CCSS.MP.6: Attend to precision.
- CCSS.MP.7: Look for and make use of structure.
- CCSS.MP.8: Look for and express regularity in repeated reasoning.

Structure of the Teacher Resource

The *New Mexico Integrated Pathway Math I* program is completely reproducible. Online materials can be provided in your Learning Management System (such as Canvas or Schoology) or in Walch's proprietary course management platform, the Curriculum Engine. The nested folder organization in the Curriculum Engine allows you to access the materials quickly and easily. The digital format also facilitates printing and copying student pages and/or making assignments online.

The Program Overview is the first section. This section helps you to navigate the materials, offers a collection of research-based Instructional Strategies along with their literacy connections and implementation suggestions, and shows the correlation between the Common Core State Standards and the CCSS Integrated Pathway: Mathematics I content map found in Appendix A of the Common Core State Standards.

The remaining materials focus on content, knowledge, and application of the six units in the CCSS Integrated Pathway Mathematics I curriculum: Relationships Between Quantities; Linear and Exponential Relationships; Reasoning with Equations; Descriptive Statistics; Congruence, Proof, and Constructions; and Connecting Algebra and Geometry Through Coordinates. The units in the *New Mexico Integrated Pathway Math I* program are designed to be flexible so that you can mix and match activities as the needs of your students and your instructional style dictate.

The Station Activities correspond to the content in the units and provide students with the opportunity to apply concepts and skills, while you have a chance to circulate, observe, speak to individuals and small groups, and informally assess and plan.

Each topic begins with a pre-assessment and ends with a progress assessment. These allow you to assess students' progress as you move from topic to topic, enabling you to gauge how well students have understood the material and to differentiate as appropriate.

***Please note*:** Throughout the Teacher Resource, page references are provided on the lower, inner corner of some pages. These references indicate the corresponding page(s) in the Student Workbook (e.g., SWB p. U1-SWB) as appropriate. These page references have been included here to facilitate assigning materials to students.

Glossary

The Glossary contains vocabulary terms and formulas from throughout the program, organized alphabetically. Each listing provides the term and the definition in both English and Spanish. The listings include the lesson number(s) where the terms can be found in the Words to Know.

Correspondence to Standards for Mathematical Practice

How Do Walch's New Mexico Mathematics Resources Address the Common Core Standards for Mathematical Practice?

Walch's programs for the *New Mexico Integrated Pathway Math I, II,* and *III* courses employ a problem-based model of instruction that supports and reinforces the eight Standards for Mathematical Practice. Although the following table focuses on Problem-Based Tasks, Walch's full programs also include hundreds of additional problems in warm-ups and practices. The Implementation Guide for each PBT highlights two or more SMPs to focus on during implementation and discussion.

CCSS Standards for Mathematical Practice		Relevant Attributes of Walch's New Mexico Math Resources
1	Make sense of problems and persevere in solving them.	Each lesson is built around a Problem-Based Task (PBT) that requires students to "make sense of problems and persevere in solving them."
2	Reason abstractly and quantitatively.	Each PBT uses a meaningful real-world context that requires students to reason both abstractly about the situation/relationships and quantitatively about the values representing the elements and relationships.
3	Construct viable arguments and critique the reasoning of others.	Since the PBT provides opportunities for multiple problem-solving approaches and varied solutions, students are required to construct viable arguments to support their approach and answer. This, in turn, provides other students the opportunity to analyze and critique their classmates' reasoning.
4	Model with mathematics.	Each PBT represents a real-world situation and requires students to model it with mathematics.
5	Use appropriate tools strategically.	PBTs require students to make choices about using appropriate tools, such as calculators, spreadsheets, graph paper, manipulatives, protractors, and compasses. The tasks do not prescribe specific tools, but instead provide opportunities for their use.
6	Attend to precision.	The real-world contexts of the PBTs require students to be precise in their solutions, both in the ways that the solutions are stated, labeled, and explained, and in the degree of precision necessary given the context (e.g., tripling chili for a crowd vs. machining a part for an airplane engine).
7	Look for and make use of structure.	The PBTs present students with complicated scenarios that must be analyzed to discern patterns and significant mathematical features.
8	Look for and express regularity in repeated reasoning.	PBTs require multiple steps, providing opportunities for students to note repeated calculations, monitor their process, and continually evaluate reasonableness of intermediate results before arriving at a solution.

Correspondence to CCSS Publishers' Criteria

How Do Walch's New Mexico Mathematics Resources Address the High School Publishers' Criteria for CCSS Mathematics?

Walch's programs for the *New Mexico Integrated Pathway Math I, II,* and *III* courses were designed to the specifications of Appendix A and the Publishers' Criteria. Focus, Coherence, and Rigor represent the fundamental assumptions of the resources.

Publishers' Criteria	Relevant Attributes of Walch's New Mexico Math Resources
FOCUS Focus strongly where the standards focus.	Each course is built around the units specified in the CCSS for Mathematics Appendix A. Units are organized around important topics in mathematics and reflect the focus of the standards. Units range from 22 lessons for Math I Unit 2, "Linear and Exponential Relationships," which address 21 Common Core standards, to three lessons for Math I Unit 6, "Connecting Algebra and Geometry Through Coordinates," which address only three Common Core standards.
COHERENCE Think across grades/courses, and link to major topics in each course.	Walch Integrated Math programs include two features contributing to coherence. Each lesson identifies necessary prerequisite skills (our forthcoming Support Supplement will point to where they were introduced, including in elementary or middle school, and provide instructional materials for remediation). Warm-Ups tie to these prerequisites in order to activate prior knowledge. Also, the major topics of each course are mapped forward to show where they are revisited and expanded upon in future courses.
RIGOR In major topics, pursue with equal intensity • conceptual understanding, • procedural skill and fluency, and • applications.	The instructional components of Walch's Integrated Math programs guide students from conceptual understanding and procedural skill developed through modeling and guided practice to the application opportunities of the PBT. The resources move students on to additional practice to refine procedural skill and fluency, and later to station activities to crystalize conceptual understanding.

Correspondence to NCTM *Principles to Actions* Teaching Practices

How Do Walch's New Mexico Mathematics Resources Address the NCTM *Principles to Actions* Mathematics Teaching Practices?

Walch's programs for the *New Mexico Integrated Pathway Math I, II,* and *III* courses were designed by experienced educators and curriculum developers, informed by best-practice research, and refined through an iterative process of implementation and feedback. Together with professional development, these materials support and sustain good teaching practices.

NCTM Mathematics Teaching Practices	Relevant Attributes of Walch's New Mexico Math Resources
Establish mathematics goals to focus learning.	Each lesson in Walch's programs addresses specified standards which can be used as goals to focus learning. Essential Questions offer further focus.
Implement tasks that promote reasoning and problem solving.	Each lesson in Walch's programs is built around a Problem-Based Task (PBT), set in a meaningful real-world context and designed to promote reasoning and problem solving. The courses include dozens of PBTs as well as warm-up and practice problems.
Use and connect mathematical representations.	Walch's High School Math programs make frequent use of, and connections among and between, equations, tables, and graphs. PBTs often require students to use and connect two or more of these representations, and the representations are modeled through guided practice.
Facilitate meaningful mathematical discourse.	Several features of the programs support mathematical discourse, including warm-up debriefs with connections to the upcoming lesson, implementation guides and optional coaching questions for the PBTs, and discussion guides for Station Activities. Explanations of PBT solutions are another opportunity for discourse. Please note: Mathematical discourse is an important topic for professional development, in conjunction with implementation of these materials.
Pose purposeful questions.	The implementation guides, coaching questions and discussion guides provide samples of purposeful questions. Note that this is another important topic for professional development.
Build procedural fluency from conceptual understanding.	The programs develop conceptual understanding through modeling, guided practice, and application, and then provide additional opportunities to practice and develop fluency.
Support productive struggle in learning mathematics.	The PBTs require "productive struggle;" implementation guides include suggestions for facilitation and monitoring, and coaching questions provide an option for additional support as appropriate, allowing students to proceed through the task and ensuring that the struggle remains productive rather than too frustrating.
Elicit and use evidence of student thinking.	Various discussions and PBTs require students to display their thinking. Implementation guides offer specific prompts and suggestions for eliciting and responding to student thinking. Professional development supports teachers in using that evidence to respond in instructionally appropriate ways.

Unit Structure

All of the instructional units have common features. Each unit begins with a table of Standards Correlations for the lessons in the unit, as well as a list of relevant online educational resources.

Each topic begins with a pre-assessment, followed by a Topic Overview listing the standards addressed by the lessons in the topic; Essential Questions; vocabulary (titled "Words to Know"); and a list of recommended websites to be used as additional resources. This Topic Overview is followed by the lesson(s) for that topic.

Each lesson begins with Warm-Up activity, followed by a list of identified prerequisite skills that students need to have mastered in order to be successful with the new material in the upcoming lesson. This is followed by an introduction, key concepts, common errors/misconceptions, scaffolded practice problems, guided practice examples, a problem-based task with coaching questions and sample responses, a closure activity, and practice.

Each topic ends with a progress assessment to evaluate students' learning. Selected topics include a Conceptual Activity before the progress assessment. The Unit Assessment, answer keys, and one or more station activities round out the unit.

All of the components are described below and on the following pages for your reference.

Standards Correlations and Connections to Future Courses

In this section, you'll find a comprehensive list of the Common Core State Standard(s) addressed in each lesson, followed by a map detailing where and how future courses will build upon the topics introduced in the unit.

Pre-Assessment

This can be used to gauge students' prior knowledge and to inform instructional planning.

Common Core State Standards for the Topic

All standards that are addressed in the entire topic are listed.

Essential Questions

These are intended to guide students' thinking as they proceed through the topic. By the end of each topic, students should be able to respond to the questions.

Words to Know

Vocabulary terms and formulas are provided as background information for instruction or to review key concepts that are addressed in the topic.

Recommended Resources

This is a list of websites that can be used as additional resources. Some websites are games; others provide additional examples and/or explanations. (*Note*: Links will be monitored and repaired or replaced as necessary.) Each Recommended Resource is also accessible through Walch's cloud-based Curriculum Engine Learning Object Repository as a separate learning object that can be assigned to students.

Conceptual Activities

Conceptual understanding serves as the foundation on which to build deeper understanding of mathematics. In an effort to build conceptual understanding of mathematical ideas and to provide more than procedural fluency and application, links to interactive open education and Desmos resources are included. (*Note*: These website links will be monitored and repaired or replaced as necessary.) These and many other open educational resources (OERs) are also accessible through the Learning Object Repository as separate objects that can be assigned to students.

Warm-Up

Each warm-up takes approximately 5 minutes and addresses either prerequisite and critical-thinking skills or previously taught math concepts.

Common Core State Standards for the Lesson

When topics are broken down into lessons, the specific standard or standards that are addressed are presented at the beginning of the instructional portion of the lesson.

Warm-Up Debrief

Each debrief provides the answers to the warm-up questions, and offers suggestions for situations in which students might have difficulties. A section titled Connection to the Lesson is also included in the debrief to help answer students' questions about the relevance of the particular warm-up activity to the upcoming instruction. Warm-Ups with debriefs are also provided in PowerPoint presentations.

Identified Prerequisite Skills

This list cites the skills necessary to be successful with the new material.

Introduction

This brief paragraph gives a description of the concepts about to be presented and often contains some Words to Know.

Key Concepts

Provided in bulleted form, this instruction highlights the important ideas and/or processes for meeting the standard.

Graphing Calculator Directions

Step-by-step instructions for using a TI-Nspire and a TI-83/84 are provided whenever graphing calculators are referenced.

Common Errors/Misconceptions

This is a list of the common errors students make when applying Key Concepts. This list suggests what to watch for when students arrive at an incorrect answer or are struggling with solving the problems.

Scaffolded Practice (Printable Practice)

This set of 10 printable practice problems provides introductory level skill practice for the lesson. This practice set can be used during instruction time.

Guided Practice

This section provides step-by-step examples of applying the Key Concepts. The three to five examples are intended to aid during initial instruction, but are also for individuals needing additional instruction and/or for use during review and test preparation.

Enhanced Instructional PowerPoint (Presentation)

Each lesson includes an instructional PowerPoint presentation with the following components: Warm-Up, Key Concepts, and Guided Practice. Selected Guided Practice examples include links to GeoGebra applets. These instructional PowerPoints are downloadable and editable.

Problem-Based Task

This activity can serve as the centerpiece of a problem-based lesson, or it can be used to walk students through the application of the standard, prior to traditional instruction or at the end of instruction. The task makes use of critical-thinking skills.

Optional Problem-Based Task Coaching Questions with Sample Responses

These questions scaffold the task and guide students to solving the problem(s) presented in the task. They should be used at the discretion of the teacher for students requiring additional support. The Coaching Questions are followed by answers and suggested appropriate responses to the coaching questions. In some cases answers may vary, but a sample answer is given for each question.

Recommended Closure Activity

Students are given the opportunity to synthesize and reflect on the lesson through a journal entry or discussion of one or more of the Essential Questions.

Problem-Based Task Implementation Guide

This instructional overview, found with selected Problem-Based Tasks in each unit, highlights connections between the task and the lesson's key concepts and Mathematical Practices. The Implementation Guide also offers suggestions for facilitating and monitoring, and provides alternative solutions.

Printable Practice (Sets A and B) and Interactive Practice (Set A)

Each lesson includes two sets of practice problems to support students' achievement of the learning objectives. They can be used in any combination of teacher-led instruction, cooperative learning, or independent application of knowledge. Each Practice A is also available as an interactive Learnosity activity with Technology-Enhanced Items.

Progress Assessment

Each lesson ends with 10 multiple-choice questions, as well as one extended-response question that incorporates critical thinking and writing components. This can be used to document the extent to which students grasp the concepts and skills addressed during instruction.

Unit Assessment

Each unit ends with 12 multiple-choice questions and three extended-response questions that incorporate critical thinking and writing components. This can be used to document the extent to which students grasped the concepts and skills of each unit.

Answer Key

Answers for all of the Warm-Ups and practice problems are provided at the end of each unit.

Station Activities

Most units include a collection of station-based activities to provide students with opportunities to practice, reinforce, and apply mathematical skills and concepts. The debriefing discussions after each set of activities provide an important opportunity to help students reflect on their experiences and synthesize their thinking.

Conceptual Tasks

These engaging tasks provide opportunities for students to deepen their understanding and develop their conceptual knowledge of math concepts. These tasks provide multiple entry points and are accessible for ALL learners.

Standards Correlations

Each lesson in this program was written specifically to address the Common Core State Standards. Each topic lists the standards covered in all the topic, and each lesson lists the standards addressed in that particular lesson. In this section, you'll find a comprehensive list mapping the lessons to the CCSS.

Guide to Common Core State Standards Annotation

As you use this program, you will come across a symbol included with the Common Core standards for some of the lessons and activities. These symbols are explained below.

Symbol: ⋆

Denotes: Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (⋆).

From https://www.walch.com/CCSS/00003

Symbol: (+)

Denotes: College and Career Readiness Standards

Advanced mathematics standards that are required in higher-level courses such as advanced statistics may also be included in lower-level courses. These additional standards are denoted by (+). According to the Common Core State Standards Initiative, "the evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6–8."

From https://www.walch.com/CCSS/00004

Connections to Future Courses

This section provides a map between topics introduced in each unit of this course and subsequent courses where each topic is revisited and built upon.

Unit 1: Relationships Between Quantities			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Interpreting Structure in Expressions**		
	1.1	Identifying Terms, Factors, and Coefficients	A–SSE.A.1a★
	1.2	Interpreting Linear and Exponential Expressions	A–SSE.A.1b★
Topic B	**Creating Equations and Inequalities in One Variable**		
	1.3	Creating Linear Equations in One Variable	A–CED.A.1★ N–Q.A.2★ N–Q.A.3★
	1.4	Creating Linear Inequalities in One Variable	A–CED.A.1★
	1.5	Creating Exponential Equations	A–CED.A.1★
Topic C	**Creating and Graphing Equations in Two Variables**		
	1.6	Creating and Graphing Linear Equations in Two Variables	A–CED.A.2★ N–Q.A.1★
	1.7	Creating and Graphing Exponential Equations	A–CED.A.2★ N–Q.A.1★
Topic D	**Representing Constraints**		
	1.8	Representing Constraints	A–CED.A.3★
Topic E	**Rearranging Formulas**		
	1.9	Rearranging Formulas	A–CED.A.4★

Unit 2: Linear and Exponential Relationships			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Graphs As Solution Sets and Function Notation**		
	2.1	Graphing the Set of All Solutions	A–REI.D.10
	2.2	Intersecting Graphs	A–REI.D.11★
	2.3	Domain and Range	F–IF.A.1
	2.4	Function Notation and Evaluating Functions	F–IF.A.2
Topic B	**Solving Linear Inequalities in Two Variables and Systems of Inequalities**		
	2.5	Solving Linear Inequalities in Two Variables	A–REI.D.12
	2.6	Solving Systems of Linear Inequalities	A–REI.D.12
Topic C	**Sequences As Functions**		
	2.7	Sequences As Functions	F–IF.A.3
Topic D	**Interpreting Graphs of Functions**		
	2.8	Identifying Key Features of Linear and Exponential Graphs	F–IF.B.4★ F–IF.B.5★
	2.9	Average Rate of Change	F–IF.B.6★ F–LE.A.1a★
	2.10	Recognizing Average Rate of Change	F–IF.B.6★ F–LE.A.1b★ F–LE.A.1c★
Topic E	**Analyzing Linear and Exponential Functions**		
	2.11	Graphing Linear Functions	F–IF.C.7a★
	2.12	Graphing Exponential Functions	F–IF.C.7e★
Topic F	**Comparing Functions**		
	2.13	Comparing Linear Functions	F–IF.C.9
	2.14	Comparing Exponential Functions	F–IF.C.9
	2.15	Comparing Linear to Exponential Functions	F–LE.A.3★
Topic G	**Building Functions**		
	2.16	Building Functions from Context	F–BF.A.1a★
	2.17	Constructing Functions from Graphs and Tables	F–LE.A.2★
Topic H	**Operating on Functions and Transformations**		
	2.18	Operating on Functions	F–BF.A.1b★
	2.19	Transformations of Linear and Exponential Functions	F–BF.B.3

Topic	Lesson number	Title	Standard(s)
Topic I	**Arithmetic and Geometric Sequences**		
	2.20	Arithmetic Sequences	F–BF.A.2★
	2.21	Geometric Sequences	F–BF.A.2★
Topic J	**Interpreting Parameters**		
	2.22	Interpreting Parameters	F–LE.B.5★

Unit 3: Reasoning with Equations			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Solving Equations and Inequalities**		
	3.1	Properties of Equality	A–REI.A.1
	3.2	Solving Linear Equations	A–REI.B.3
	3.3	Solving Linear Inequalities	A–REI.B.3
	3.4	Solving Exponential Equations	A–REI.A.1
Topic B	**Solving Systems of Equations**		
	3.5	Solving Systems of Linear Equations by Substitution and Elimination	A–REI.C.5 A–REI.C.6
	3.6	Solving Systems of Linear Equations by Graphing	A–REI.C.6

Unit 4: Descriptive Statistics			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Working with a Single Measurement Variable**		
	4.1	Representing Data Sets	S–ID.A.1★
	4.2	Comparing Data Sets	S–ID.A.2★
	4.3	Interpreting Data Sets	S–ID.A.3★
Topic B	**Working with Two Categorical and Quantitative Variables**		
	4.4	Summarizing Data Using Two-Way Frequency Tables	S–ID.B.5★
	4.5	Solving Problems Given Functions Fitted to Data	S–ID.B.6a★
	4.6	Analyzing Residuals	S–ID.B.6b★
	4.7	Fitting Linear Functions to Data	S–ID.B.6c★
Topic C	**Interpreting Linear Models**		
	4.8	Interpreting Slope and *y*-intercept	S–ID.C.7★
	4.9	Calculating and Interpreting the Correlation Coefficient	S–ID.C.8★
	4.10	Distinguishing Between Correlation and Causation	S–ID.C.9★

Unit 5: Congruence, Proof, and Constructions			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Introducing Transformations**		
	5.1	Defining Terms	G–CO.A.1
	5.2	Transformations As Functions	G–CO.A.2
	5.3	Applying Lines of Symmetry	G–CO.A.3
Topic B	**Defining and Applying Rotations, Reflections, and Translations**		
	5.4	Defining Rotations, Reflections, and Translations	G–CO.A.4
	5.5	Applying Rotations, Reflections, and Translations	G–CO.A.5
Topic C	**Constructing Lines, Segments, and Angles**		
	5.6	Copying Segments and Angles	G–CO.D.12
	5.7	Bisecting Segments and Angles	G–CO.D.12
	5.8	Constructing Perpendicular and Parallel Lines	G–CO.D.12

Topic	Lesson number	Title	Standard(s)
Topic D	**Constructing Polygons**		
	5.9	Constructing Equilateral Triangles Inscribed in Circles	G–CO.D.13
	5.10	Constructing Squares Inscribed in Circles	G–CO.D.13
	5.11	Constructing Regular Hexagons Inscribed in Circles	G–CO.D.13
Topic E	**Exploring Congruence**		
	5.12	Describing Rigid Motions and Predicting the Effects	G–CO.B.6
	5.13	Defining Congruence in Terms of Rigid Motions	G–CO.B.6
Topic F	**Congruent Triangles**		
	5.14	Triangle Congruency	G–CO.B.7
	5.15	Explaining ASA, SAS, and SSS	G–CO.B.8

Unit 6: Connecting Algebra and Geometry Through Coordinates			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Slope and Distance**		
	6.1	Using Coordinates to Prove Geometric Theorems with Slope and Distance	G–GPE.B.4 G–GPE.B.5
	6.2	Working with Parallel and Perpendicular Lines	G–GPE.B.5
Topic B	**Lines and Line Segments**		
	6.3	Calculating Perimeter and Area	G–GPE.B.7★

TOPICS FROM FUTURE COURSES: NEW MEXICO INTEGRATED PATHWAY MATH I

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 1	Identifying and interpreting the effects of terms, factors, and coefficients of linear, exponential, and quadratic expressions	Math II Unit 3	includes more complex quadratic expressions and introduces the standard form of a quadratic expression
		Math III Units 1 and 2	extended to include higher-order polynomials
		Math III Unit 3	extended to rational expressions
	Creating and graphing with linear and simple exponential equations	Math II Unit 2	creating and graphing functions by using quadratic, square root, cube root, absolute value, step, and piecewise functions
		Math II Unit 3	extended to quadratic and rational functions and to graphing higher-order polynomial functions
		Math III Unit 2	extended to rational and radical functions
		Math III Unit 3	introduces students to graphing logarithmic and periodic functions
		Math III Unit 5	addressed using more complex exponential equations
		Math III Unit 6	extended to applying the creating and graphing principles
	Constraints in terms of linear relationships	Math II Unit 3, Math III Unit 6	addressed using linear, exponential, quadratic, and logarithmic models
	Rearranging linear formulas	Math II Unit 3	extended to rearranging quadratic expressions
		Math III Unit 6	extended to rearranging all function families

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 2	Function notation (concepts regarding domain, range, intersecting functions, and graphing solution sets)	Math II Unit 2	extended to describing transformations of functions
		Math III Unit 6	includes combinations of functions
	Linear inequalities	Math II Unit 3	extended to quadratic inequalities and rational inequalities
		Math III Unit 6	extended to linear, exponential, quadratic, and logarithmic functions
	Sequences	Math III Unit 2	applied to more complex sequences and series
	Analyzing the features of graphs of linear functions	Math II Unit 2	extended to exponential and quadratic functions
		Math III Unit 2	applied to polynomial functions
		Math III Unit 3	extended to periodic functions
		Math III Unit 5	expanded to logarithmic functions
	Domain	Math IV	restricts the domain of trigonometric functions in order to find the inverses of these functions

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 3	Reasoning with equations (linear equations)	Math II Unit 3	extended to quadratic functions
		Math III Unit 3	extended to rational and radical functions
	Systems of equations (linear-linear)	Math II Unit 3	includes linear-quadratic systems
		Math III Unit 2	extended to linear-polynomial, polynomial-polynomial, and absolute value-polynomial systems
		Math III Unit 3	extended to linear-rational and linear-radical systems
		Math IV	includes systems with vectors and matrices

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 4	Descriptive statistics	Math III Unit 1	extended to using statistics to draw conclusions

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 5	Transformations and connections between functions and geometric transformations; congruence	Math II Unit 5	includes similarity transformations
		Math II Unit 6	extended to coordinate geometry and proofs
	Perimeter and area	Math II Unit 6	extended to area and volume
		Math III Unit 6	includes density and cross sections of three-dimensional objects
	Geometric figures (points, lines, and planes)	Math II Unit 5	extended to parallel lines cut by a transversal and other relationships between angle pairs, other special triangles, and parallelograms
		Math II Unit 6	includes circles and special segments
		Math III Unit 6	extended to three-dimensional objects in modeling and design

Math I	Topic introduced	Course/unit where addressed	How addressed
Unit 6	Connecting algebra and geometry	Math II Unit 2	extended to transformations of quadratic functions
		Math III Unit 4	extended to proofs of the laws of sines and cosines
		Math III Units 4 and 5	includes transformations of families of functions, evaluating trigonometric functions, and graphing trigonometric functions

Conceptual Activities

Use these interactive open education and/or Desmos resources to build conceptual understanding of mathematical ideas. (*Note*: Activity links will be monitored and repaired or replaced as necessary.)

Unit 1

- Desmos. "Function Carnival."

  https://www.walch.com/ca/01006

  This activity focuses attention on graphs as expressing relationships between variables. It lays the informal groundwork for the more formal definitions and properties of functions.

- Desmos. "Marbleslides: Lines."

  https://www.walch.com/ca/01008

  Restrict, reposition, and rotate lines at will using slope-intercept form, and describe transformations using words and/or symbols.

- Desmos. "Put the Point on the Line."

  https://www.walch.com/ca/01009

  The focus of this activity is slope. Participants are asked to estimate, calculate, and notice proportionality as they place points on an imaginary line.

- Desmos, Inc. "Polygraph: Linear Inequalities."

  https://www.walch.com/ca/10000

  In this activity, students will engage in vocabulary-rich conversations about linear inequalities. Key vocabulary terms that may appear in student questions include *shading*, *above*, *below*, *boundary*, *solid*, *dotted*, *horizontal*, *vertical*, *slanted*, *axis*, and *quadrant*.

Unit 2

- Desmos. "Avi and Benita's Repair Shop."

  https://www.walch.com/ca/01014

  Compare linear and exponential growth in the context of daily payments. One plan increases by $100 each day, while another grows by doubling the previous day's payment. This activity is appropriate for students who have studied linear functions but may not have an experience with exponential growth.

- Desmos. "Card Sort: Functions."

  https://www.walch.com/ca/01005

  Sort graphs, equations, and contexts according to whether each one represents a function.

- Desmos. "Card Sort: Linear Functions."

  https://www.walch.com/ca/01010

  Notice and use properties of linear functions to make groups of three. Different properties will lead to different groupings by different participants.

- Desmos. "Function Carnival, Part 2."

  https://www.walch.com/ca/01007

  This activity follows up on "Function Carnival" by using the contexts in that activity to develop an understanding of function notation.

- Desmos. "Game, Set, Flat."

  https://www.walch.com/ca/01015

  Develop understanding of the exponential relationship that describes a bouncing tennis ball. Learn to examine successive terms in a sequence to determine if it represents an exponential relationship or not, and how to construct the exponential equation itself.

- Desmos. "Marbleslides: Exponentials."

  https://www.walch.com/ca/01016

  Restrict, reposition, and otherwise transform exponential curves at will by modifying the basic form $y = b^x$, and use precision in describing these transformations using words and/or symbols.

- Desmos. "Match My Line."

  https://www.walch.com/ca/01013

  Work through a series of scaffolded linear graphing challenges to develop proficiency with direct variation, slope-intercept, point-slope, and other linear function forms.

- Desmos. "Polygraph: Exponentials."

  https://www.walch.com/ca/01019

  This Custom Polygraph is designed to spark vocabulary-rich conversations about exponentials, including how they differ from linear functions. Key vocabulary terms that may appear in student questions include *increasing*, *decreasing*, *intercept*, *rate*, *asymptote*, and *curve*.

- Desmos. "What Comes Next?"

  https://www.walch.com/ca/01020

  Predict "what comes next" for linear and exponential functions based first on graphs and then on tables of values, then explore connections between graphs, tables, and equations of linear and exponential functions.

Unit 3

- Desmos. "Card Sort: Linear Systems."

  https://www.walch.com/ca/01000

  In this activity, students practice what they've learned about solving systems of linear equations. The activity begins with a review of the graphical meaning of a solution to a system. Later, students consider which algebraic method is most efficient for solving a given system. Finally, students practice solving equations using substitution and elimination. Prior to beginning this activity, students should have experience solving systems of linear equations graphically and algebraically.

- Desmos. "The Intersection."

  https://www.walch.com/ca/01011

  Predict the point of intersection for a system of two linear equations: first without a grid, then with one. With the grid in play, participants are able to use the slope of the lines (formally or informally) to improve the accuracy of their predictions.

- Desmos. "Solutions to Systems of Linear Equations."

  https://www.walch.com/ca/01001

  This activity will help students understand what it means for a point to be a solution to a system of equations—both graphically and algebraically.

- Desmos. "Systems of Two Linear Equations."

  https://www.walch.com/ca/01002

  This resource gives a progression of written explanations, equations, and graphs to explain what the algebraic or graphical solution to a system of equations represents.

Unit 4

- Desmos. "LEGO Prices."

  https://www.walch.com/ca/01012

  Use the concept of linear regression to predict the cost of a LEGO set with *x* pieces. (This activity does NOT use the calculator, just the concept. Participants draw the line on the graph, and Desmos calculates the equation.)

- Desmos. "Predicting Movie Ticket Prices."

  https://www.walch.com/ca/01018

  Build a model to describe the relationship between average movie ticket prices and time, then use that model to make predictions about past and future ticket prices. Participants also interpret the parameters of their equation in context.

- Desmos. "Polygraph: Histograms."

  https://www.walch.com/ca/01024

  This activity is designed to spark vocabulary-rich conversations about histograms. Key vocabulary terms that may appear in student questions include *shape*, *center*, *spread*, *roughly symmetric*, *skew right*, *skew left*, *mean*, *median*, *range*, *peak*, *unimodal*, and *bimodal*.

Unit 5

- Desmos. "Polygraph: Transformations."

  https://www.walch.com/ca/01025

  This activity is designed to spark vocabulary-rich conversations about transformations. Key vocabulary terms that may appear in student questions include *translation*, *rotation*, *reflection*, *dilation*, *scale factor*, *preimage*, and *image*.

- Desmos. "Symmetry."

  https://www.walch.com/ca/01026

  In this activity, students develop an informal understanding of symmetry of functions. By the end, students should be able to identify the symmetry of a function (reflectional vs. rotational) by considering its graph.

Unit 6

- Illustrative Mathematics. "A Midpoint Miracle."

  https://www.walch.com/ca/10003

  Students will prove a theorem about quadrilaterals that is somewhat difficult to prove with a straightedge and ruler but relatively easy to prove using coordinates. This task requires that students be comfortable using the formula for the midpoint of a line segment and the parallel line criterion.

Station Activities Guide

Introduction

Each unit includes a collection of station-based activities to provide students with opportunities to practice and apply the mathematical skills and concepts they are learning. You may use these activities in addition to the instructional topics, or, especially if the pre-test or other formative assessment results suggest it, instead of direct instruction in areas where students have the basic concepts but need practice. The debriefing discussions after each set of activities provide an important opportunity to help students reflect on their experiences and synthesize their thinking. Debriefing also provides an additional opportunity for ongoing, informal assessment to guide instructional planning.

Implementation Guide

The following guidelines will help you prepare for and use the activity sets in this section.

Setting Up the Stations

Each activity set consists of four or five stations. Set up each station at a desk, or at several desks pushed together, with enough chairs for a small group of students. Place a card with the number of the station on the desk. Each station should also contain the materials specified in the teacher's notes, and a stack of student activity sheets (one copy per student). Place the required materials (as listed) at each station.

When a group of students arrives at a station, each student should take one of the activity sheets to record the group's work. Although students should work together to develop one set of answers for the entire group, each student should record the answers on his or her own activity sheet. This helps keep students engaged in the activity and gives each student a record of the activity for future reference.

Forming Groups of Students

All activity sets consist of four or five stations. You might divide the class into four or five groups by having students count off from 1 to 4 or 5. If you have a large class and want to have students working in small groups, you might set up two identical sets of stations, labeled A and B. In this way, the class can be divided into eight groups, with each group of students rotating through the "A" stations or "B" stations.

Assigning Roles to Students

Students often work most productively in groups when each student has an assigned role. You may want to assign roles to students when they are assigned to groups and change the roles occasionally. Some possible roles are as follows:

- Reader—reads the steps of the activity aloud
- Facilitator—makes sure that each student in the group has a chance to speak and pose questions; also makes sure that each student agrees on each answer before it is written down
- Materials Manager—handles the materials at the station and makes sure the materials are put back in place at the end of the activity
- Timekeeper—tracks the group's progress to ensure that the activity is completed in the allotted time
- Spokesperson—speaks for the group during the debriefing session after the activities

Timing the Activities

The activities in this section are designed to take approximately 10 minutes per station. Therefore, you might plan on having groups change stations every 10 minutes, with a two-minute interval for moving from one station to the next. It is helpful to give students a "5-minute warning" before it is time to change stations.

Since each activity set consists of four or five stations, the above time frame means that it will take about 50 to 60 minutes for groups to work through all stations.

Guidelines for Students

Before starting the first activity set, you may want to review the following "ground rules" with students. You might also post the rules in the classroom.

- All students in a group should agree on each answer before it is written down. If there is a disagreement within the group, discuss it with one another.
- You can ask your teacher a question only if everyone in the group has the same question.
- If you finish early, work together to write problems of your own that are similar to the ones on the activity sheet.
- Leave the station exactly as you found it. All materials should be in the same place and in the same condition as when you arrived.

Debriefing the Activities

After each group has rotated through every station, bring students together for a brief class discussion. At this time, you might have the groups' spokespersons pose any questions they had about the activities. Before responding, ask if students in other groups encountered the same difficulty or if they have a response to the question. The class discussion is also a good time to reinforce the essential ideas of the activities. The questions that are provided in the teacher's notes for each activity set can serve as a guide to initiating this type of discussion.

You may want to collect the student activity sheets before beginning the class discussion. However, it can be beneficial to collect the sheets afterward so that students can refer to them during the discussion. This also gives students a chance to revisit and refine their work based on the debriefing session. If you run out of time to hold class discussions, you might want to have students journal about their experiences and follow up with a class discussion the next day.

Digital Enhancements Guide

Introduction

With this program, you have access to the following digital components, described here with guidelines and suggestions for implementation.

Digital Instruction PowerPoints (Presentations)

These optional versions of the Warm-Ups, Warm-Up Debriefs, Introductions, Key Concepts, and Guided Practices for each lesson run on PowerPoint. (*Please note*: Computers may render PowerPoint images differently. For best viewing and display, use a PowerPoint Viewer and adjust your settings to optimize images and text.)

Each PowerPoint begins with the lesson's Warm-Up and is followed by the Warm-Up Debrief, which reveals the answers to the Warm-Up questions.

In the notes section of the last Warm-Up slide, you will find the "Connections to the Lesson," which describes concepts students will glean or skills they will need in the upcoming lesson. The "Connections" help transition from the Warm-Up to instruction.

GeoGebra Applets (Interactive Practice Problems)

One or two interactive GeoGebra applets are provided for most lessons. The applets model the mathematics in the Guided Practice examples for these lessons. Links to these applets are also embedded within the Instructional PowerPoints. With an Internet connection, simply click on the "Play" button slide that follows selected examples.

Once you've accessed the GeoGebra applet, please adjust your view to maximize the image. Each applet illustrates the specific problem addressed in the Guided Practice example. The applets allow you to walk through the solution by visually demonstrating the steps, such as defining points and drawing lines. Variable components of the applets (usually fill-in boxes or sliders) allow you to substitute different values in order to explore the mathematics. For example, "What happens to the line when we increase the amount of time?" or "What if we cut the number of students in half?" This experimentation and discussion supports development of conceptual understanding.

GeoGebra for PC/MAC

GeoGebra is not required for using the applets, but can be downloaded for free for further exploration at the following link:

https://www.geogebra.org/download

Curriculum Engine Learning Object Repository

Walch's Curriculum Engine comes loaded with thousands of curated learning objects that can be used to build formative and summative assessments, as well as practice worksheets, instructional components, and an item bank. District leaders and teachers can search for items by standard and create assessments or worksheets in minutes using the three-step assessment builder.

For more information about the Curriculum Engine, or for additional support, please contact us at (207) 828-8800 or success@bwwalch.com.

Standards for Mathematical Practice Implementation Guide

Introduction

The eight Standards for Mathematical Practice describe features of lesson design, teaching pedagogy, and student actions that will lead to a true conceptual understanding of the mathematics standards. The Walch lessons, practice problems, and Problem-Based Tasks lend themselves to teaching through this framework. When the Walch resources are combined with high-level questioning and engaging teacher decisions in the classroom, it will lead to high-level math instruction and student achievement.

Here is a brief description of the SMPs and how they can be applied in the classroom:

CCSS.MP.1: Make sense of problems and persevere in solving them.

Students will read, interpret, and understand complicated mathematical and real-world problems, and they will be willing to try multiple methods with the ultimate goal of determining the correct answer. Strategies such as annotation and student discourse can lead to improvement on this standard. Presenting students with higher-level problems is essential to ensuring students achieve maximum understanding. Teacher prompts that can enhance this standard include:

- What is the problem asking you to solve?
- What are some (other) strategies you could use to solve this problem?
- Compare your answer with a classmate's answer. Who is correct? Why?

CCSS.MP.2: Reason abstractly and quantitatively.

Mathematical reasoning with numbers and variables is essential to understanding the connections among the standards. Students must be able to discover and formalize general rules using numbers and variables, and apply them to determine numerical quantities in other situations. Teacher prompts that can enhance this standard include:

- Substitute realistic numbers into the situation.
- What operation/strategy would you use?
- Will your strategy work for any number?
- For which categories of numbers (negative integers, all real numbers, etc.) will your strategy work?

CCSS.MP.3: Construct viable arguments and critique the reasoning of others.

Many students are most concerned with the "what" aspects of mathematics, i.e. "what" do we do or "what" is the answer. However, math educators must develop the "why" of mathematics. Students must learn to question algorithms, challenge answers, and justify their reasoning in order to truly understand the concepts behind their answers. Teacher prompts that can enhance this standard include:

- How did you determine your answer?
- Why did you choose that strategy?
- Defend your answer based on a real-world situation.

CCSS.MP.4: Model with mathematics.

An important goal of mathematics instruction is for students to be able to apply mathematics to the world around them. Students should be able to link a real problem to a mathematical concept, identify quantities that are modeled well with mathematics, and use mathematics to find a solution. Emphasizing this standard will help students represent and interpret information using physical, visual, and abstract models. Encourage students to use any or all of their learning experiences to gain a deep and flexible understanding of mathematics. Teacher prompts that can enhance this standard include:

- Can you represent this situation with a visual model?
- How will it help you solve the problem?
- What information is needed to solve this problem?
- Is there another way to solve this problem?
- While working to solve this problem, what do you notice/wonder?

CCSS.MP.5: Use appropriate tools strategically.

There are many available tools suitable for mathematics, such as calculators, manipulatives, formulas, rulers, computers, and developed mathematical strategies. Choosing and using the correct tool to work through a problem is an important skill for mathematicians. Teacher prompts that can enhance this standard include:

- Can you graph this equation in the calculator to see a relationship?
- What formula or strategy might help you determine the answer to this question?
- How can you represent the situation using handheld tools (rulers, protractors, etc.) to determine an answer?

CCSS.MP.6: Attend to precision.

When using mathematics to solve problems, an answer can be considered correct only if it is sufficiently precise and accurate for the situation to which it pertains. When applying mathematics, it is vital to clearly define the question, the reasoning, the answer, and the explanation. Vocabulary, units, numerical responses, and pictures must be represented precisely in questions and answers to ensure that the mathematical solutions represent the true answer to a question. Teacher prompts that can enhance this standard include:

- What does your answer represent in a real-world context?
- Is your answer reasonable based on your initial estimate?
- What units of measure help describe your numerical answer?

CCSS.MP.7: Look for and make use of structure.

Structure, whether geometric, algebraic, statistical, or numerical, is an important aspect of mathematical reasoning that students often overlook. Teachers often explicitly refer to geometric and other visual structures as explanations of mathematical concepts, but algebraic and numerical structures can often be just as important in analyzing and interpreting mathematical situations. These structures yield clues as to the meaning of expressions, equations, graphs, and other representations. As students interpret these structures, they will gain a greater understanding of the mathematical concepts. Teacher prompts that can enhance this standard include:

- What do the characteristics of the graph tell us about the situation?
- What do each of the variables and numbers in the equation/formula represent?
- How are these situations the same and different based on their representations?

CCSS.MP.8: Look for and express regularity in repeated reasoning.

Just as patterns appear in real life, patterns appear throughout the subject of mathematics. Recognizing and applying these patterns, and applying the reasoning contained within, is one of the most important skills teachers can instill in their students. Rather than teaching isolated algorithms to determine answers, have students discover relationships, create their own algorithms, and apply the reasoning to other situations. These skills can be applied throughout their education and will enrich their lives after high school. Teacher prompts that can enhance this standard include:

- What relationship do you notice in the graph/table/numbers?
- Why did you choose to use this process to solve this word problem/equation?
- How can you apply this process in other situations?

Instructional Strategies

Ensuring Access for All Students

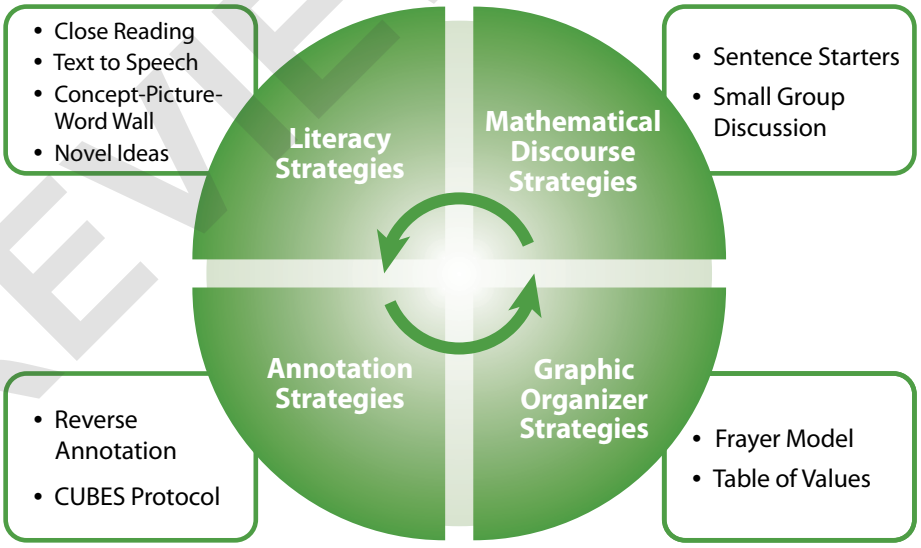
Introduction

The increased focus on literacy in math instruction can help some students navigate mathematical contexts, but for struggling readers, it can further complicate calculations. English language learners struggle to master difficult mathematical concepts while simultaneously processing a new language. Students with learning and behavioral disabilities struggle with the math concepts in their own contexts. This is where teachers and the strategies they select for their classrooms become essential.

The strategies presented here can help all students succeed in math, literacy, school, and, ultimately, in life. These instructional strategies provide teachers with a wide range of instructional support to aid English as a Second Language (ESL) students, students with disabilities (SWD), and struggling readers. These strategies provide support for the Mathematics Standards and the Standards of Mathematical Practice (SMP), English Language Development (ELD) Standards, English Language Arts Standards, and WIDA English Language Development Standards.

Within each lesson throughout this course, you will find suggested instructional strategies. These instructional strategies are research-based strategies and best practices that work well for all students.

The instructional strategies detailed here fall into four main categories: Literacy, Mathematical Discourse, Annotation, and Graphic Organizers. These strategies provide teachers with research-based strategies to address the needs of all students.

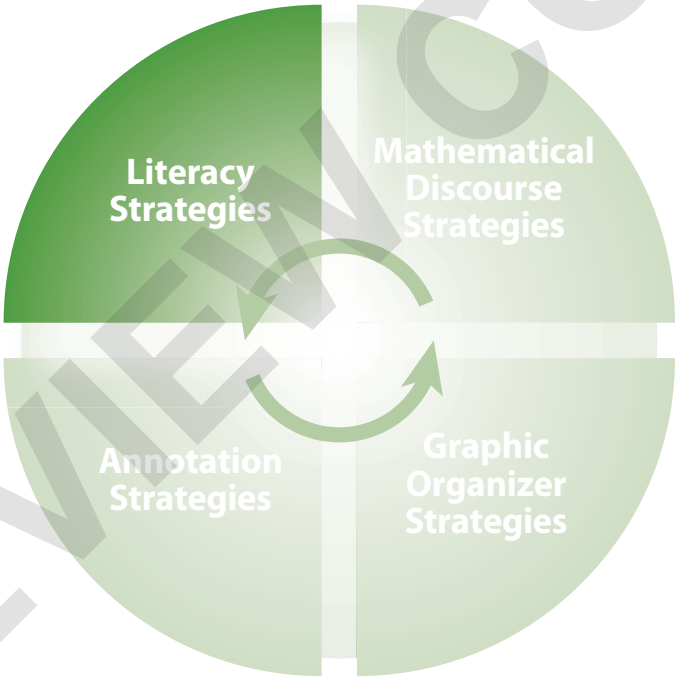


Source

- WIDA: https://www.walch.com/rr/09052

Understanding the Language of Mathematics: Literacy

Mathematics has its own language consisting of words, notations, formulas, and visuals. In education, the language of mathematics is often regarded solely in the context of word problems and articles. This neglects the vocabulary and other mathematical representations students must be able to interpret. The strategies presented here help students navigate the language of mathematics so that they can understand text and feel confident speaking in and listening to mathematical discussions. For students with disabilities, the stress on repetition and different representations in this approach is essential to their ability to grasp the math concepts. For ESL students, repetition and different representations can strip out some of the English language barriers to understanding the language of mathematics, as well as provide multiple means of accessing the content. Literacy strategies include Close Reading, Text-to-Speech, Concept-Picture-Word Walls, and Novel Ideas.



Literacy Strategies

Close Reading with Guiding Questions

What is Close Reading with Guiding Questions?

Close Reading with Guiding Questions is a process that allows students to preview mathematical reading and problems by answering questions related to the text in advance and reviewing their responses during and/or after reading. Multiple reading protocols can be used in conjunction with guiding questions to enhance their effectiveness.

How do you implement Close Reading with Guiding Questions in the classroom?

When utilizing a textbook, task, or article in a math class, literacy struggles are often a strong barrier to entry into the mathematical ideas. Asking students to answer accessible questions before and/or as they read can lead them to the key information.

Prior to implementation, the teacher should determine the most important information students need to obtain from a text, whether it is a math problem to solve, a task to complete, or an informational lesson or article to read. Then, the teacher should come up with some questions to guide students before they read. These questions can:

- assess and relate prior knowledge
- define key vocabulary words
- discuss non-mathematical concepts in the text

The teacher should also prepare some questions to guide students as they read. These questions can:

- point out key concepts within the text
- relate the text and concepts to future learning
- assist students in identifying key facts in the text
- highlight the importance of text features (graphics, headings, etc.) in the text

To ensure the questions are accessible for students and to encourage reflection and debate after reading, many of these questions should be designed as either "True/False" or "Always True/Sometimes True/Never True." Students can represent their reasoning for their answer in writing, numbers, or graphic/pictorial representations. Students should complete the guiding questions and reading individually, with discussion to follow.

After students complete the reading, they should be given some time to individually evaluate their initial answers. Then, in partners or in groups, they can discuss their answers and come to final conclusions that will help them find the important information initially identified by the teacher. After deciphering the text through close reading, students will be able to complete the given activity.

When would I use Close Reading with Guiding Questions in the classroom?

Close Reading with Guiding Questions can be used for any activity in which literacy could be a barrier to learning or demonstrating mastery of mathematical concepts. The number of questions and length of the discussions can be altered based on the length, importance, and difficulty of the text and concept. As students become more accustomed to mathematical literacy, the text complexity can be increased, but the adherence to close reading strategies must be maintained to ensure students can access the mathematical concepts. The length of time spent on the literacy aspect can be shortened as students become more skilled, but the questioning and discussions must occur to ensure students are properly interpreting the text in the mathematical context.

How can I use Close Reading with Guiding Questions with students needing additional support?

For struggling readers, including ESLs, Close Reading with Guiding Questions can help make an intimidating lesson, word problem, or task much more accessible. Questions focusing more on Tier 2 and Tier 3 vocabulary, text features, and real-world concepts can help struggling readers relate to the text and learn how to decipher the text in context. Discussions around the questions will help students grasp the math concepts.

Allowing struggling readers to explain their answers using words, numbers, or graphics/pictures ensures that they can express their opinion and rationale despite a potential lack of vocabulary. Through these representations and the ensuing discussion, students will begin to learn the necessary vocabulary to be successful.

What other standards does Close Reading with Guiding Questions address?

Standards of Mathematical Practice:

- SMP.1
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7

Sources

- Anne Adams, Jerine Pegg, and Melissa Case. "Anticipation Guides: Reading for Mathematics Understanding."

  https://www.walch.com/rr/09053

- Diane Staehr Fenner and Sydney Snyder. "Creating Text Dependent Questions for ELLs: Examples for 6th to 8th Grade."

  https://www.walch.com/rr/09054

Literacy Strategies

Text-to-Speech Technology

What is Text-to-Speech Technology?

Text-to-Speech Technology is an adaptive technology that reads text aloud from a text source for students. It is usually accessed through an application or program on a computer, smartphone, or tablet. Some new programs utilize Mathematical Markup Language (MathML) to read mathematical notation in a common, understandable manner for students. Many programs also highlight the words and notation on the screen as the audio plays, which helps students relate the written representation to the words they hear. The use of Text-to-Speech Technology allows students who struggle with literacy to hear the words and notation and access the text in a different way.

How do you implement Text-to-Speech Technology?

A classroom community focused on everyone's learning and a growth mindset is the first step in implementing Text-to-Speech Technology. One of the main barriers to implementation is encouraging students to use the program. Once they do, they will realize how the audio can help them understand the difficult mathematical texts and interpret the math content within them. After students realize the benefits of Text-to-Speech Technology, it can become part of the regular routine for group and independent work.

The use of headphones can be very important for effective use of Text-to-Speech Technology. Students can use the technology to listen to lessons and texts at their own pace. Extra noise from other students working or other students listening at different paces can confuse students attempting to use Text-to-Speech Technology, and headphones can help mitigate these distractions. Many teachers are nervous about the potential disruption headphones can cause in class. However, well-managed use of headphones can help students successfully utilize the technology to learn.

When would I use Text-to-Speech Technology in the classroom?

Text-to-Speech Technology can be used at any time throughout the year, and if the program speaks in MathML, it can be used with any lesson. Without MathML, effective use could be limited to word problems without unusual notation. For example, if x^2 is read as "x-two" instead of "x-squared" or "x to the second power," that could confuse students more.

During a lesson or small group discussion, Text-to-Speech Technology could detract from students' ability to listen, question, and process information. However, during warm-ups, independent work, or assessments, Text-to-Speech Technology can help students process the information and access the activity. It can become a routine for students to automatically listen to the question, problem, or directions first, and then attempt the activity.

How can I use Text-to-Speech Technology with students needing additional support?

Text-to-Speech Technology is an important adaptation and accommodation for struggling readers. Students who have read-aloud accommodations sometimes don't receive them because they are either embarrassed to accept them or because of staffing restrictions. These students can use Text-to-Speech Technology to supplement their math instruction by having text automatically read to them in a manner in which they can process it.

Additionally, for ESL students, hearing the English mathematical language, especially referring to mathematical representations and notation, can help put English words to the ideas they see. Some Text-to-Speech Technology can translate written and mathematical text into other languages, so students can hear the text in their natural language and see the English highlighted on the screen as they hear it. In this way, students are learning English vocabulary as well as learning the mathematical content in a language they can understand.

What other standards does Text-to-Speech Technology address?

Standards of Mathematical Practice:

- SMP.1
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7

Source

- Steve Noble. "Using Mathematics eText in the Classroom: What the Research Tells Us." https://www.walch.com/rr/09055

Literacy Strategies

Concept-Picture-Word Wall

more> than
Concept/Word Cards

What is a Concept-Picture-Word Wall?

A Concept-Picture-Word Wall is a classroom display, often a bulletin board or a set of posters, that exposes students to important vocabulary words they will use in math class.

Posting vocabulary words in class helps reinforce the words students will see in textbooks, videos, websites, and test questions on math concepts. These Tier 3 vocabulary words are often not used in everyday language, and the exposure to the words visually through Concept-Picture-Word Walls can help students connect them to the math content.

How do you implement Concept-Picture-Word Walls in the classroom?

Just seeing the vocabulary on a Concept-Picture-Word Wall by itself will help students; more importantly, referring to the words as the teacher uses them in class helps students connect the visual to the application. A simple gesture to the wall makes a very explicit reference to the word as it is used and allows students to connect the unfamiliar word to its meaning in context. Additionally, students can be taught to refer to the wall as they use the words in class, and they can be asked to make sure they say at least 3 words from the wall during each class period in small-group discourse or as answers to whole-class questions. The comfort gained from using these Tier 3 words will help students to use appropriate math vocabulary while solving problems and will help students connect concepts more explicitly.

Postings on the Concept-Picture-Word Wall can be arranged strategically to connect concepts, units of study, or groups of words where appropriate. Having three sections of the Concept-Picture-Word Wall—for example, an "In the Future" section, a "Live in the Present" section, and a "Remember the Past" section——can help students see and remember the vocabulary throughout the entire course. Even without regular use of some words, just seeing the words before a unit can help instill a familiarity with the vocabulary. Leaving the words on the Concept-Picture-Word Wall after a unit is taught can help students connect "old" concepts to the current lesson and ensure that students still have access to the vocabulary.

When would I use Concept-Picture-Word Walls in the classroom?

Concept-Picture-Word Walls can be used for the entire year. The actual words might have to change, or at least be moved to different areas of the Concept-Picture-Word wall. The more exposure students have to the words, the more familiar and comfortable they will become. The constant exposure to the math context is beneficial for students throughout the entire course, especially for words with multiple meanings (bias, tangent, etc.) that could exist as Tier 2 words in everyday conversation but are Tier 3 words in the math classroom.

How can I use Concept-Picture-Word Walls with students needing additional support?

For all students learning mathematics, knowing and using the math vocabulary is often a major barrier. This is a problem especially for ESL students, who are learning the English language along with math content. If teachers try to simplify the words too much for students, it does them a disservice as they seek out information from other teachers, textbooks, and online sources that use the proper vocabulary. Most tests, especially state tests, will expect students to have knowledge of the Tier 3, math-specific vocabulary. The more students see these words, the more familiarity they will have when they apply them.

Concept-Picture-Word Walls can also be written in multiple languages. Especially for students who are on-grade-level in their native language, a multi-lingual Concept-Picture-Word Wall can help students connect the content they already know in another language to the English vocabulary necessary for success on English-language math activities and tests.

This website can help you get started on an English-Spanish Concept-Picture-Word Wall: https://www.walch.com/rr/09056

What other standards do Concept-Picture-Word Walls address?

Standards of Mathematical Practice:

- SMP.1
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7

Source

- Janis M. Harmon, Karen D. Wood, Wanda B. Hedrick, Jean Vintinner, and Terri Willeford. "Interactive Word Walls: More Than Just Reading the Writing on the Walls."
  https://www.walch.com/rr/09057

Literacy Strategies

Novel Ideas

What is Novel Ideas?

Novel Ideas is a classroom activity that explores students' understanding of important Tier 2 vocabulary words they will use in math class. Instead of asking students to look up vocabulary words in the dictionary, Novel Ideas allows students to have conversations with their peers about vocabulary words in class. This reinforces the mathematical vocabulary students will see in textbooks, videos, websites, and test questions. These Tier 2 vocabulary words are often used in everyday language, but have specific meaning in mathematics. Exposure to the words through Novel Ideas can help students connect them to the math content.

How do you implement Novel Ideas in the classroom?

While building a rich representation of math content words and connecting the words to other words and concepts has inherent merit, it is more important to consider that pre-teaching the words before they are used in class helps students connect to the application. The understanding gained from discussing these Tier 2 words will help students apply them in a mathematical context to solve problems and connect concepts.

Here is a step-by-step process for implementing Novel Ideas:

1. Students separate into groups of four.
2. Students copy the teacher generated prompt/sentence starters and number their papers 1–8.
3. One student offers an idea, another echoes it, and all write it down.
4. After three minutes, students draw a line under the last item in the list.
5. All students stand, and the teacher calls one student from a group to read the group's list.
6. The student starts by reading the prompt/sentence starters, "We think a _____ called _____ may be about ... ," and then adds whatever ideas the team has agreed on.
7. The rest of the class must pay attention because after the first group has presented all their ideas, the teacher asks them to sit down and calls on a student from another team to add that team's "novel ideas only." Ideas that have already been presented cannot be repeated.
8. As teams complete their turns and sit down, each seated student should record novel ideas from other groups below the line that marks the end of his or her team's ideas.

When would I use Novel Ideas in the classroom?

Novel Ideas can be used for the entire year. The more students are exposed to mathematical vocabulary, the more familiar and comfortable they become, leading to increased usage of these math terms in their conversation and writing. Using math vocabulary in context is beneficial for students throughout the entire course, especially for words with multiple meanings (bias, tangent, etc.) that could exist as Tier 2 words in everyday conversation but are Tier 3 words in the math classroom.

How can I use Novel Ideas with students needing additional support?

Most tests, especially state tests, will expect students to have knowledge of the Tier 3, math-specific vocabulary. The more students use these words in conversation, the more familiarity they will have when they apply them. Understanding Tier 2 words also helps students avoid misconceptions in mathematics. Twice a week before the start of a lesson, allow students to use sentence starters in small groups that include all students. Prepare the sentence starter "When I hear the word _____, I think about _____" to share out with whole class. This will allow students who know the vocabulary words to share their knowledge, and will allow other students to hear the meaning of the vocabulary words. This strategy is particularly helpful for ESL students.

What other standards does Novel Ideas address?

Standards of Mathematical Practice:

- SMP.1
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7

Sources

- Colorín Colorado. "Selecting Vocabulary Words to Teach English Language Learners."

  https://www.walch.com/rr/09058

- Elsa Billings and Peggy Mueller, WestEd. "Quality Student Interactions: Why Are They Crucial to Language Learning and How Can We Support Them?"

  https://www.walch.com/rr/09059

Novel Ideas Sentence Starters

Slope

- When I hear the word climb, I think about ...
- When I hear the word steep, I think about ...

Volume

- When I hear the word filling, I think about ...

Equations

- When I hear the word balance, I think about ...
- When I hear the word equal, I think about ...

Graphing

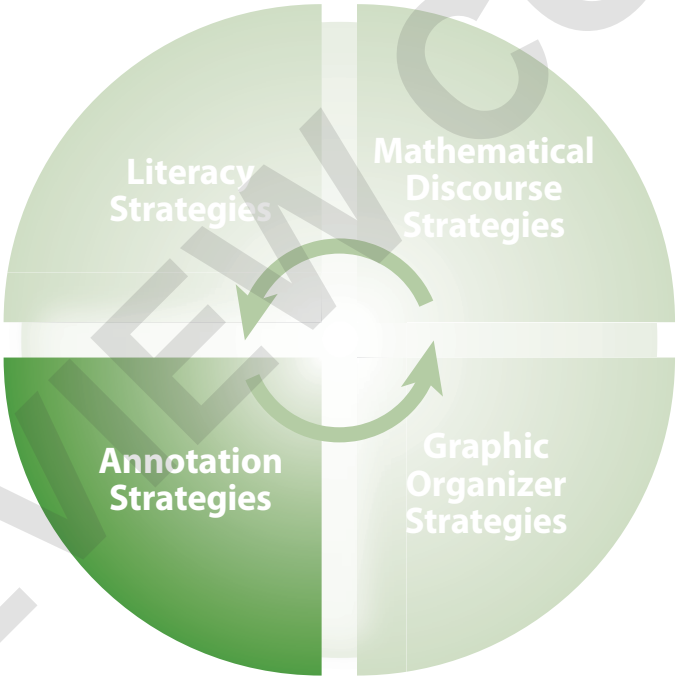
- When I hear the word grid, I think about ...
- When I hear the word graph, I think about ...

Scatter Plots

- When I hear the word scattered, I think about ...

Understanding Mathematical Content: Annotation

Understanding mathematical content is an extremely important skill, both in the math classroom and in life. When students read word problems, articles, charts, graphs, equations, tables, or other forms of mathematical text, they must be able to decode and extract meaning from the text. Annotation can help. The strategies presented here help students identify and focus on key characteristics and facts from various forms of text while ignoring the non-essential information. For students with disabilities, many of whom struggle with the distractions inherent in many high-school level texts, making notes and drawing pictures to explain a problem can help them focus. ESL students will be pointed to certain Tier 3 vocabulary words and determine which Tier 2 vocabulary words they must learn to be proficient in math class and in the English language. Annotation strategies include Reverse Annotation and CUBES protocol.



Annotation Strategies

Reverse Annotation Protocol

What is Reverse Annotation?

Reverse Annotation is a strategy that asks students to identify and write down key information from math problems. This is especially helpful for problems given on a computer or tablet, where students can't annotate directly on the problem. A template is given at the end of this section.

How do you implement Reverse Annotation in the classroom?

Many annotation strategies ask students to write, underline, or mark directly on the text of a problem. While those forms of annotation are also beneficial, they are not always possible with technology. Whether the problem is given on paper or using technology, having students write the answers to these questions will ensure that they are thinking strategically and specifically about the strategies and information needed to solve the problem.

The three questions at the top of the Reverse Annotation template are the key to understanding mathematical problems. For every problem given in class, ask students:

1. What is the problem asking us to solve?
2. What key words tell us the mathematical steps we need to perform?
3. What information in the problem can help us figure it out?

After answering the initial questions, students should make a guess, or estimate, of what they think the answer will be. This helps grow their number sense, and provides an initial, reasonable solution to guide their work. Students can then use the strategies they selected to solve the problem and evaluate their solution using the questions at the bottom of the template.

When students first begin to use Reverse Annotation, the teacher should walk them through the steps individually to ensure they can accurately identify the question, key words, and important information. Teachers can also lead students through the estimation process, making a game out of which student has the closest estimate.

Work through each step individually for several "easy" problems first, so that difficult math doesn't interfere with the process. Increase the problem difficulty incrementally as students begin to master the process. This may seem like a long process at first, but the ultimate result is worth the time investment.

When would I use Reverse Annotation in the classroom?

Reverse Annotation can be used to solve any math problem, and is especially helpful for word problems. When Reverse Annotation is initially implemented, the steps should be discussed in detail. As students become accustomed to Reverse Annotation and begin thinking about problems in this manner automatically, the individual steps become less important and can be scaffolded out to improve efficiency. Students should reach the point where they immediately ask themselves the three

initial questions when they first see a problem. However, the teacher should ensure that students are truly evaluating all the key information before routine discussions of the individual steps are removed.

How can I use Reverse Annotation with students needing additional support?

Annotation strategies can help students identify key information, even when certain vocabulary words are not known. As teachers introduce the content-specific Tier 3 vocabulary to their classes, annotation strategies such as reverse annotation can help students use these words to apply appropriate strategies while problem solving. Answering the three initial questions can help students organize the key facts and vocabulary, and the identification of key information can simplify the problem. This strategy is especially beneficial for ESL students.

Using reverse annotation with graphic organizers benefits ESL students by removing a lot of the confusing wording and allowing them to focus on the important pieces of a problem. When using Reverse Annotation, all students, including ESL students, will begin to think about problem solving in a way that encourages them to use the appropriate information to find a solution.

What other standards does the Reverse Annotation Protocol address?

Standards of Mathematical Practice:

- SMP.1
- SMP.2
- SMP.5
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.SL.9–10.3
- ELA–LITERACY.SL.9–10.2
- ELA–LITERACY.RST.9–10.4

Source

- Alliance for Excellent Education. "Six Key Strategies for Teachers of English Language Learners." https://www.walch.com/rr/09060

Reverse Annotation Template

Name: _____ Problem/Assignment: _____

Analyze the Problem

What is the problem asking us to solve?	
What key words will tell us the mathematical steps we need to perform?	
What information in the problem can help us figure it out?	

Initial estimate of solution:

Work Space

Remember to box in your solution!

Name: _____ Problem/Assignment: _____

Check It Over

How close was your estimate?	
Does your answer make sense? Is it reasonable? How do you know?	
Did you perform the calculations correctly?	
What does your answer mean in context?	

Annotation Strategies

CUBES Protocol

What is the annotation strategy CUBES?

CUBES is an annotation strategy in which students use different written designs to highlight the key aspects of word problems. It can help them choose the correct mathematical strategy to solve the problem accurately.

How do you implement CUBES in the classroom?

The steps for CUBES are:

1. **C**: **C**ircle all the key numbers.
2. **U**: **U**nderline the question.
3. **B**: **B**ox in the key words that will determine the operation(s) necessary and write the mathematical symbol for the operation(s).
4. **E**: **E**valuate the information given to determine the strategy needed. **E**liminate any unnecessary information.
5. **S**: **S**olve the problem, **s**how your work, and check your answer.

As students learn to use CUBES, walk them through the steps individually to ensure they can accurately identify the key numbers, question, key words, unnecessary information, and strategy. Work through each step individually for several "easy" problems first, so that difficult math doesn't interfere with the process. Increase the problem difficulty incrementally as students begin to master the process. This may seem like a long process at first, but the ultimate result is worth the time investment.

A graphic organizer can help students master the process, especially when problems are given on a computer or tablet where students can't always annotate directly on the problem. Students can write down the key numbers and circle them, write down the question and underline it, and so on. This will encourage students to truly think about the different pieces of the problem they are identifying, and how these pieces will guide the strategy and affect the solution.

When would I use CUBES in the classroom?

CUBES can be used to solve any math problem, and is especially helpful for word problems. When CUBES is initially implemented, the steps should be discussed in detail. As students become accustomed to using CUBES and begin thinking about problems in this manner automatically, the individual steps become less important and can be scaffolded out to improve efficiency. However, the teacher should ensure that students are truly evaluating all the key information before routine discussions of the individual steps are removed.

How can I use CUBES with students needing additional support?

Design features can help students identify key words and features, even when certain vocabulary words are not known. As teachers introduce the content-specific Tier 3 vocabulary to their classes, annotation strategies such as CUBES can help students use these words to apply appropriate strategies while problem solving. Using circles, underlines, and boxes can help students organize the key facts and vocabulary, and the elimination of unnecessary information can simplify the problem. This strategy is especially beneficial for ESL students.

Combining CUBES with graphic organizers also benefits ESL students by removing a lot of the confusing wording and allowing them to focus on the important facts of a problem. When using CUBES with a graphic organizer, all students, including ESL students, will begin to think about problem solving in a way that helps encourage them to use the appropriate information to find a solution.

What other standards does the CUBES Protocol address?

Standards of Mathematical Practice:

- SMP.1
- SMP.2
- SMP.5
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.SL.9–10.3
- ELA–LITERACY.SL.9–10.2
- ELA–LITERACY.RST.9–10.4

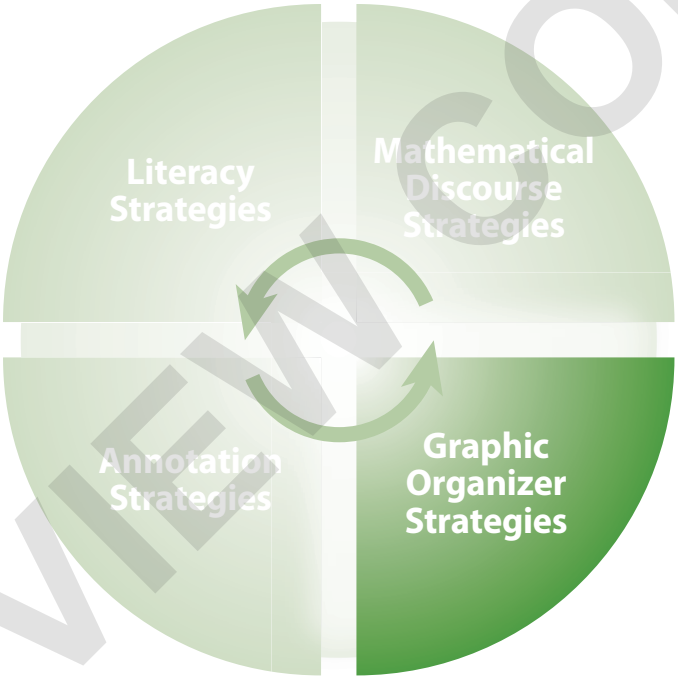
Source

- Margaret Tibbett. “Comparing the effectiveness of two verbal problem solving strategies: Solve It! and CUBES.”

  https://www.walch.com/rr/09061

Organizing Mathematical Content: Graphic Organizers

Organizing mathematical content is a crucial skill for problem solving, exploring other possible methods for finding solutions, and managing math content. All students need strategies for organizing content to build conceptual understanding. For students with disabilities, visual representations and graphic organizers can help them clarify their thoughts and focus on the math. ESL students also benefit from visual representations and graphic organizers. Organizing mathematical knowledge with visuals can help ESL students navigate math content while learning the language. Graphic organizers include Frayer Models and Tables of Values.



Graphic Organizers

Frayer Models

What is a Frayer Model?

A Frayer Model is a graphic organizer that can help students understand new vocabulary words and concepts by exploring their characteristics. A Frayer model lists the definition of a word or concept, describes some key facts, and gives examples and non-examples. Examples and non-examples can come from a mathematical or real-world context.

How do you implement Frayer Models in the classroom?

Students can learn to create Frayer Models the first week of school, and the process can be used throughout the year each time students experience a new word or concept.

While it is important for teachers to give students precise mathematical definitions with appropriate content vocabulary, it is maybe more important for students to understand the application of mathematical words and concepts in their own context. As students learn new information, small group discussions and think-pair-share activities are great ways for students to formulate their own definitions, review the characteristics and facts they have learned, and discuss examples and non-examples.

Discussions of the examples and non-examples can help lead to the mathematical definition. For example, if students use a Frayer Model to define a quadratic function, they would notice that all examples have a highest exponent of 2, and all non-examples would not have a highest exponent of 2. All examples would have parabolic graphs, and all non-examples would have other graphs. Through these comparisons, students will understand the definition of quadratics using different representations, and they will be able to apply it in different contexts.

When would I use Frayer Models in the classroom?

Frayer Models can be used at different points during instruction. They are appropriate as introductions to new concepts, summaries to ensure understanding of new concepts, or as note-organizers throughout the lesson for students to fill in as they learn new concepts. At first, students might need help figuring out how to list and differentiate between the definition, facts and characteristics, examples, and non-examples. As students adapt to the process, they will be able to categorize information on their own or in small groups. As they compare newer Frayer Models to previous models, they will also be able to see how concepts build upon each other.

How can I use Frayer Models with students needing additional support?

Frayer Models can be a point of reference for students as they progress throughout the year. As students determine their own definitions for math-specific words and concepts, and use the examples

and non-examples to determine the key facts, they will be able to put them in their own context and apply them to solve complicated problems. As math concepts build upon each other both within a unit and throughout the year, the use of Frayer Models to remind students of their initial definitions of words or concepts can help solidify their understanding. Using Frayer Models as part of a Word Wall or Concept Wall, or having a consistent notebook process to reference past Frayer models, can help consistently reinforce learning.

What other standards do Frayer Models address?

Standards of Mathematical Practice:

- SMP.1
- SMP.2
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.1
- ELA–LITERACY.SL.9–10.1
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4

Source

- Deborah K. Reed. "Building Vocabulary and Conceptual Knowledge Using the Frayer Model." https://www.walch.com/rr/09062

Frayer Model

Definition	Characteristics
WORD	
Examples from Life	Non-Examples

Graphic Organizers

Tables of Values

What is a Table of Values?

A Table of Values is an organized way to list numbers that represent different categories of values. These values can be represented as ordered pairs, graphs, word problems, or lists. Tables can help students see and compare values in a different way.

How do you implement Tables of Values in the classroom?

Tables can be used throughout the year to support various mathematical standards. Some standards mention tables specifically, and in others, tables can be an effective support to help students organize and understand the meaning and application of values.

Tables can be set up with numerical values in rows or columns. The key to understanding the values lies in the headings. The headings must be specific enough to show students the meaning and/or application of the numerical values, but not so wordy that they interfere with the clarity of the numbers in the table. For example:

x (year)	*y* (population in millions)
1960	219
1970	230
1980	258
1990	312
2000	342

Mean (statistical average)	50	45
Median (middle value)	52	43
Quartile 1 (median of the lower 50%)	40	38
Quartile 3 (median of the upper 50%)	72	80
Range (difference of max and min values)	80	61
Interquartile Range (difference of quartiles)	32	42
Standard Deviation (measure of spread of data)	7.24	10.23

When would I use Tables of Values in the classroom?

Various mathematical topics can be represented by tables. For example:

- An (x, y) table of values to represent coordinates on a graph or independent and dependent variables for a given context
- A table to represent coefficients and/or constants in an equation
- A table to show different statistical measures when comparing sets of data
- A table to compare output values for the same input given different functions

Each time numbers or values are being listed, compared, or graphed, a table can help students differentiate between the values. Tables are easy to create, and students can be encouraged to create them as another representation to clarify and compare numbers for nearly any topic.

How can I use Tables of Values with students needing additional support?

Tables of Values can help students focus on numerical values and their meaning in context without distraction. They clarify what each number represents, what numbers can be compared, and what ordered pairs can be graphed to give a visual representation. Additionally, headings can be used to either highlight the relevant facts from a context or to describe mathematical vocabulary.

In general, graphic organizers benefit students by removing much of the confusing wording and focusing on the important facts and numbers of a problem.

What other standards do Tables of Values address?

Standards of Mathematical Practice:

- SMP.1
- SMP.2
- SMP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

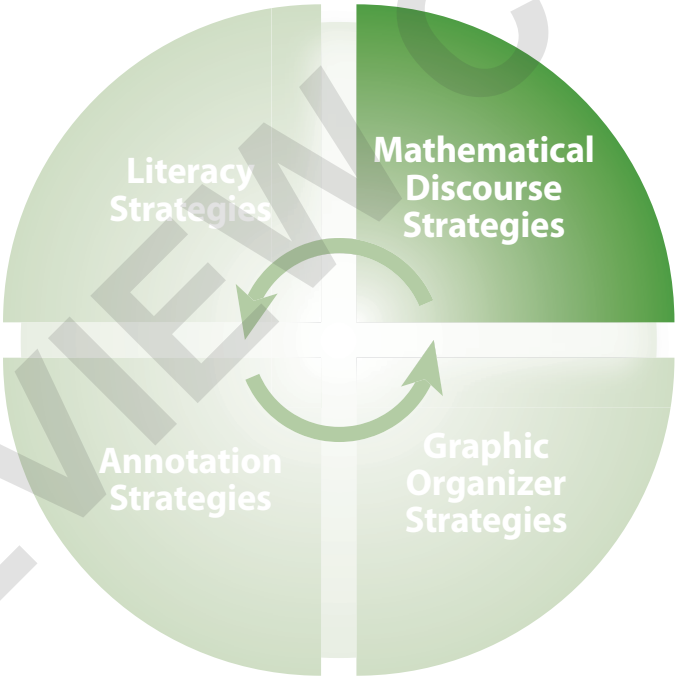
- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.1
- ELA–LITERACY.SL.9–10.1
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4

Source

- Alliance for Excellent Education. "Six Key Strategies for Teachers of English Language Learners." https://www.walch.com/rr/09060

Communicating Mathematical Content: Mathematical Discourse

Reading, writing, speaking, and listening are all important ways to learn and express information, but the last two ways are often slighted in the math classroom. The mathematical discourse strategies presented here promote speaking and listening in a math-focused literacy context. Working these strategies into the daily routine of a classroom can help students become comfortable speaking and listening in a mathematical context, which will help them become comfortable with the mathematical content. Routines and structures are essential to support students with disabilities, as they often benefit from following a routine. This can lead to developing capability in their mathematical skills. These strategies also remove the barrier to entry for many ESL students, as structure and routine can help them focus on the math content rather than English language deficiencies. Mathematical Discourse strategies include Sentence Starters and Small Group Discussion.



Mathematical Discourse Strategies

Sentence Starters

What is a Sentence Starter?

A Sentence Starter is a common phrase or mathematical sentence frame that can help students begin and sustain academic conversations around mathematical content. It helps guide students through the discussion and bring out pertinent ideas that can lead to greater understanding.

How do you implement Sentence Starters in the classroom?

Many people view math class as a place to calculate solutions to math problems. However, to ensure the conceptual understanding and proper application of a math concept, students need to be able to explain the concepts and reasoning behind a solution to a problem. As many students are not accustomed to having academic conversations about math, sentence starters can help begin and continue these conversations in a productive manner.

There are two main types of sentence starters for mathematical discussions: discourse starters and math starters. For example, a poster with these or other sentence starters can be displayed from the beginning of the year, and the expectation can be set that any answer to a question or comment in a discussion should be framed using one of these starters. As students become accustomed to framing mathematical conversations in this way, they can expand on the given sentence starters and create some of their own. They will begin to realize how these statements ensure that their conversations revolve around math, enhance understanding of the concept, and force them not only to state, but also to explain their thinking. They will gain confidence from the ability to engage, as the first step has already been taken for them.

When would I use Sentence Starters in the classroom?

Sentence Starters can be used throughout the entire school year with any concept. However, they are most important to use at the beginning of the school year to build a mathematical community in the classroom centered on a comfort with mathematical discourse. Especially at the beginning of the year, students should be encouraged to use these sentence starters for every math statement. Appropriate settings include during small group discussion, while responding to whole class questions, and when writing explanations for problem solutions.

Modifications can be introduced so that students must use certain mathematical vocabulary within the sentences, or must use certain sentence starters at different points in conversations or for different conversation types and situations. However the starters are implemented, it is important for students to realize that these are intended to enhance and focus their conversations, not limit them.

How can I use Sentence Starters with students needing additional support?

Often, students are reluctant to talk about math concepts because they either lack confidence in their knowledge, are afraid to be "wrong," or don't know how to start or continue the conversation. Sentence starters can help students overcome this reluctance. The non-threatening, easy-to-interpret sentence starters remove the barrier to entry for students who don't know how to engage, and the respectful, mathematical focus promoted by sentence starters can help build confidence and provide a structure so that students will not fear being wrong.

For ESL students specifically, sentence starters can provide the English language support to help students engage with and discuss the math. The support of sentence structure removes language barriers to entry for students who don't fully understand English sentence structure.

Discourse Starters	Math Starters
I agree/disagree with ... because ...	My answer was ... because ...
I understand/don't understand ...	The next step is ... because ...
First/Next/Finally I ... because ...	I used (insert formula/equation/concept) because ...
I noticed that ...	
I wonder ...	My answer is right/reasonable because ...

What other standards do Sentence Starters address?

WIDA English Language Development Standards

- ELD Standard 3

Standards of Mathematical Practice:

- SMP.1
- SMP.3
- SMP.6

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.1
- ELA–LITERACY.SL.9–10.1
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4

Source

- AVID. "Sentence Starters." https://www.walch.com/rr/09064

Mathematical Discourse Strategies

Small Group Discussion

What is Small Group Discussion?

Small Group Discussion is a structured way for students to verbalize their mathematical thinking in a comfortable setting to solve a problem, build conceptual understanding, or summarize a concept.

How do you implement Small Group Discussion?

Small Group Discussion in math class depends on a trusting relationship between the teacher and the students. From there, students can build trusting relationships among themselves. Once this trust has been built, students will feel free to explore mathematical topics in groups, take risks, and engage in a productive struggle toward understanding or a solution.

Once these relationships have been established, certain structures should be established for Small Group Discussion to be effective. Discussion norms can be set by the class to ensure discussions are respectful and productive, and discussions should have predetermined time limits. The group composition is also important and should be based on instructional measures. For different activities, homogeneous groups, heterogeneous groups, or groups based on specific data by standard could be appropriate. Students should always be aware that the groups were chosen to maximize their learning.

Another structure that can be effective for Small Group Discussion is assigning group roles. These roles can include group leader, note taker, timekeeper, resource manager, culture keeper, or other roles determined to be appropriate for the classroom context. During the discussion, assigning each student a letter within the group (A, B, C, D, etc.) can help structure the discussion. Different roles can specify certain time limits for talk, which sentence starters to use, or other structured aspects of the discussion.

When implementing a Small Group Discussion, the question or task should inspire students to think in different ways about a concept. Through the structured format of the discussion, students will compare their ideas and arrive at an answer or explanation of the concept. Within the trusting framework of the class and group, students can focus on the common goal of the discussion and develop their thinking around the math concept. These rich discussions will enhance their understanding.

When would I use Small Group Discussion in the classroom?

Small Group Discussion can be used for nearly any topic, and it can be used at a variety of times in the classroom. The questions and tasks may need to change depending on when it is used. Opening activities for lessons can be Small Group Discussions where students explore properties of new math concepts or review/build upon their prior learning. Turn and talks throughout the lesson can be structured as Small Group Discussions if a consistent framework is in place. At the end of class, a Small Group Discussion can be used to come to a common understanding about an essential question from the lesson.

Depending on when the Small Group Discussion is used in class, and what the goal of the discussion is, the discussion reporting may vary. For a warm-up, each group might be asked to share their thinking. For a guided practice, recording answers on chart paper and a gallery walk could be appropriate. For a closing activity, individual written responses to a question could be appropriate.

How can I use Small Group Discussion with students needing additional support?

As discussed in other Mathematical Discourse strategies, struggling students are reluctant to talk about math concepts because they lack confidence in their knowledge and don't always have the needed vocabulary in their toolbox. Structured discussions with effective grouping can help students through these barriers. After a trusting and respectful classroom environment has been established, struggling students often feel more comfortable sharing their ideas with just a few classmates rather than the whole class. Additionally, adding structure can help students engage by providing the expectation that they participate in the process.

The intentional grouping of students can also help them succeed using Small Group Discussion. At times, heterogeneous groups could be appropriate so that stronger students can help struggling students, and at other times, homogeneous groups could be appropriate so the teacher can work with an entire group of struggling students. ESL students can be grouped with other students with the same dominant language to help remove the language barrier from the conversation.

What other standards does Small Group Discussion address?

WIDA English Language Development Standards:

- ELD Standard 3

Standards of Mathematical Practice:

- SMP.1
- SMP.3
- SMP.6

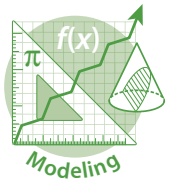
Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.1
- ELA–LITERACY.SL.9–10.1
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4

Source

- Jessie C. Store. "Developing Mathematical Practices: Small Group Discussions." https://www.walch.com/rr/09065

Modeling Strategies



Mathematical Modeling

What is Mathematical Modeling?

Mathematical modeling is generally understood as the process of applying mathematics to a real-world problem with a view of understanding the connection. According to the CCSSM, mathematical modeling is the ability to apply concepts learned in class to real-world applications and to use the model to analyze a situation, draw conclusions, and make predictions.

How do you implement Mathematical Modeling in the classroom?

Modeling can be implemented by demonstrating how to make or generate mathematical representations or models, how to validate them, and how to use them to solve real-world problems. There are many ways to show understanding in a math classroom, such as using words, drawings or sketches, physical models, computer programs, or math formulas.

The following is a list of questions and answers suggested in order to create a mathematical modeling classroom environment:

- **Why?** What are we looking for? Identify the need for the model.
- **Find?** What do we want to know? List the data we are seeking.
- **Given?** What do we know? Identify the available relevant data.
- **Assume?** What can we assume? Identify the circumstances that apply.
- **How?** How should we look at this model? Identify the parameters.
- **Predict?** What will our model predict? Identify the equations that will be used, the calculations that will be made, and the answers that will result.
- **Valid?** Are the predictions valid? Identify tests that can be made to validate the model; i.e., is it consistent with its principles and assumptions?
- **Verified?** Are the predictions good? Identify tests that can be made to verify the model; i.e., is it useful in terms of the initial reason it was done? *(inspired by Carson and Cobelli, 2001)*

Teachers should expect these questions to recur often during the modeling process, and should regard this list as a fairly general approach to ways of thinking about mathematical modeling.

In a classroom where mathematical modeling is the expectation, teachers will need to establish that students are responsible for coming up with methods for solving the problems presented and that the teacher will only assist and facilitate.

When would I use Mathematical Modeling in the classroom?

It should come as no surprise that many students find mathematics boring. The most common question posed to any mathematics teacher is "When will I ever need to use this?" Often teachers fail to find problems in which students are interested or to even take student interest into account when planning a lesson. Problems that spark students' interest and curiosity will increase their attention and desire to learn. These types of real-world problems provide students an opportunity to think and respond as a mathematician. Students should be exposed to rigorous learning tasks that allow opportunities for mathematical modeling in the classroom.

How can I use Mathematical Modeling with struggling students?

When struggling readers, which includes ELLs and students with learning disabilities, are exposed to rigorous math learning tasks, there must be a level of scaffolding that includes coaching and guided questions that help to make a word problem or learning task much more accessible. Teachers should come up with questions to guide the students before and during the engagement of the task. Teachers should also:

- assess prior knowledge;
- define Tier 2 and 3 vocabulary words;
- discuss non-mathematical concepts in the task; and
- assist students in identifying key concepts and facts within the tasks.

Allowing struggling readers to explain their answers using words, numbers, or graphics/pictures ensures that they can express their opinion and rationale despite a potential lack of vocabulary. Through these representations and the ensuing discussion, students will begin to learn the necessary math concepts to be successful.

What other standards does Mathematical Modeling address?

WIDA English Language Development Standards:

- ELD Standard 3

Standards of Mathematical Practice:

- SMP.1
- SMP.2
- SMP.4
- SMP.5

English Language Development for Mathematics:

- ELD–A.9–12: Explain (Interpretive)
- ELD–MA.9–12: Explain (Expressive)

English Language Arts standards:

- ELA–LITERACY.SL.9–10.2
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7
- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9

Sources

- Carson, Ewart, and Cobelli, Claudio. *Modeling Methodology for Physiology and Medicine*. Academic Press, San Diego, CA, 2001.
- Dym, Clive L. *Principles of Mathematical Modeling*. Amsterdam: Elsevier Academic Press, 2008.
- MathisFun.com. "Mathematical Models." https://www.walch.com/rr/09066
- Oswalt, Selena. "Mathematical Modeling in the High School Classroom." LSU Digital Commons. Accessed February 25, 2021. https://www.walch.com/rr/09067

PROGRAM OVERVIEW

Mathematical Modeling Implementation Guide

Introduction

Walch resources support the framework of the Standards for Mathematical Practice (SMPs) and the NCTM Principles of Teaching Practices. Implementing strategies and support from both practices lead to true conceptual understanding of the math standards. One of which includes mathematical modeling, the process of designing and revising representations to solve a problem.

Mathematical modeling is essential to building a deep conceptual understanding of math concepts for students. Teaching students to model boosts engagement, builds student confidence in math concepts, helps them to make sense of problems, and allows them to make connections to the world around them for better understanding. Students then make decisions about the information, create models, interpret the results, and form conclusions.

A Mathematical Modeling Framework

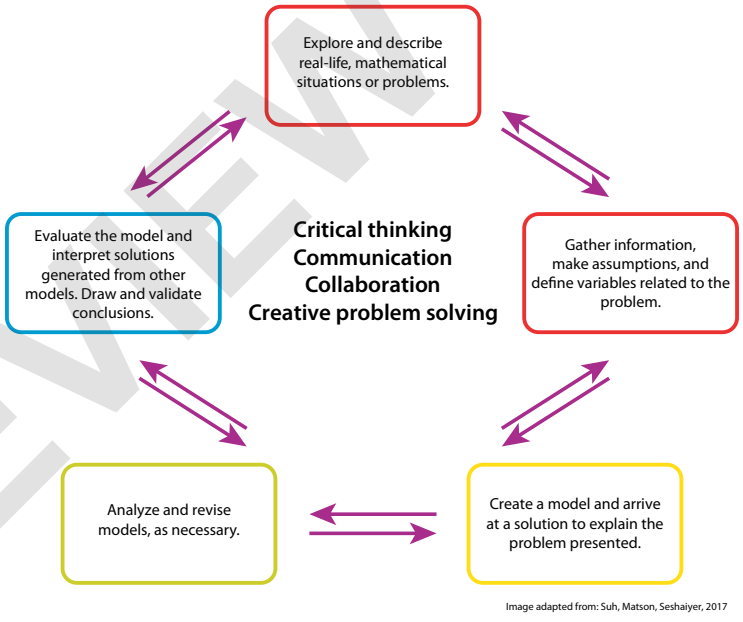


Image adapted from: Suh, Matson, Seshaiyer, 2017

The following is a brief description of how this framework can be applied in the classroom.

Critical Thinking

Students will explore and describe real-life mathematical situations or problems. We want students to discover new ways of thinking and ideas in mathematics. Students do this by developing questions to ask, gathering information, and coming up with solutions. Fostering critical thinking in the classroom

not only makes students better at math, but also prepares them for the real world. Below are some ideas and probing questions teachers may use to implement critical thinking.

- Allow for pair-share and small group discussions.
- Encourage students to think and form their own conclusions.
- Encourage the revision of their own thinking and the thinking of others.
- Ask students to think out loud as they work.
- Create a classroom environment that embraces and values student ideas.

Ask students:

- What is the problem asking you to solve?
- Can you think of other strategies you could use to solve this problem?
- What conclusions can you make from this particular problem?
- Will this strategy work in all problems like this? Why or why not? How can we test that?
- Explain how you got to your answer.
- Explain your reasoning.
- How would you respond to a different answer to the same problem?

Communication

When students gather information, make assumptions, and define variables related to the problem, communication allows for them to show their understanding of the math content. Encourage discourse by allowing students to explain their thinking and challenge each other. This encourages students to justify their reasoning. If students communicate their thinking in various ways (including written and oral responses) while doing math, it will improve their understanding of math concepts.

Teachers can do the following to foster communication in the classroom:

- Ask open-ended questions.
- Encourage oral and visual (written and pictorial) communication through journal writing.
- Provide students with detailed feedback.

Ask students:

- Can you explain your thinking?
- How did you get your answer?
- What strategies did you use?
- What information was necessary for you to solve this problem?

Collaboration

Collaboration is an essential component of student success. It allows students to rely on each other during their problem solving. During collaboration, students work in groups, share ideas, ask questions, and discuss math concepts and additional solution strategies while supporting and defending their thinking. Collaboration is most beneficial to students with the use of effective grouping strategies such as assigning students to heterogeneous groups or random grouping.

The following procedures and probing questions can help you implement collaboration in your classroom.

- Establish a classroom culture where all ideas are valued.
- Establish expectations and routines of collaborative learning.
- Discuss "math talk" passages with students.
- Allow students to teach each other.
- Incorporate an accountability piece for students.
- Arrange student seating to support collaboration (group seating).
- Create heterogeneous student groups with varying skill levels.
- Randomize student groups.
- Keep group sizes between 3 and 6 students.
- Assign group roles.

Ask students:

- Come up with as many strategies to solve the problem as you can.
- Explain how you made your calculations.
- Why did you choose that strategy? Why did that strategy work?
- Describe in your own words how your peer-solved the problem.
- Can you make any connections between your strategies?
- Were there any methods that were better than others when solving this problem? Why or why not?
- What did you learn from your group?
- Defend your reasoning behind that solution.

Creative Problem Solving

Creative problem solving is the ability for students to perform math tasks that allow for challenges that increase their conceptual understanding. While performing these tasks, we want students to use mathematical modeling. We want students to evaluate their models and to interpret solutions from other models.

In creative problem solving, students solve problems using different approaches and models, draw on prior knowledge, and justify their thinking. This results in students becoming better problem solvers and increases their understanding of math concepts. Problem solving should be integrated into their math learning and should not be separated.

Here are some tips for implementing creative problem solving.

- Encourage students to challenge different approaches and strategies from their peers as well as the teacher.
- Encourage discourse.
- Allow appropriate wait time for student responses.
- Refrain from telling students how to solve the problem. Instead, allow students to engage and come to their own solutions.
- Allow students to struggle productively.

Ask students:

- How is the information in the problem important to determining the solution?
- How did you go about solving this?
- Can you explain why you chose that model and strategy?
- Are there other ways to model this particular problem? Can you model the problem another way?
- Why did you make that calculation?
- Justify your solution.
- What generalizations can you make about the math concepts based on this particular problem?

Recommended Resource

- Georgia Department of Education. "Scaffolding Instruction for English Learners: A Georgia Mathematics Instructional Resource Guide."

  https://www.walch.com/rr/09047

  The purpose of this document is to provide mathematics teachers and leaders with evidence-based, pragmatic scaffolds and supports for English Learners (ELs). This guide is a useful tool to help teachers provide high-quality instruction aligned to Georgia's K-12 Mathematics Standards.

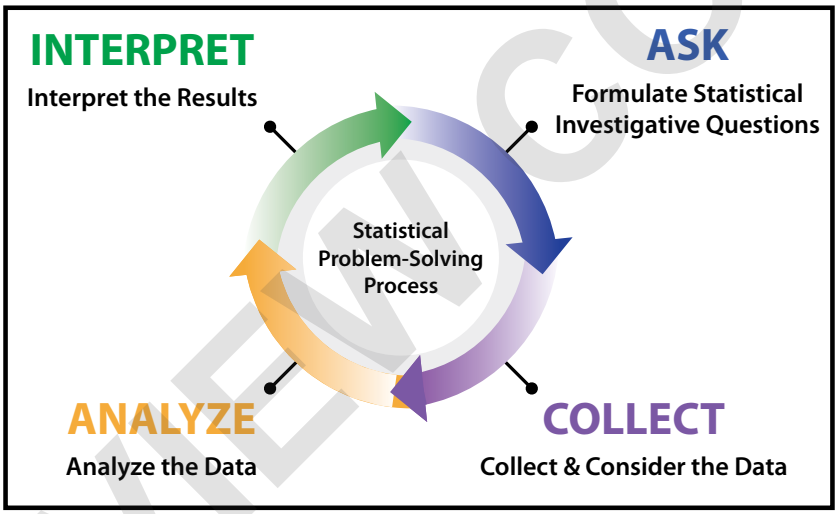
Source

National Council of Teachers of Mathematics. "Problem Solving." Accessed January 11, 2023. https://www.walch.com/rr/09048.

Statistical Reasoning Implementation Guide

Introduction

Statistical reasoning allows students to make sense of ideas, information, and the changing world through questioning and exploration. It provides the foundation necessary for students to fully understand the concept. Statistical reasoning is a continuous cycle consisting of students asking questions, collecting, analyzing, and interpreting data. In order to guide students in this sense-making process, Walch resources support this four-step statistical problem-solving strategy to help students develop their understanding in statistical reasoning.



Source: Georgia Department of Education

Here is a brief description of how this framework can be applied in the classroom.

Formulate Statistical Investigative Questions

Students will form and ask investigative questions that allow for various answers. These questions will clarify the problem and lead to questions that can be answered with the data. Best practices and teacher prompts that can foster this framework include:

- Using a student-centered approach.
- Having students prepare ahead of time with an assigned reading to familiarize themselves with words and techniques.

Ask students:

- What do you think?
- What do you notice? What do you wonder?
- What criteria need to be met in order for the question to be statistical?
- How did you determine your question?
- What changes would you make to the question?

Collect and Consider the Data

Students will collect data by creating a plan in order to collect real and relevant data. Making sure the data is relevant to students will increase engagement and lead to more math talk and discussion. Strategies include:

- Refraining from presenting students with procedures.
- Allowing students to use real data sets and to generate their own data.
- Encouraging students to discuss the questions and possible ideas.

Ask students:

- What do you notice about the data?
- In what other ways can the data be collected?
- What are some other methods you can use to collect the data? How do these different methods affect your data collection?
- How can you represent your data? Can you represent it with a visual?
- Are there representations better fit for particular findings? Justify your answer.

Analyze the Data

Students will analyze the data by selecting methods that are appropriate. Exploration of various methods will allow for students to make connections and draw conclusions based on the data. This will deepen their understanding of statistical reasoning. Strategies include:

- Allowing students to use technology tools to explore and analyze their findings.
- Refraining from giving students all the information. Allow students to form their own analysis of the data.
- Creating a classroom environment in which student ideas are valued.

Ask students:

- What conclusions can you draw from the data?
- Do you notice any trends in the data? How can you tell?
- What is the relationship between the data points?
- What evidence may help you distinguish between results?
- Do you agree or disagree? Justify your thinking.
- How can we test that conclusion?
- What do you do about outliers in your data? What do they tell you?
- If extreme values are removed, what happens to the data representation?
- Compare your data with a classmate's. What do you notice?

Interpret the Results

Students will interpret and discuss the results by relating all findings to the original question. Students will discuss these findings and justify their reasoning. Best practices and teacher prompts include:

- Encouraging discourse. Encourage students to present their ideas, answer classmates' questions, and support their responses.
- Focusing on key ideas instead of procedures and calculated answers.
- Making sure students have answered their "I wonder" questions.

Ask students:

- What do the results tell you about the original question?
- Have your "I wonder" questions been answered?
- What conclusions can you make from the results?
- Compare your interpretations to those of your classmates. What connections can you make?
- What do your interpretations represent in a real-world context?

Source

Garfield, Joan and Ben-Zvi, Dani. "Helping Students Develop Statistical Reasoning: Implementing a Statistical Reasoning Learning Environment." Accessed Jan. 11, 2023. https://www.walch.com/rr/09049

Suggested Pacing Guide

Overview

The lessons in this program are designed to be covered over one, two, or three days, depending on the depth and complexity of the content. The pacing guide that follows provides *suggested* pacing, but this should be viewed as a flexible guideline only. If students need an extra day to solidify understanding, a one-day lesson may be spread over two days, or a two-day lesson may be spread over three days. Conversely, if students grasp a concept more quickly than suggested by the pacing guide, it is fine to shorten the duration of the lesson and move on to the next.

Each lesson features several components. The estimated time for each component is given in parentheses. Components include: a Warm-Up, Instruction (including an Introduction, Key Concepts, Scaffolded Practice, and Guided Practice), a Problem-Based Task, and Practice (Sets A and B).

Please note: PowerPoint versions of each lesson's Warm-Up and Instruction are provided. These PowerPoints include the Warm-Up and Warm-Up Debrief, the Introduction, Key Concepts, and Guided Practice examples, some of which have links to corresponding GeoGebra applets. Estimated times reflect presentation of these PowerPoint versions and applets.

Guided Practice, Scaffolded Practice, and Practice

For each Guided Practice in the Teacher Resource (TRB), corresponding worksheets are provided in the Student Workbook (SWB). Scaffolded Practice worksheets are also included to introduce the skills for each lesson. Each lesson also includes two sets of Practice problems. Guided Practice, Scaffolded Practice, and Practice problems may be assigned from the SWB worksheets for in-class work or homework assignments.

Please note: When a lesson is spread across more than one class period, the Guided Practice examples will also be spread across the same number of class periods. Assign Practice problems that correspond to the Guided Practice example(s) you select for each day, to ensure appropriate prior instruction.

Suggested Progression of One-, Two-, and Three-Day Lessons

One Day

- Warm-Up PowerPoint and Debrief (10 minutes)
- Introduction and Key Concepts (10 minutes)
- One or two Guided Practice examples, including PowerPoint applet presentation for selected examples as appropriate (15 minutes)
- Problem-Based Task and discussion (15 minutes)
- Homework: Practice problems from the SWB

Two Days

Day 1

- Warm-Up PowerPoint and Debrief (10 minutes)
- Introduction and Key Concepts (15 minutes)
- Selected Scaffolded Practice problems and two or three Guided Practice examples, including PowerPoint applet presentation for selected examples as appropriate (15 minutes)
- In-class practice: selected Scaffolded Practice and Practice problems, or a conceptual task or activity
- Homework: selected Practice problems from the SWB

Day 2

- Review Practice problems (10 minutes)
- Selected Scaffolded Practice problems and one or two remaining Guided Practice examples, including PowerPoint applet presentation for selected examples as appropriate (10 minutes)
- Problem-Based Task and discussion (30 minutes)
- In-class practice: selected Scaffolded Practice and Practice problems, or a conceptual task or activity
- Homework: the remainder of the Practice problems

Three Days

Day 1

- Warm-Up PowerPoint and Debrief (10 minutes)
- Introduction and Key Concepts (25 minutes)
- Selected Scaffolded Practice problems and one or two Guided Practice examples, including PowerPoint applet presentation for selected examples as appropriate (15 minutes)
- Homework: selected Practice problems

Day 2

- Review Practice problems (15 minutes)
- Two or three remaining Guided Practice examples, including PowerPoint applet presentation for selected examples as appropriate (15 minutes)
- Problem-Based Task and discussion (20 minutes)
- Homework: the remaining Practice problems

Day 3

- Review Practice problems (15 minutes)
- In-class practice: selected Scaffolded Practice and Practice problems, or a conceptual task or activity
- Homework: the remaining Practice problems

Assessments, Conceptual Tasks and Activities, and Station Activities

Each topic includes a Pre-Assessment and a Progress Assessment, and each unit concludes with a Unit Assessment. Units also feature sets of Station Activities, and selected topics include Conceptual Task along with links to interactive web-based Conceptual Activities, to complement instruction.

Pre-Assessments are short, multiple-choice assessments with five problems, designed to evaluate prior knowledge of the upcoming topic. These are brief and should not factor into a student's grade. Pre-Assessments are often given on the same day as the preceding topic's Progress Assessment, to prepare for the upcoming topic.

Progress Assessments include 10 multiple-choice problems and one extended-response problem. Progress Assessments may not take an entire class period. The additional time may be used to review before the assessment, work through the Conceptual Activities, or to begin the next topic after the assessment.

Unit Assessments include 12 multiple-choice problems and three extended-response problems, and generally require a full class period.

Conceptual Tasks provide opportunities for students to deepen their understanding and develop their conceptual knowledge of math concepts.

Conceptual Activities are digital math resources that allow students to explore mathematical ideas with engaging, real-world problems and interactive games.

Station Activities generally require a full class period for the students to rotate through each station and then to engage in a class discussion at the end.

Unit 1: Relationships Between Quantities (24 Days)

Unit Overview: This unit begins with students exploring the relationships between the parts that make up an expression. This leads into using individual expressions to write equations in one variable, then in two variables. Finally, students work with formulas that have multiple variables. This unit begins with definitions, calculations, and graphing, and progresses in depth to reasoning abstractly with variables instead of numbers while still applying mathematical concepts.

Topic A: Interpreting Structure in Expressions

Before embarking on an in-depth study of mathematics, students must have a common language for and understanding of how expressions are created, modified, and simplified. In this topic, students reexamine the terminology used in discussing the fundamental elements of algebra. With this common language, students go on to explore how an expression is affected when its smaller elements are changed. This is the foundation of modeling in algebra—using and manipulating expressions to describe contexts. This topic lays the groundwork for the rest of the unit, which builds on this common terminology.

Day	Area of study/content	Standard(s)
Day 1 Day 2	Topic A Pre-Assessment 1.1: Identifying Terms, Factors, and Coefficients	A–SSE.A.1a★
Day 3	1.2: Interpreting Linear and Exponential Expressions	A–SSE.A.1b★
Day 4	Topic A Progress Assessment Topic B Pre-Assessment	

Topic B: Creating Equations and Inequalities in One Variable

From expressions, students go on to create linear equations and inequalities in one variable that model real-world situations. Students gain practice with translating contexts into symbolic representations by writing linear equations and using these equations to solve problems. Similarly, students build upon these skills as they learn how to identify and translate simple exponential contexts into exponential equations that can be solved. This topic provides a foundation for working with linear and exponential equations in two variables as seen in the next topic, and for exploring equations in multiple variables as encountered later in the unit.

Day	Area of study/content	Standard(s)
Day 5	Station Activities Set 1: Ratios and Proportions	N–Q.A.1★ A–CED.A.1★
Day 6 Day 7	1.3: Creating Linear Equations in One Variable	A–CED.A.1★ N–Q.A.2★ N–Q.A.3★
Day 8	1.4: Creating Linear Inequalities in One Variable	A–CED.A.1★
Day 9 Day 10	1.5: Creating Exponential Equations	A–CED.A.1★
Day 11	Station Activities Set 2: Solving Inequalities, or a conceptual activity (e.g., Desmos, etc.)	A–CED.A.1★
Day 12	Topic B Progress Assessment/Conceptual Task Topic C Pre-Assessment	

Topic C: Creating and Graphing Equations in Two Variables

Students extend their understanding of linear and exponential equations to include equations in two variables. Students see that equations in two variables can be graphed. Students learn to set up a coordinate plane with appropriate axes, scales, and labels, which prepares them for graphing in the remainder of the course and throughout their studies in mathematics.

Day	Area of study/content	Standard(s)
Day 13 Day 14	1.6: Creating and Graphing Linear Equations in Two Variables	A–CED.A.2★ N–Q.A.1★
Day 15 Day 16	1.7: Creating and Graphing Exponential Equations	A–CED.A.2★ N–Q.A.1★
Day 17	Topic C Progress Assessment/Conceptual Task Topic D Pre-Assessment	

Topic D: Representing Constraints

Students learn that equations and inequalities can have restrictions, and that solutions may be valid under only certain conditions. This shows students that even though a solution "works" mathematically, it might not be reasonable for the given set of equations or inequalities that arise from a context. This topic provides the basis for building functions from context and understanding domain and range, which are presented later in the course.

Day	Area of study/content	Standard(s)
Day 18	1.8: Representing Constraints	A–CED.A.3★
Day 19	Topic D Progress Assessment	

Topic E: Rearranging Formulas

In this topic, students use what they have learned about solving equations to rearrange an equation that contains several variables in order to solve it for a given variable.

Day	Area of study/content	Standard(s)
Day 20 Day 21	Topic E Pre-Assessment 1.9: Rearranging Formulas	A–CED.A.4★
Day 22	Topic E Progress Assessment	
Day 23	Station Activities Set 3: Solving Equations	A–CED.A.1★ A–CED.A.2★
Day 24	**Unit Assessment**	

Unit 2: Linear and Exponential Relationships (59 days)

Unit Overview: This unit builds on the concepts of functions that were first introduced in Grade 8. Students extend their understanding of functions to include exponential relationships. Students learn how to analyze and model relationships from contexts, graphs, tables, and equations using what they know about exponential and linear relationships.

Topic A: Graphs As Solution Sets and Function Notation

Students investigate the concept of a graph as representing the solution set to an equation in two variables. To describe that graph, students are introduced to using function notation, evaluating functions, identifying domain and range, and analyzing the graphs of two functions that meet at one or more points. This topic serves to familiarize students with the language and behavior of functions and how to distinguish them from relations. These notions serve as the basis for the remainder of the unit, as students go on to study functions in greater depth.

Day	Area of study/content	Standard(s)
Day 1 Day 2	Topic A Pre-Assessment 2.1: Graphing the Set of All Solutions	A–REI.D.10
Day 3 Day 4	2.2: Intersecting Graphs	A–REI.D.11★
Day 5 Day 6	2.3: Domain and Range	F–IF.A.1
Day 7 Day 8	2.4: Function Notation and Evaluating Functions	F–IF.A.2
Day 9	Station Activities Set 2: Relations Versus Functions/ Domain and Range	F–IF.A.1 F–IF.A.2 F–BF.A.1a★
Day 10	Topic A Progress Assessment Topic B Pre-Assessment	

Topic B: Solving Linear Inequalities in Two Variables and Systems of Inequalities

Students build on their knowledge of graphs as solutions to equations, and understand the concept that the solution to an inequality is a half plane. Similar to systems of equations, systems of inequalities are used to find the set of points that are valid for both inequalities. Students learn how to graph the solution set of a system of inequalities in two variables.

Day	Area of study/content	Standard(s)
Day 11 Day 12 Day 13	2.5: Solving Linear Inequalities in Two Variables	A–REI.D.12
Day 14 Day 15 Day 16	2.6: Solving Systems of Linear Inequalities	A–REI.D.12
Day 17	Topic B Progress Assessment/Conceptual Task Topic C Pre-Assessment	

Topic C: Sequences As Functions

Students learn what a sequence is and how a sequence can be described as a function either recursively or explicitly. This continues to reinforce the students' comprehension of linear and exponential relationships, and prepares them for studying arithmetic and geometric sequences later in the unit.

Day	Area of study/content	Standard(s)
Day 18 Day 19 Day 20	2.7: Sequences As Functions	F–IF.A.3
Day 21	Topic C Progress Assessment Topic D Pre-Assessment	

Topic D: Interpreting Graphs of Functions

Students learn the vocabulary used to describe functions and develop their understanding of this terminology in context. Students learn what to look for when analyzing a function in terms of its graph and the context of the situation that the graph depicts.

Day	Area of study/content	Standard(s)
Day 22 Day 23 Day 24	2.8: Identifying Key Features of Linear and Exponential Graphs	F–IF.B.4★ F–IF.B.5★
Day 25 Day 26	2.9: Average Rate of Change	F–IF.B.6★ F–LE.A.1a★
Day 27 Day 28	2.10: Recognizing Average Rate of Change	F–IF.B.6★ F–LE.A.1b★ F–LE.A.1c★
Day 29	Topic D Progress Assessment/Conceptual Task Topic E Pre-Assessment	

Topic E: Analyzing Linear and Exponential Functions

Students learn how to graph linear and exponential functions, as well as how to identify the intercepts and asymptotes (for exponential functions). This lays the foundation for comparing the attributes of two functions later in the unit.

Day	Area of study/content	Standard(s)
Day 30 Day 31	2.11: Graphing Linear Functions	F–IF.C.7a★
Day 32 Day 33	2.12: Graphing Exponential Functions	F–IF.C.7e★
Day 34	Station Activities Set 3: Comparing Exponential Models	F–IF.A.2 F–IF.C.7d★ (+) F–IF.C.7e★
Day 35	Topic E Progress Assessment Topic F Pre-Assessment	

Topic F: Comparing Functions

Students compare attributes of linear functions and then exponential functions that are presented in different ways; i.e., as tables, graphs, and/or equations. Then, students analyze linear functions and compare them to exponential functions. This skill prepares students for building functions from real-world contexts in the next topic.

Day	Area of study/content	Standard(s)
Day 36	2.13: Comparing Linear Functions	F–IF.C.9
Day 37	2.14: Comparing Exponential Functions	F–IF.C.9
Day 38	2.15: Comparing Linear to Exponential Functions	F–LE.A.3★
Day 39	Topic F Progress Assessment/Conceptual Task Topic G Pre-Assessment	

Topic G: Building Functions

Students create linear and exponential equations to model the relationship between two quantities given a context, a graph, or a table of values. This readies students to operate on functions by using transformations.

Day	Area of study/content	Standard(s)
Day 40 Day 41	2.16: Building Functions from Context	F–BF.A.1a★
Day 42 Day 43	2.17: Constructing Functions from Graphs and Tables	F–LE.A.2★
Day 44	Topic G Progress Assessment/Conceptual Task Topic H Pre-Assessment	

Topic H: Operating on Functions and Transformations

Students perform operations with functions and then analyze the effects of vertical translations on linear and exponential functions. This experience allows students to perform more complex operations and transformations in later units.

Day	Area of study/content	Standard(s)
Day 45	2.18: Operating on Functions	F–BF.A.1b★
Day 46	2.19: Transformations of Linear and Exponential Functions	F–BF.B.3
Day 47	Station Activities Set 4: Interpreting Exponential Functions	F–IF.A.2 F–IF.C.7e★
Day 48	Topic H Progress Assessment Topic I Pre-Assessment	

Topic I: Arithmetic and Geometric Sequences

Students build on their earlier work with sequences as functions to extend, create, and analyze arithmetic and geometric sequences.

Day	Area of study/content	Standard(s)
Day 49 Day 50	2.20: Arithmetic Sequences	F–BF.A.2★
Day 51 Day 52	2.21: Geometric Sequences	F–BF.A.2★
Day 53	Station Activities Set 5: Sequences	F–BF.A.2★ F–LE.A.2★
Day 54	Topic I Progress Assessment Topic J Pre-Assessment	

Topic J: Interpreting Parameters

Students understand the impact of changing the parameters of linear equations in slope-intercept form and exponential functions in general form. They examine how those parameters relate to the context of the problem, the graph, and the equation.

Day	Area of study/content	Standard(s)
Day 55 Day 56	2.22: Interpreting Parameters	F–LE.B.5★
Day 57	Topic J Progress Assessment	
Day 58	Station Activities Set 1: Comparing Linear Models	A–CED.A.2★ A–REI.D.10 A–REI.D.11★ F–IF.C.7a★
Day 59	**Unit Assessment**	

Unit 3: Reasoning with Equations (23 Days)

Unit Overview: This unit combines the terminology developed in Unit 1 with the algebraic skills learned in Unit 2 to guide students to solve more challenging linear equations, pushing them beyond the two-step equations that were first learned in Grade 8. Students also extend what they know about linear equations and apply this understanding toward solving linear inequalities. The unit then delves deeper into linear equations by exploring how systems of linear equations are constructed, solved, and graphed.

Topic A: Solving Equations and Inequalities

Students further develop their ability to solve linear equations by solving equations of greater difficulty. Afterward, students go on to solve linear inequalities and exponential equations.

Day	Area of study/content	Standard(s)
Day 1 Day 2	Topic A Pre-Assessment 3.1: Properties of Equality	A–REI.A.1
Day 3 Day 4 Day 5	3.2: Solving Linear Equations	A–REI.B.3
Day 6 Day 7 Day 8	3.3: Solving Linear Inequalities	A–REI.B.3
Day 9 Day 10 Day 11	3.4: Solving Exponential Equations	A–REI.B.3
Day 12	Topic A Progress Assessment/Conceptual Task Topic B Pre-Assessment	

Topic B: Solving Systems of Equations

This topic develops students' previous work with systems of equations first learned in Grade 8. Students explore three methods for solving systems of equations: by substitution, by elimination, and by graphing. Students develop their understanding of how to write a system of equations and what the solution means in terms of the context of the problem.

Day	Area of study/content	Standard(s)
Day 13 Day 14 Day 15	3.5: Solving Systems of Linear Equations by Substitution and Elimination	A–REI.C.5
Day 16	Station Activities Set 1: Solving Systems by Substitution and Elimination	A–REI.C.5
Day 17	Station Activities Set 2: Solving Systems by Graphing	A–REI.C.6
Day 18 Day 19 Day 20	3.6: Solving Systems of Linear Equations by Graphing	A–REI.C.6
Day 21	Station Activities Set 3: Using Systems in Applications	A–CED.A.2★ A–CED.A.3★ A–REI.C.5 A–REI.C.6 A–REI.D.11★
Day 22	Topic B Progress Assessment/Conceptual Task	
Day 23	**Unit Assessment**	

Unit 4: Descriptive Statistics (27 Days)

Unit Overview: This unit develops students' skills and understanding of how to represent, compare, and analyze data using mathematical techniques and terminology. Students move from making statements based on inspection to drawing conclusions based on mathematics. Students begin by analyzing single-count variables, and move toward analyzing multiple variables and two-way frequency tables. Students continuously practice analyzing data sets in context by describing and interpreting the sets using measures of center and spread, and later incorporate any deviations. Students also learn how to fit linear models to data and use these models to evaluate and make predictions.

Topic A: Working with a Single Measurement Variable

Students learn how to represent quantitative data using histograms, dot plots, and box-and-whisker plots. Students also learn how to use the graphical representations they create from data sets to analyze the data. This topic serves as the foundation for describing and analyzing data sets—skills that will be carried through the remainder of this unit and into data analysis units in future math courses.

Day	Area of study/content	Standard(s)
Day 1 Day 2	Topic A Pre-Assessment 4.1: Representing Data Sets	S–ID.A.1★
Day 3 Day 4	4.2: Comparing Data Sets	S–ID.A.2★
Day 5 Day 6	4.3: Interpreting Data Sets	S–ID.A.3★
Day 7	Station Activities Set 1: Displaying and Interpreting Data	S–ID.A.1★ S–ID.A.2★ S–ID.A.3★
Day 8	Topic A Progress Assessment Topic B Pre-Assessment	

Topic B: Working with Two Categorical and Quantitative Variables

Students move from analyzing single-count variables to analyzing multiple variables in a two-way frequency table. Then, students learn how to examine the relationship between two variables by plotting the data points and fitting curves to the data. Students come to understand that when a relationship between two variables exists, the data are correlated, and the relationship is not necessarily causal. This topic prepares students to further investigate linear models in the next topic.

Day	Area of study/content	Standard(s)
Day 9 Day 10	4.4: Summarizing Data Using Two-Way Frequency Tables	S–ID.B.5★
Day 11 Day 12	4.5: Solving Problems Given Functions Fitted to Data	S–ID.B.6a★
Day 13 Day 14	4.6: Analyzing Residuals	S–ID.B.6b★
Day 15 Day 16 Day 17	4.7: Fitting Linear Functions to Data	S–ID.B.6c★
Day 18	Station Activities Set 2: Line of Best Fit	S–ID.B.6a★ S–ID.B.6b★ S–ID.B.6c★ S–ID.C.7★
Day 19	Topic B Progress Assessment/Conceptual Task Topic C Pre-Assessment	

Topic C: Interpreting Linear Models

Students delve deeper into analysis and interpretation of linear models. They also examine the strength of a linear model by exploring the correlation coefficient. Students learn the difference between correlation and causation.

Day	Area of study/content	Standard(s)
Day 20 Day 21	4.8: Interpreting Slope and *y*-intercept	S–ID.C.7★
Day 22 Day 23	4.9 Calculating and Interpreting the Correlation Coefficient	S–ID.C.8★
Day 24 Day 25	4.10: Distinguishing Between Correlation and Causation	S–ID.C.9★
Day 26	Topic C Progress Assessment/Conceptual Task	
Day 27	**Unit Assessment**	

Unit 5: Congruence, Proof, and Constructions (34 Days)

Unit Overview: This unit further develops the concept of transformations presented to students in Grade 8 by applying transformations to functions. The unit also introduces students to geometric constructions in order to show the relationships between lines and angles. Students continue to develop their understanding of these relationships as they explore the concepts of congruency and congruent triangles. Throughout the unit, students are asked to prove relationships that exist in geometric figures using theorems and postulates.

Topic A: Introducing Transformations

Students begin the topic by being introduced to the language of geometry. This language is the foundation of the unit. Students then apply this terminology to connect algebra and geometry through the examination of transformations as they relate to functions. Students take the familiar topic of symmetry and learn to visualize applying lines of symmetry to verbal descriptions of geometric figures. These concepts serve as a springboard for further developing the concepts of reflections, rotations, and congruency presented later in the unit.

Day	Area of study/content	Standard(s)
Day 1 Day 2	Topic A Pre-Assessment 5.1: Defining Terms	G–CO.A.1
Day 3 Day 4	5.2: Transformations As Functions	G–CO.A.2
Day 5 Day 6	5.3: Applying Lines of Symmetry	G–CO.A.3
Day 7	Topic A Progress Assessment Topic B Pre-Assessment	

Topic B: Defining and Applying Rotations, Reflections, and Translations

Students take the general concepts of symmetry and transformations learned in the previous topic and apply specific rotations, reflections, and translations in a given order. Students explore the order of transformations, and verify that the order is important. Students see that these rigid transformations allow the figures to be manipulated back onto themselves while remaining congruent. This topic sets the stage for exploring congruent triangles later in the unit.

Day	Area of study/content	Standard(s)
Day 8	5.4: Defining Rotations, Reflections, and Translations	G–CO.A.4
Day 9 Day 10	5.5: Applying Rotations, Reflections, and Translations	G–CO.A.5
Day 11	Topic B Progress Assessment/Conceptual Task Topic C Pre-Assessment	

Topic C: Constructing Lines, Segments, and Angles

Students begin by constructing copies of segments and angles without measuring them. Students explore the relationships within constructions using only a compass and a straightedge. This topic helps students begin to examine the spatial relationships in congruency.

Day	Area of study/content	Standard(s)
Day 12 Day 13	5.6: Copying Segments and Angles	G–CO.D.12
Day 14	5.7: Bisecting Segments and Angles	G–CO.D.12
Day 15	5.8: Constructing Perpendicular and Parallel Lines	G–CO.D.12
Day 16	Topic C Progress Assessment Topic D Pre-Assessment	

Topic D: Constructing Polygons

Students develop their spatial exploration of geometric figures by inscribing triangles, squares, and regular hexagons in circles. This exploration further prepares students for the study of congruent triangles.

Day	Area of study/content	Standard(s)
Day 17 Day 18	5.9: Constructing Equilateral Triangles Inscribed in Circles	G–CO.D.13
Day 19	5.10: Constructing Squares Inscribed in Circles	G–CO.D.13
Day 20	5.11: Constructing Regular Hexagons Inscribed in Circles	G–CO.D.13
Day 21	Topic D Progress Assessment Topic E Pre-Assessment	

Topic E: Exploring Congruence

Students combine their knowledge of transformations from the beginning of the unit with their understanding of congruent angles and segments in order to explore congruent figures. This topic leads to investigating congruent triangles in the next topic.

Day	Area of study/content	Standard(s)
Day 22 Day 23	5.12: Describing Rigid Motions and Predicting the Effects	G–CO.B.6
Day 24	5.13: Defining Congruence in Terms of Rigid Motions	G–CO.B.6
Day 25	Station Activities Set 1: Rotations and Reflections	G–CO.A.1 G–CO.A.3 G–CO.A.4 G–CO.A.5
Day 26	Topic E Progress Assessment/Conceptual Task Topic F Pre-Assessment	

Topic F: Congruent Triangles

Students address one of geometry's central concepts: congruent triangles. Students learn what attributes triangles must have in order to be congruent. As the topic progresses, students see what minimum requirements are necessary to determine whether two triangles are congruent, and they understand why these criteria are enough to make the determination.

Day	Area of study/content	Standard(s)
Day 27 Day 28	5.14: Triangle Congruency	G–CO.B.7
Day 29 Day 30 Day 31	5.15: Explaining ASA, SAS, and SSS	G–CO.B.8
Day 32	Station Activities Set 2: Corresponding Parts, Transformations, and Proof	G–CO.A.1 G–CO.A.2 G–CO.A.5 G–CO.B.6 G–CO.B.7 G–CO.B.8
Day 33	Topic F Progress Assessment/Conceptual Task	
Day 34	**Unit Assessment**	

Unit 6: Connecting Algebra and Geometry Through Coordinates (12 Days)

Unit Overview: This unit connects the domains of algebra and geometry through coordinate proof.

Topic A: Slope and Distance

Students constructed parallel and perpendicular lines in the previous unit. In this topic, they use those skills to explore how to prove that two lines are parallel or perpendicular using coordinates and algebra.

Day	Area of study/content	Standard(s)
Day 1	Topic A Pre-Assessment Station Activities Set 1: Parallel Lines, Slopes, and Equations	G–GPE.B.4 G–GPE.B.5
Day 2 Day 3	6.1: Using Coordinates to Prove Geometric Theorems with Slope and Distance	G–GPE.B.4 G–GPE.B.5
Day 4	Station Activities Set 3: Coordinate Proof with Quadrilaterals	G–GPE.B.4 G–GPE.B.5
Day 5 Day 6	6.2: Working with Parallel and Perpendicular Lines	G–GPE.B.5
Day 7	Station Activities Set 2: Perpendicular Lines	G–GPE.B.4 G–GPE.B.5
Day 8	Topic A Progress Assessment/Conceptual Task Topic B Pre-Assessment	

Topic B: Lines and Line Segments

Students use the distance formula to calculate the perimeter and area of figures given the coordinates of the vertices of the figures, again connecting algebra and geometry.

Day	Area of study/content	Standard(s)
Day 9 Day 10	6.3: Calculating Perimeter and Area	G–GPE.B.7★
Day 11	Topic B Progress Assessment	
Day 12	**Unit Assessment**	

End-of-Course Assessment	
Day 13	This Learnosity-based interactive assessment is available through Canvas or Walch's Curriculum Engine. For more information, contact us at success@bwwalch.com.

Graphic Organizers

Overview

Graphic organizers can be a versatile tool in your classroom. Organizers offer an easy, straightforward way to visually present a wide range of material. Research suggests that graphic organizers support learning in the classroom for all levels of learners. Gifted students, students on grade level, and students with learning difficulties all benefit from the use of graphic organizers. They reduce the cognitive demand on students by helping them access information quickly and easily. Using graphic organizers, learners can understand content more clearly and can take concise notes. Ultimately, learners find it easier to retain and apply what they've learned.

Graphic organizers help foster higher-level thinking skills. They help students identify main ideas and details in their reading. They make it easier for students to see patterns such as cause and effect, comparing and contrasting, and chronological order. Organizers also help students master critical-thinking skills by asking them to recall, evaluate, synthesize, analyze, and apply what they've learned. Research suggests that graphic organizers contribute to better test scores because they help students understand relationships between key ideas, and enable them to be more focused as they study.

Types of Graphic Organizers

There are four main purposes for using graphic organizers in mathematics and a variety of tools within each category:

Purpose 1: Organizing, Categorizing, and Classifying	**Purpose 2: Problem Solving**	**Purpose 3: Understanding Mathematical Information**	**Purpose 4: Communicating Mathematical Information**
Tables Flowcharts Webs Venn Diagrams	Number Lines Geometric Drawings Factor Trees Attribute Tables Cause and Effect Maps Coordinate Plane Probability Trees	Frayer Model Semantic Map/ Concept Map Compare-and-Contrast Diagram	Line Graphs Bar Charts

Tables

A table is simply a grid with rows and columns. Tables are useful because information stored in a table is easy to find—much easier than the same information embedded in text.

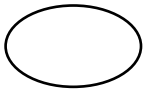
Usually, a table has a row (horizontal) for each item being listed. The columns (vertical) provide places for details about the listed items—the things they have in common. The places where the rows and columns meet are called cells. In each cell, we write information that fits both the topic of the row (the thing being listed) and the topic of the column (the aspect being examined). To create a table, we make rows and columns to fit the number of items and attributes.

Flowcharts

Flowcharts are graphic organizers that show the steps in a process. Flowcharts can be very simple—just a series of boxes with one step in each box. However, there is also a more formal type of flowchart. These flowcharts use special symbols to show different things, such as starting and stopping points, or points where decisions must be made. These symbols make flowcharts especially useful for showing complicated processes.

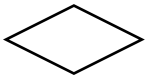
Each step in a flowchart is written in a box. The boxes are connected by arrows to show the sequence of steps. The boxes aren't all rectangular; different shapes are used to indicate different actions. The shapes and symbols are a kind of visual shorthand. Whenever a certain symbol is used, it always has the same meaning.

- Circles and ovals show starting and stopping points. They often contain the words start or stop. The "start" circle or oval has no arrows in and one arrow out. The "stop" circle or oval has one arrow in and no arrows out.

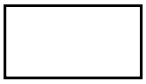


- Arrows show the direction in which the process is moving.

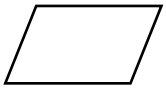
- Diamonds show points where a decision must be made or a question must be answered. The question can usually be answered either "yes" or "no."



- Rectangles and squares show steps where a process or an operation takes place.



- Parallelograms show input or output, such as writing or printing a result or solution.



Webs

Webs are graphic organizers that help take notes, identify important ideas, and show relationships between and among pieces of information. In a web, the main idea is written in the center circle. Details are recorded in other circles with lines to connect related topics. Circles or lines can be added or deleted as necessary.

Number Lines

In its simplest form, a number line is any line that uses equally spaced marks to show numbers. Number lines are used to visualize equalities and inequalities, positive and negative numbers, and measurements of all kinds. They can "map" math problems, especially ones that involve negative numbers or distances.

Geometric Drawings

A geometric drawing is a representation on paper (or some other surface) of a geometric figure. The geometric drawings we make can never be as perfect as the geometric figures they represent, but as long as they are reasonably accurate, they can help us visualize the figures. In fact, it's often impossible to solve a geometry problem without making a drawing.

Factor Trees

There are several ways to find factors. One that helps to visually keep track of all the factors is called a factor tree. This is a diagram with a tree-like shape. It uses "branches" to show the factors of a number.

All whole numbers other than 1 can be written as the product of factors. A prime number is a number that has only two factors, itself and 1. An example of a prime number is 13. Its only factors are 13 and 1. A composite number is a number that has more than two factors. An example of a composite number is 6. Its factors include 6, 3, 2, and 1. Prime factors are factors that are also prime numbers. The greatest common factor (GCF) of two numbers is the largest number that is a factor of both numbers.

Coordinate Plane

This is the plane determined by a horizontal number line, called the x-axis, and a vertical number line, called the y-axis, intersecting at a point called the origin. A coordinate plane can be used to illustrate locations and relationships using ordered pairs of numbers.

Venn Diagrams

A set is a list of objects in no particular order. Items in a set can be numbers, but they can also be letters or words. Venn diagrams are a visual way of showing how sets of things can include one another, overlap, or be distinct from one another.

Venn diagrams are often used to compare and contrast things. But they are also a useful tool to sort and classify information. You can use Venn diagrams to take notes on material that shows relationships between things or ideas. You can also use them to solve certain types of word problems. When a word problem names two or three different categories and asks you how many items fall into each category, a Venn diagram can be a useful problem-solving tool.

A Venn diagram begins with a rectangle representing the universal set. Then each set in the problem is represented by a circle. Circles can be separate, overlapping, or one within another. When two circles overlap, it means that the two sets intersect. Some members of one set are also members of the other set.

Venn Diagrams AND Compare-and-Contrast Diagrams

The Venn diagram is an organizing device for planning comparisons and contrasts. A completed Venn diagram helps students categorize and organize similarities and differences, and provides a blueprint for a comparison-and-contrast exercise. The compare-and-contrast diagram provides a structure to identify or list similarities and differences between two objects.

Attribute Tables

To solve logic problems, you need a way to keep track of the subjects and which attributes they have or don't have. An attribute table can help. This is a table with a row for each subject in the problem, and a column for each attribute. The rows and columns meet to form cells. Because the attributes in logic problems are usually exclusive, you can use Xs or check marks (✔) to show which attribute belongs to which subject.

Cause and Effect Maps

Cause and effect maps help you work through information to make sense of it. Write each cause in the oval. Write all its effects in the boxes. Add or delete ovals and boxes as needed.

Frayer Model

The Frayer Model is a word categorization activity that helps learners to develop their understanding of concepts. Using this model, students provide a definition, list characteristics, and provide examples and non-examples of the concept.

Semantic Map

A semantic word map allows students to conceptually explore their knowledge of a new term or concept by mapping it with other related words, concepts, or phrases that are similar in meaning. Semantic maps portray the schematic relations that compose a concept. It assumes that there are multiple relations between a concept and the knowledge that is associated with the concept.

Line Graphs

Line graphs are often used to show how things change over time. They clearly show trends in data and can let you make predictions about future trends, too. Line graphs use two number lines, one horizontal and one vertical. The horizontal number line is called the *x*-axis. The vertical line is called the *y*-axis. The *x*-axis often shows the passage of time. The *y*-axis often shows a quantity of some kind, such as height, speed, cost, and so forth.

Bar Charts

Bar charts are useful when you want to compare things or to show how one thing changes over time. They are a good way to show overall trends. Bar charts use horizontal or vertical bars to represent data. Longer bars represent higher values. Different colors can be used to show different variables. When you look at a bar chart, it's easy to see which element has the greatest value—the one with the longest bar.

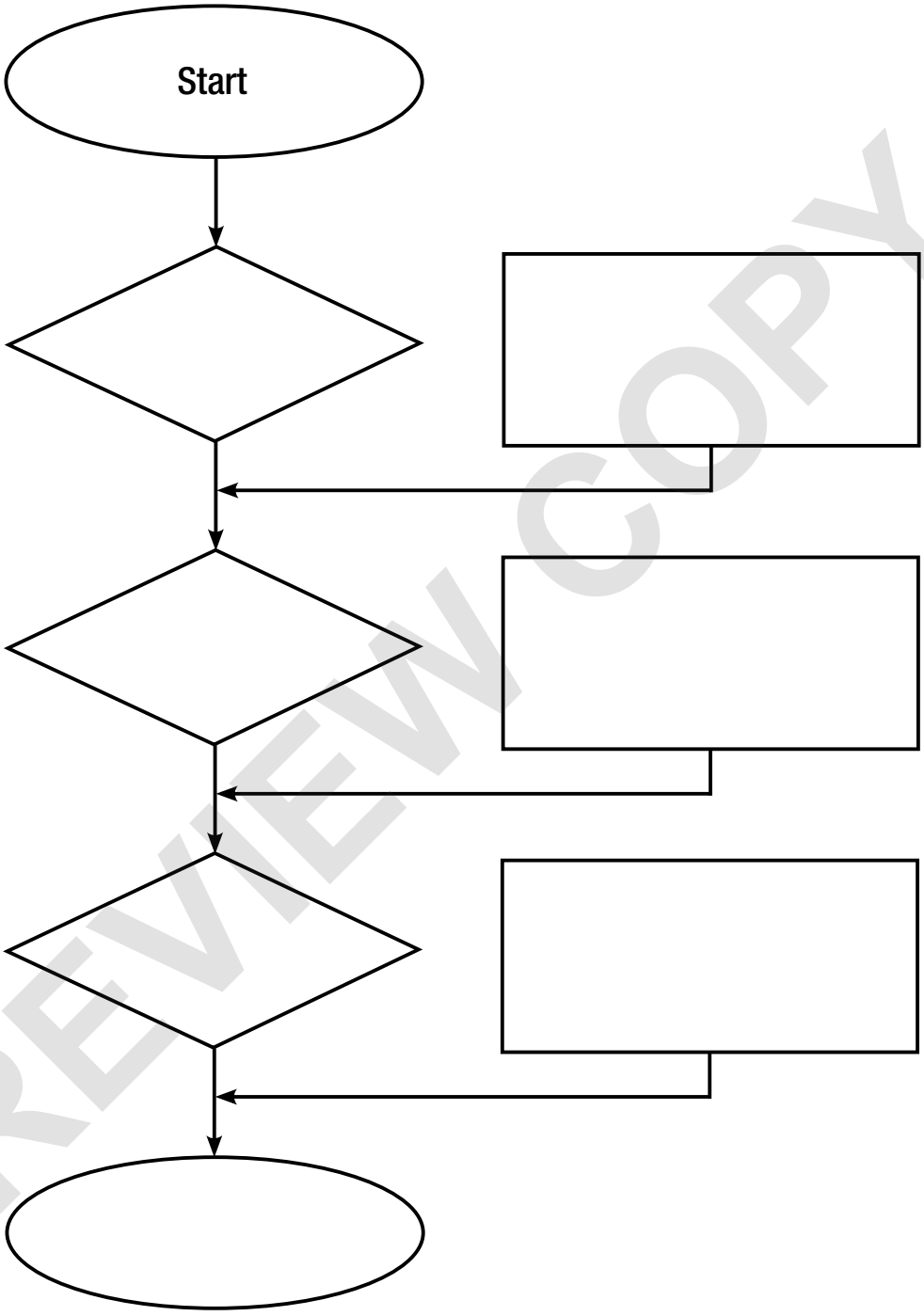
Bar charts have an *x*-axis (horizontal) and a *y*-axis (vertical). If the graph is being used to show how something changes over time, the *x*-axis has numbers for the time period. If the graph is being used to compare things, the *x*-axis shows which things are being compared. The *y*-axis has numbers that show how much of each thing there is.

Probability Trees

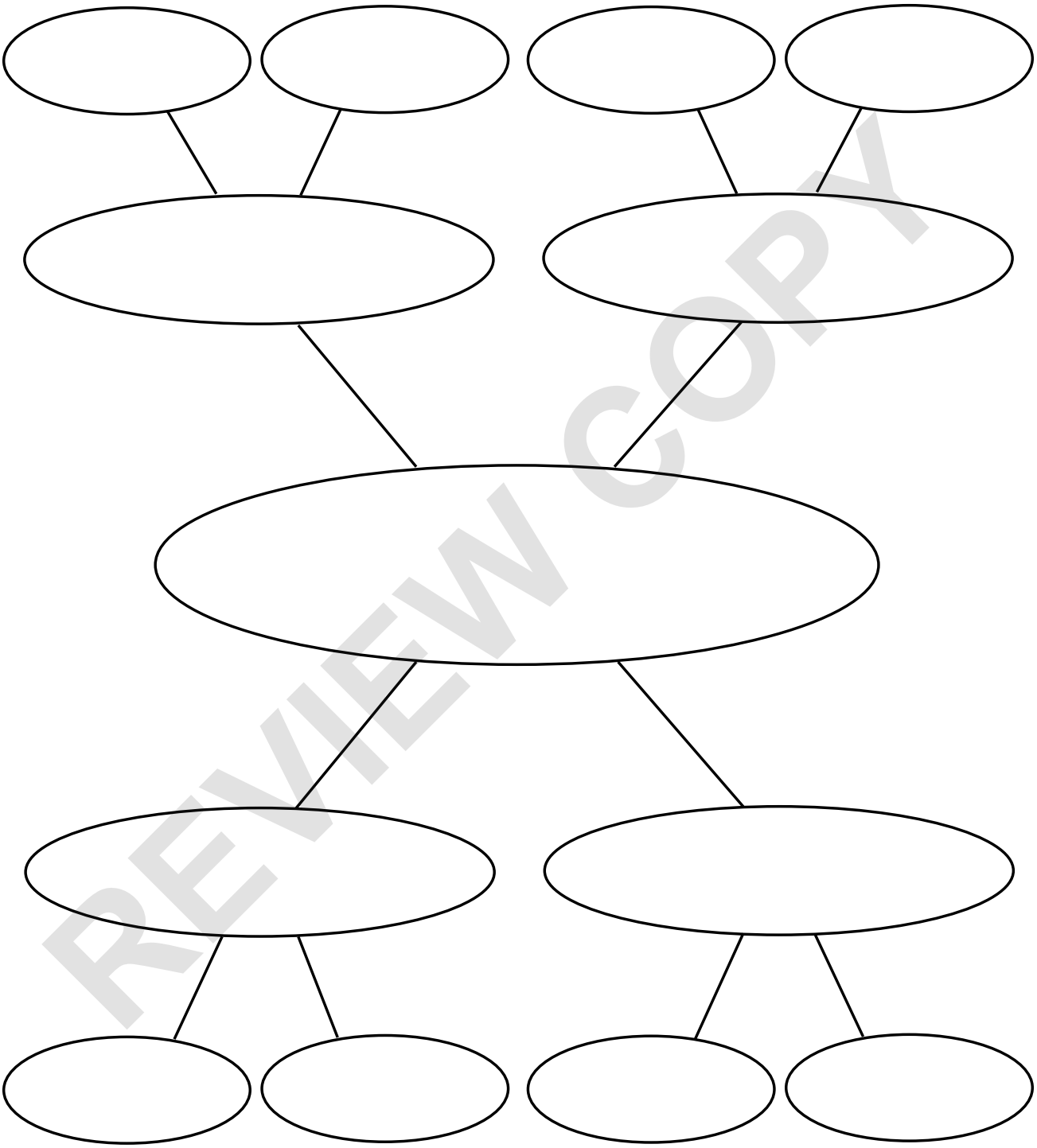
When we have probability problems with many possible outcomes, or events that depend on one another, probability trees can help. Probability trees show all the possible outcomes of an event. Whenever a problem calls for figuring out how many possible outcomes there are, and the probability that any one of them will happen, a probability tree can be useful.

Table

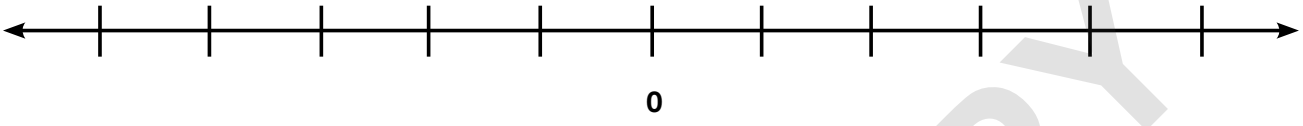
Flowchart



Web



Number Line



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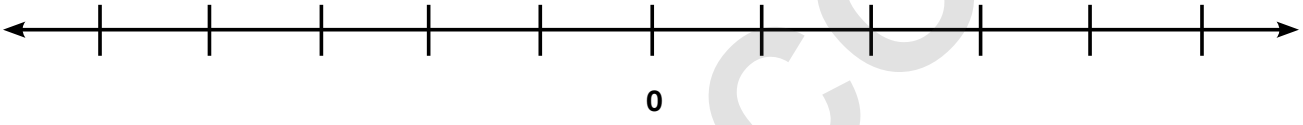

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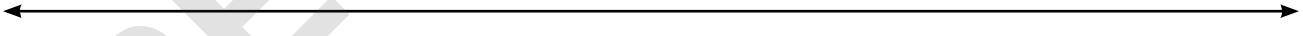
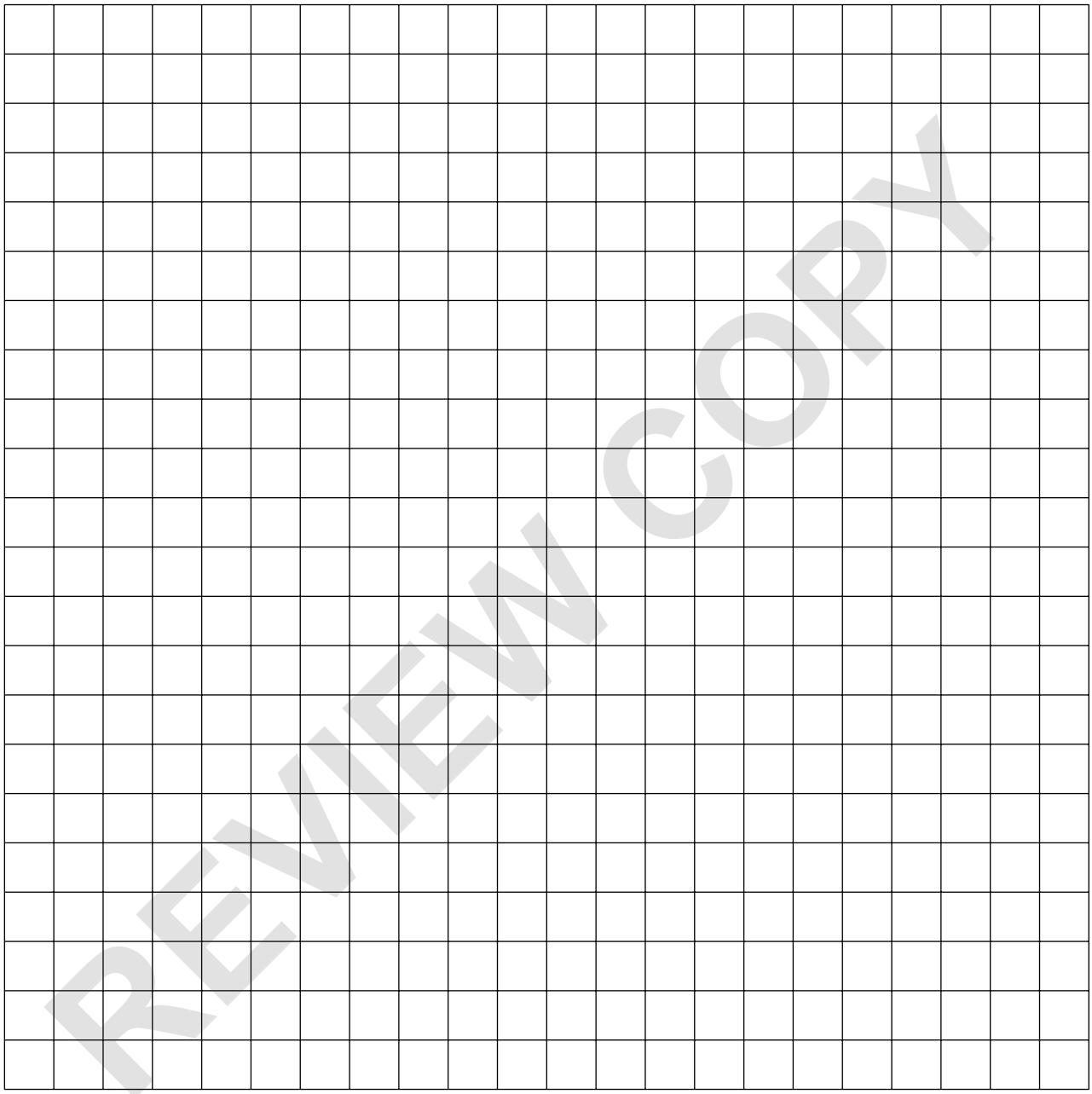
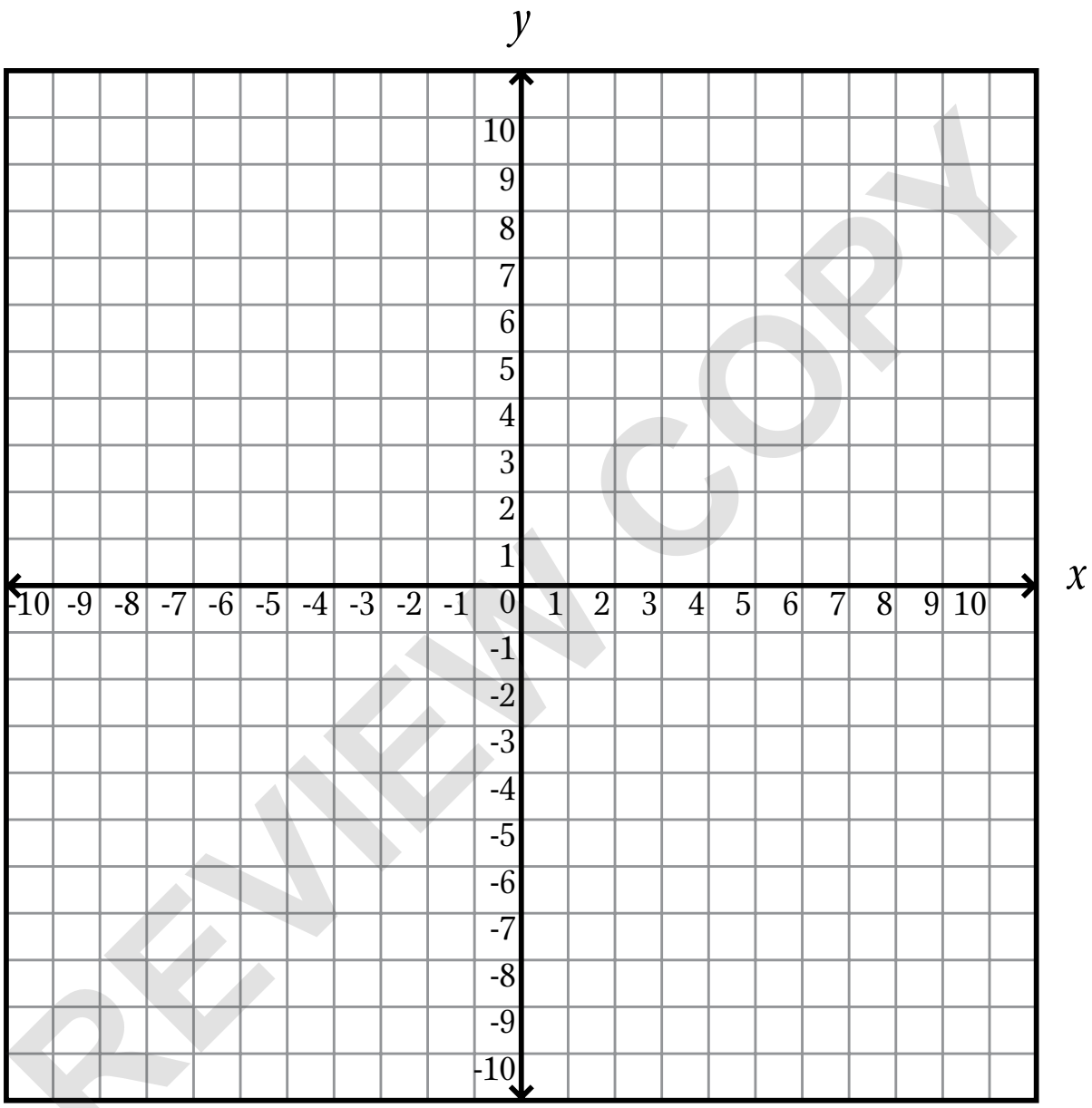


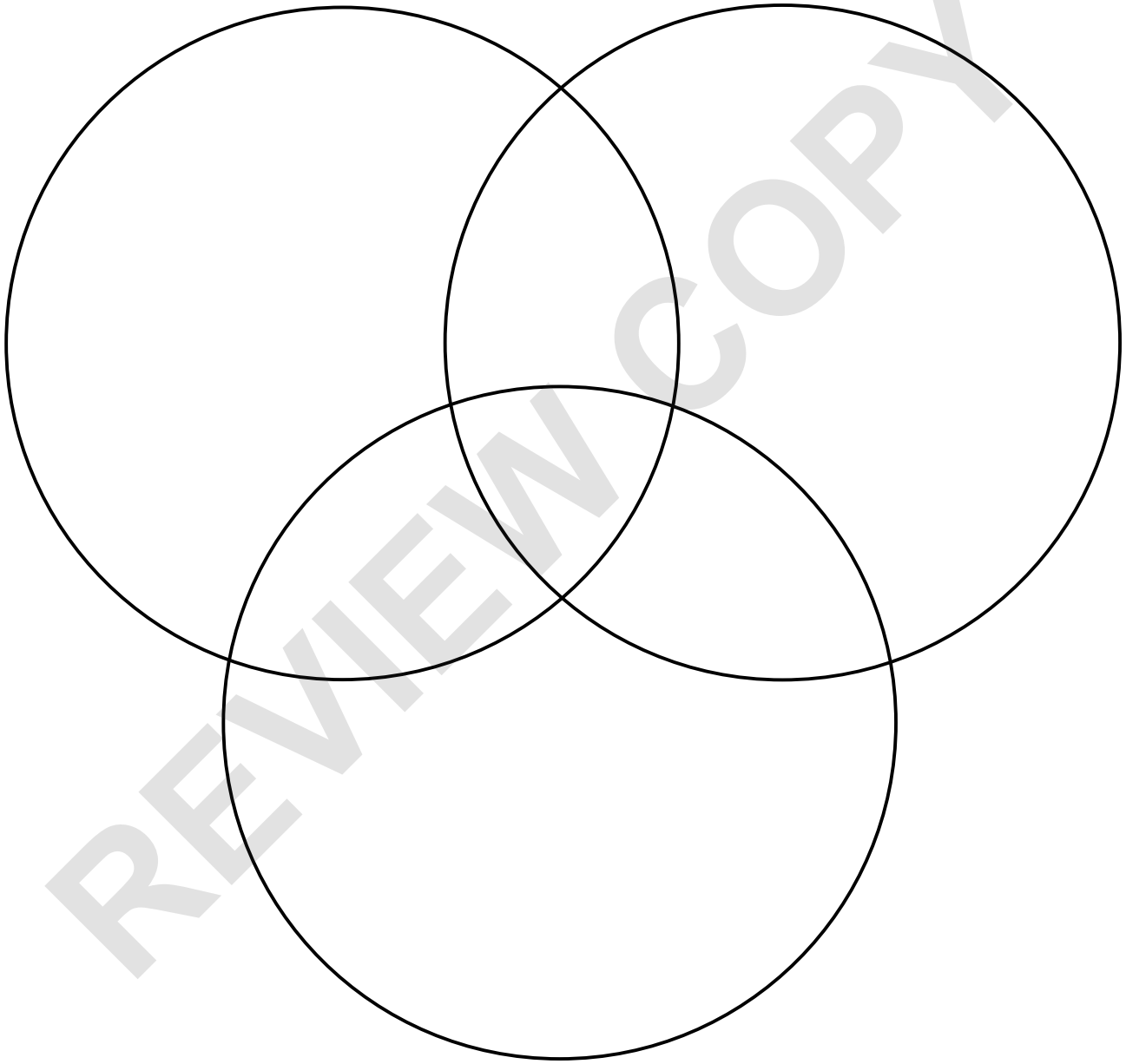
Geometric Drawing



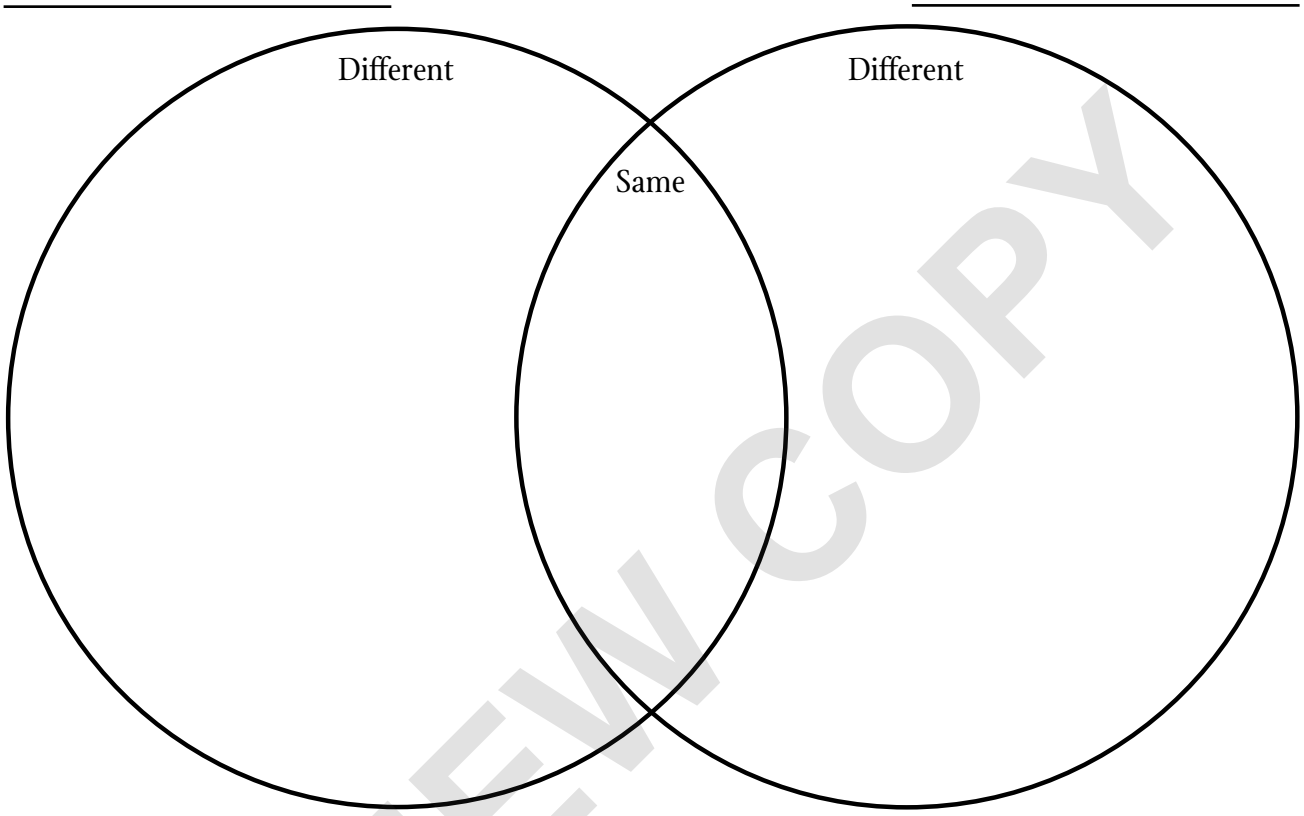
Coordinate Plane



Venn Diagram



Venn Diagram



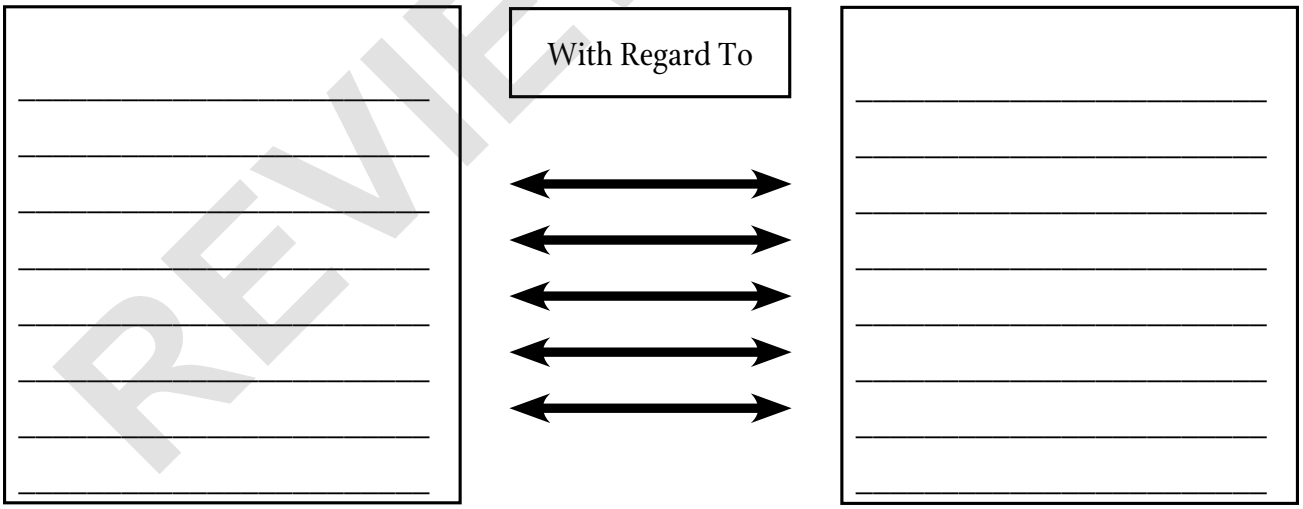
Compare-and-Contrast Diagram

Item 1 _____ Item 2 _____

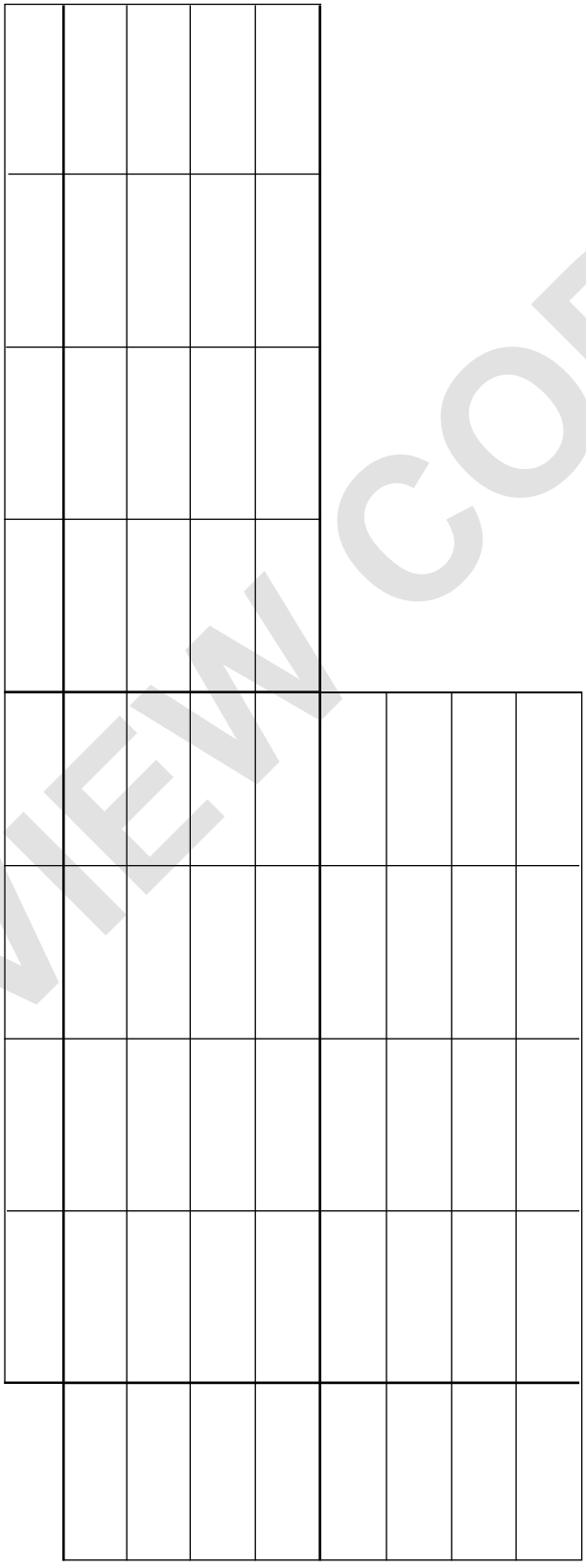
How Alike?

How Different?

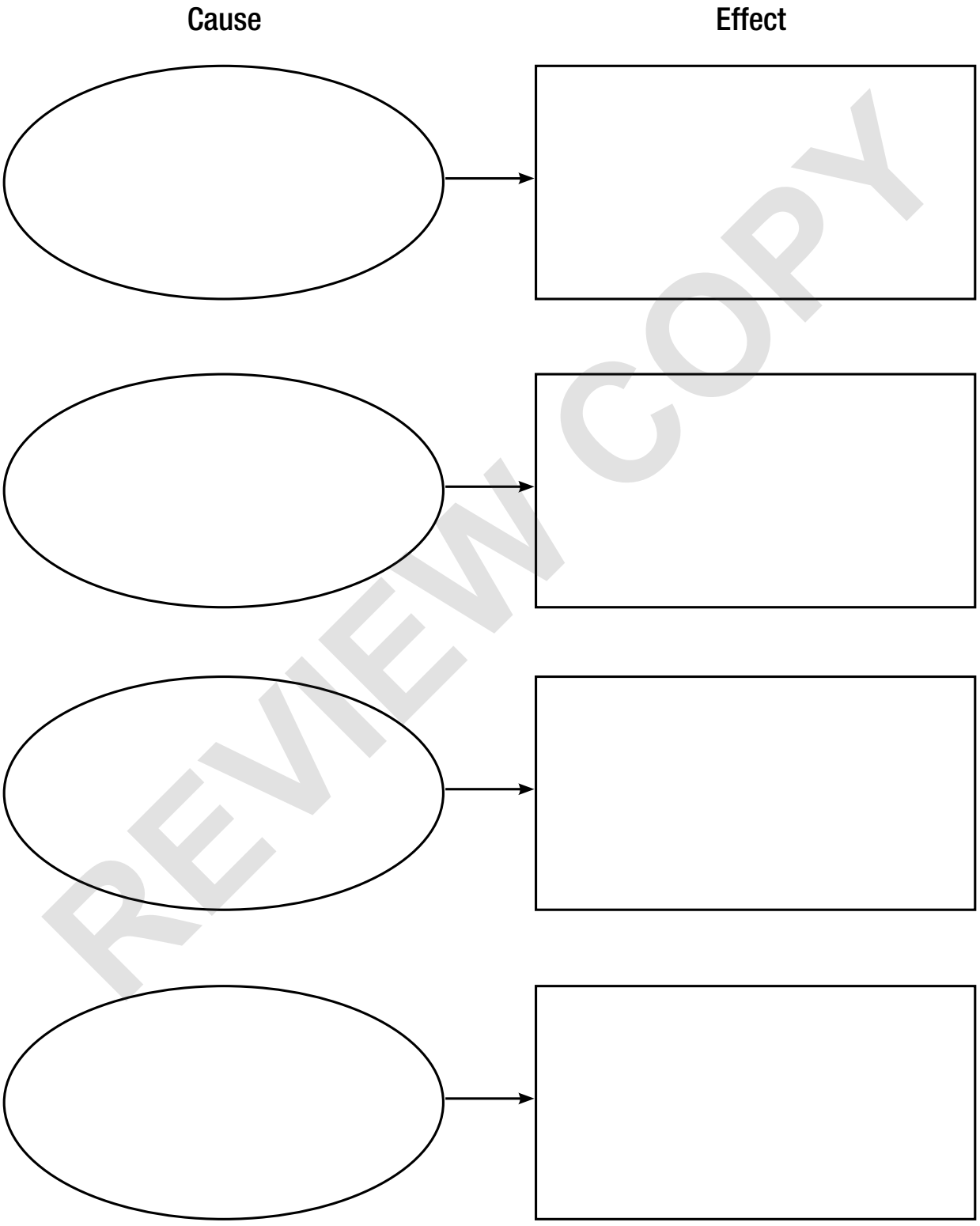
With Regard To



Attribute Table



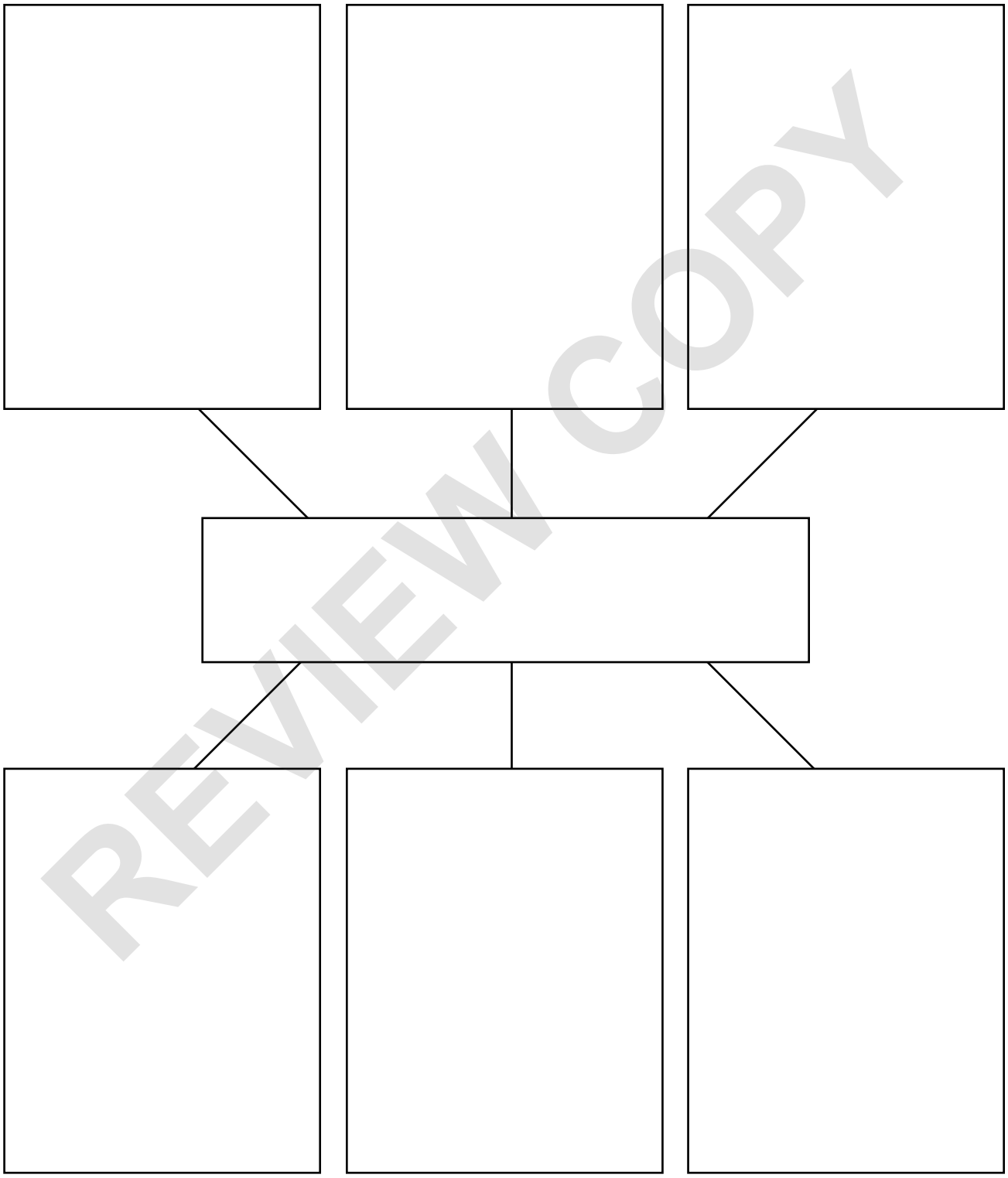
Cause and Effect Map



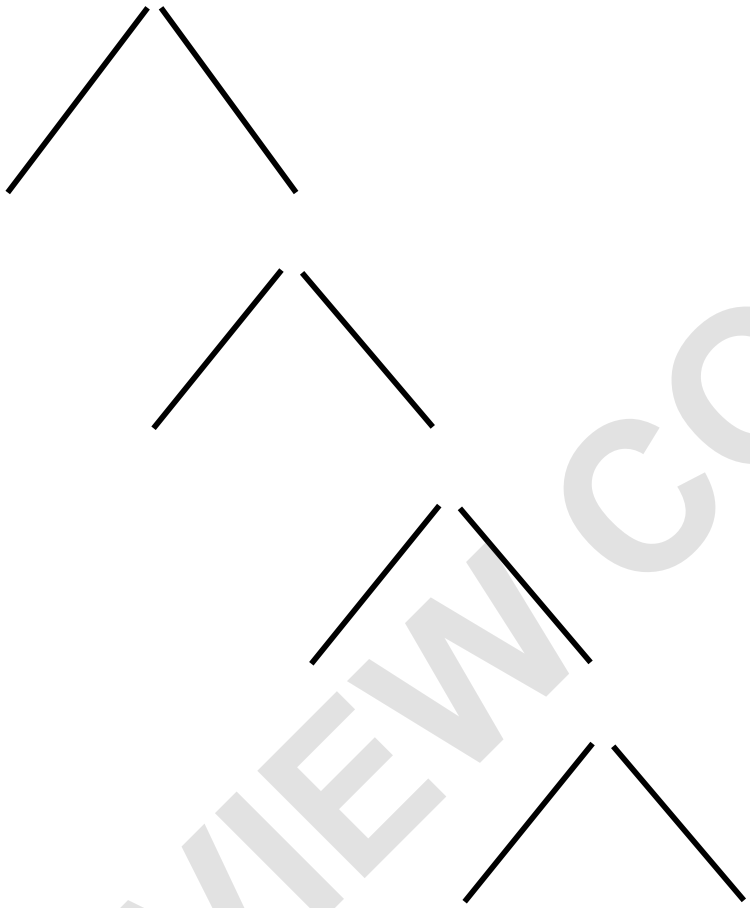
Frayer Model

Definition	Characteristics
WORD	
Examples from Life	Non-Examples

Semantic Map/Concept Map

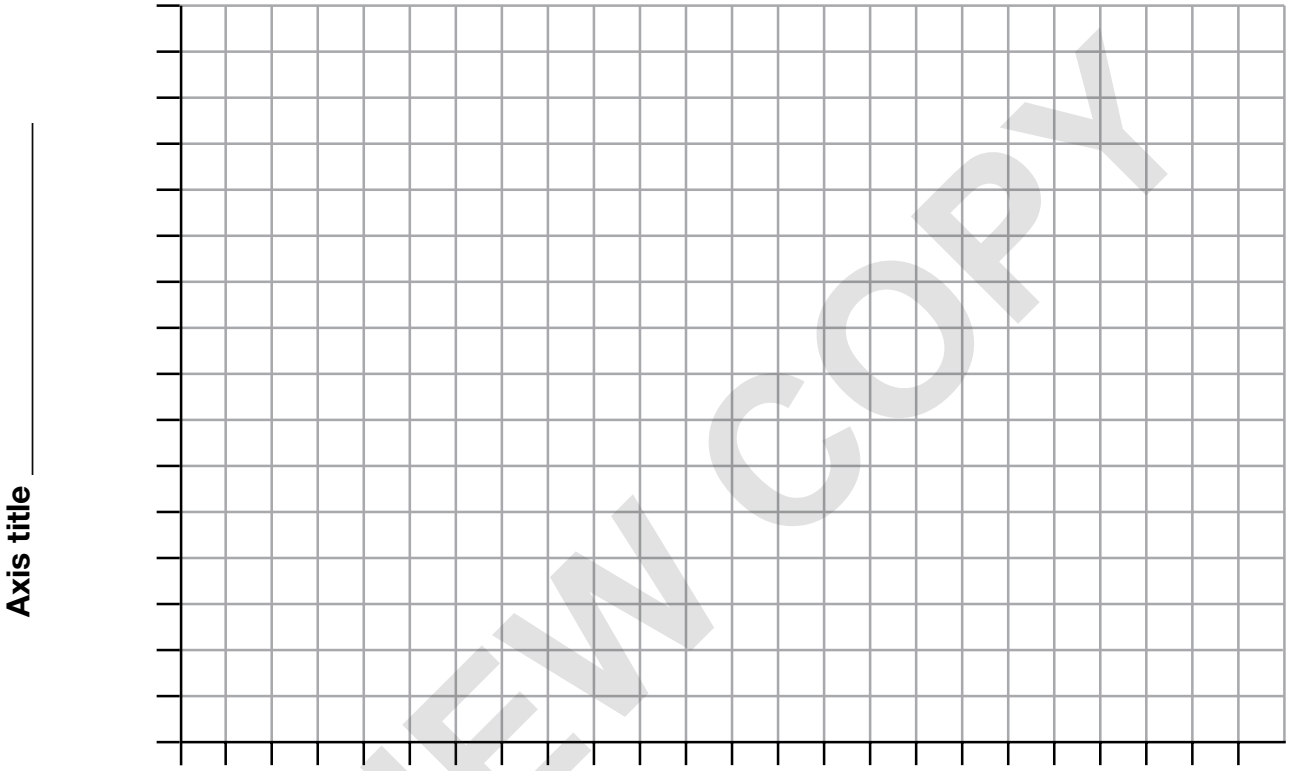


Factor Tree



Line Graph

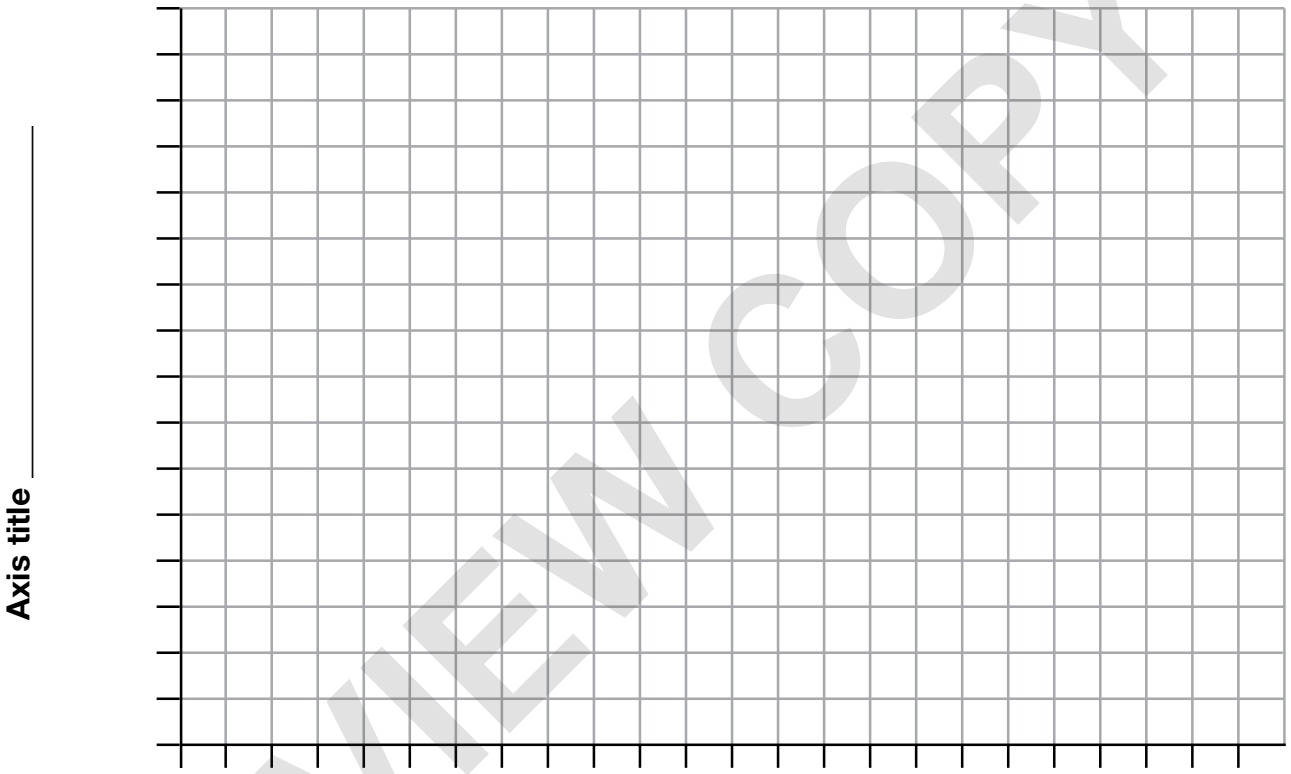
Graph title _____



Axis title _____

Bar Chart/Histogram

Graph title _____



KEY

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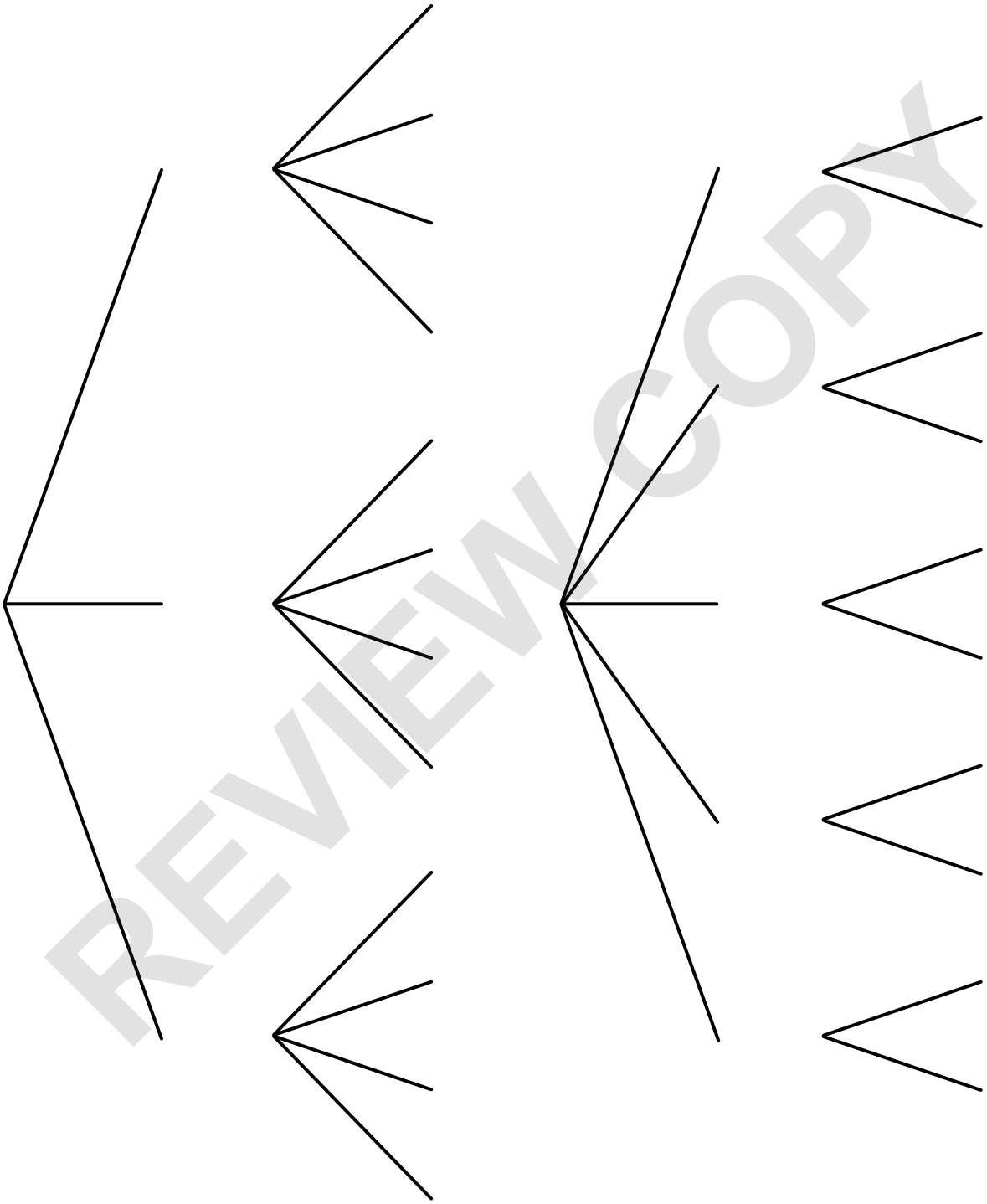
Axis title _____

Probability Trees



ALGEBRA

General	
(x, y)	Ordered pair
$(x, 0)$	x-intercept
$(0, y)$	y-intercept

Symbols	
$\approx$	Approximately equal to
$\neq$	Is not equal to
$\lvert a \rvert$	Absolute value of a
$\sqrt{a}$	Square root of a

Linear Equations	
$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$ax + b = c$	One variable
$y = mx + b$	Slope-intercept form
$ax + by = c$	General form
$y - y_1 = m(x - x_1)$	Point-slope form

Arithmetic Sequences	
$a_n = a_1 + (n - 1)d$	Explicit formula
$a_n = a_{n-1} + d$	Recursive formula

Geometric Sequences	
$a_n = a_1 \bullet r^{n-1}$	Explicit formula
$a_n = a_{n-1} \bullet r$	Recursive formula

Exponential Equations	
$y = ab^x$	General form
$y = ab^{\frac{x}{t}}$	Exponential equation
$y = a(1 + r)^t$	Exponential growth
$y = a(1 - r)^t$	Exponential decay

$A = P\left(1 + \frac{r}{n}\right)^{nt}$	Compounded interest formula
Compounded...	n (number of times per year)
Yearly/annually	1
Semiannually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

Functions	
$f(x)$	Notation, "f of x"
$f(x) = mx + b$	Linear function
$f(x) = b^x + k$	Exponential function
$(f + g)(x) = f(x) + g(x)$	Addition
$(f - g)(x) = f(x) - g(x)$	Subtraction
$(f \bullet g)(x) = f(x) \bullet g(x)$	Multiplication
$(f \div g)(x) = f(x) \div g(x)$	Division

Properties of Equality	
Property	**In symbols**
Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$ and $c \neq 0$, then $a \bullet c = b \bullet c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Properties of Operations	
Property	**General rule**
Commutative property of addition	$a + b = b + a$
Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of multiplication	$a \bullet b = b \bullet a$
Associative property of multiplication	$(a \bullet b) \bullet c = a \bullet (b \bullet c)$
Distributive property of multiplication over addition	$a \bullet (b + c) = a \bullet b + a \bullet c$

Properties of Inequality
Property
If $a > b$ and $b > c$, then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \bullet c > b \bullet c$.
If $a > b$ and $c < 0$, then $a \bullet c < b \bullet c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Laws of Exponents	
Law	**General rule**
Multiplication of exponents	$b^m \bullet b^n = b^{m+n}$
Power of exponents	$\left(b^m\right)^n = b^{mn}$ $\left(bc\right)^n = b^n c^n$
Division of exponents	$\frac{b^m}{b^n} = b^{m-n}$
Exponents of zero	$b^0 = 1$
Negative exponents	$b^{-n} = \frac{1}{b^n}$ and $\frac{1}{b^{-n}} = b^n$

DATA ANALYSIS

$IQR = Q_3 - Q_1$	Interquartile range
$Q_1 - 1.5(IQR)$	Lower outlier formula
$Q_3 + 1.5(IQR)$	Upper outlier formula
$y - y_0$	Residual formula

GEOMETRY

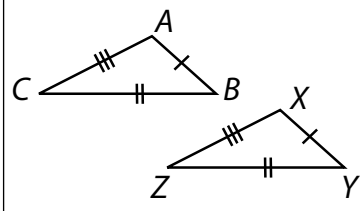
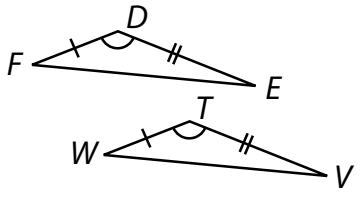
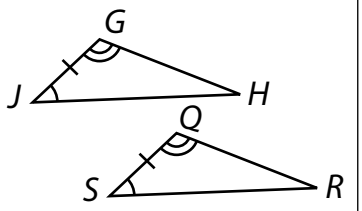
Symbols	
$d\left(\overset{\frown}{ABC}\right)$	Arc length
$\angle$	Angle
$\odot$	Circle
$\cong$	Congruent
$\overleftrightarrow{PQ}$	Line
$\overline{PQ}$	Line Segment
$\overrightarrow{PQ}$	Ray
$\parallel$	Parallel
$\perp$	Perpendicular
$\bullet$	Point
$\triangle$	Triangle
A'	Prime
$^\circ$	Degrees

Translations	
$T_{(h, k)} = (x + h, y + k)$	Translation

Reflections	
$r_{x\text{-axis}}(x, y) = (x, -y)$	Through the x-axis
$r_{y\text{-axis}}(x, y) = (-x, y)$	Through the y-axis
$r_{y = x}(x, y) = (y, x)$	Through the line $y = x$

Rotations	
$R_{90}(x, y) = (-y, x)$	Counterclockwise 90° about the origin
$R_{180}(x, y) = (-x, -y)$	Counterclockwise 180° about the origin
$R_{270}(x, y) = (y, -x)$	Counterclockwise 270° about the origin

Congruent Triangle Statements

Side-Side-Side (SSS)	Side-Angle-Side (SAS)	Angle-Side-Angle (ASA)
		
$\triangle ABC \cong \triangle XYZ$	$\triangle DEF \cong \triangle TVW$	$\triangle GHJ \cong \triangle QRS$

Pythagorean Theorem
$a^2 + b^2 = c^2$

Distance Formula	
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula

Area	
$A = lw$	Rectangle
$A = \frac{1}{2}bh$	Triangle

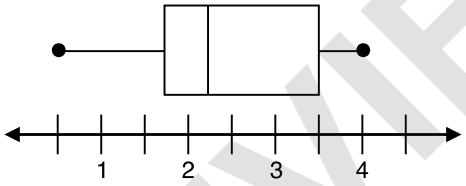
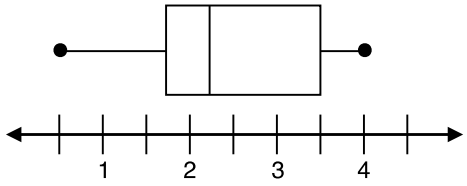
MEASUREMENTS

Length
Metric
1 kilometer (km) = 1000 meters (m)
1 meter (m) = 100 centimeters (cm)
1 centimeter (cm) = 10 millimeters (mm)
Customary
1 mile (mi) = 1760 yards (yd)
1 mile (mi) = 5280 feet (ft)
1 yard (yd) = 3 feet (ft)
1 foot (ft) = 12 inches (in)

Volume and Capacity
Metric
1 liter (L) = 1000 milliliters (mL)
Customary
1 gallon (gal) = 4 quarts (qt)
1 quart (qt) = 2 pints (pt)
1 pint (pt) = 2 cups (c)
1 cup (c) = 8 fluid ounces (fl oz)

Weight and Mass
Metric
1 kilogram (kg) = 1000 grams (g)
1 gram (g) = 1000 milligrams (mg)
1 metric ton (MT) = 1000 kilograms (kg)
Customary
1 ton (T) = 2000 pounds (lb)
1 pound (lb) = 16 ounces (oz)

English	Unit/Topic	Español
	A	
acute angle an angle measuring less than 90˚ but greater than 0˚	5/A	**ángulo agudo** ángulo que mide menos de 90˚ pero más de 0˚
algebraic expression a mathematical statement that includes numbers, operations, and variables to represent a number or quantity	1/A	**expresión algebraica** declaración matemática que incluye números, operaciones y variables para representar un número o una cantidad
altitude the perpendicular line segment from a vertex of a figure to its opposite side; height	5/C	**altitud** línea perpendicular desde un vértice de una figura hasta su lado opuesto; altura
angle two rays or line segments sharing a common endpoint. Angles can be measured in degrees or radians; written as $\angle A$.	5/A 5/C	**ángulo** dos semirrectas o segmentos de línea que comparten un extremo común. Los ángulos se pueden medir en grados o radianes; se expresa como $\angle$.
angle of rotation the angle through which a figure is rotated about a center point; the measure of the angle created by connecting the center of rotation to a point on the preimage and to the corresponding point on the image	5/E	**ángulo de rotación** el ángulo a través del cual se hace girar una figura alrededor de un punto central; la medida del ángulo creado por la conexión del centro de rotación a un punto de la imagen inversa y para el punto correspondiente en la imagen
angle-side-angle (ASA) if two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent	5/F	**ángulo-lado-ángulo (ASA)** si dos ángulos y el lado incluido de un triángulo son congruentes con los dos ángulos y el lado incluido de otro triángulo, entonces los dos triángulos son congruentes
arc length the distance between the endpoints of an arc; written as $d\left(\overset{\frown}{ABC}\right)$	5/A	**longitud de arco** distancia entre los extremos de un arco; se expresa como $d\left(\overset{\frown}{ABC}\right)$
area the amount of space inside the boundary of a two-dimensional figure	6/B	**área** cantidad de espacio dentro del límite de una figura bidimensional
arithmetic sequence a linear function with a domain of positive consecutive integers in which the difference between any two consecutive terms is equal	2/C 2/I	**secuencia aritmética** función lineal con dominio de enteros consecutivos positivos, en la que la diferencia entre dos términos consecutivos es equivalente

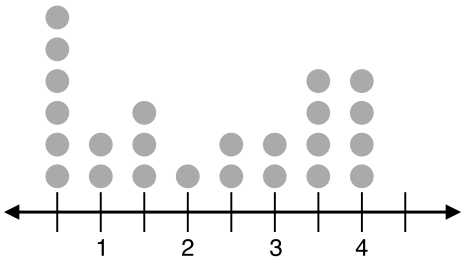
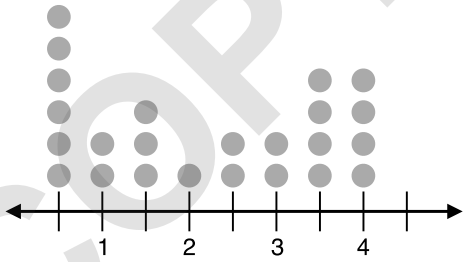
English	Unit/Topic	Español
asymptote a line that a function gets closer and closer to as one of the variables increases or decreases without bound	2/D 2/E	**asíntota** una línea que una función se acerca cada vez más cerca de una de las variables aumenta o disminuye sin límite
B		
base the factor being multiplied together in an exponential expression; in the expression a^b, a is the base	1/A	**base** factor que se multiplica en forma conjunta en una expresión exponencial; en la expresión a^b, a es la base
bisect to cut in half	5/C	**bisecar** cortar por la mitad
boundary line the graph of the line that represents a linear inequality and that divides the coordinate plane into two half planes, one of which contains all the solutions of the inequality	2/B	**línea de límite** la gráfica de la línea que representa una desigualdad lineal y que divide el plano de coordenadas en dos medios planos, uno de los cuales contiene todas las soluciones de la desigualdad
box plot a plot showing the minimum, maximum, first quartile, median, and third quartile of a data set; the middle 50% of the data is indicated by a box. Example: 	4/A	**diagrama de caja** diagrama que muestra el mínimo, máximo, primer cuartil, mediana y tercer cuartil de un conjunto de datos; se indica con una caja el 50% medio de los datos. Ejemplo: 
C		
causation a relationship between two events where a change in one event is responsible for a change in the second event	4/C	**causalidad** relación entre dos eventos en la que un cambio en un evento es responsable por un cambio en el segundo evento
circle the set of points on a plane at a fixed distance, or radius, from a given point, the center	5/A 5/D	**círculo** conjunto de puntos en un plano a determinada distancia, o radio, de un único punto, el centro
circular arc on a circle, the set of points between the endpoints of two radii	5/A	**arco circular** en un círculo, conjunto de puntos no compartidos entre los extremos de dos radios
clockwise rotating a figure in the direction that the hands on a clock move	5/B 5/E	**sentido horario** rotación de una figura en la dirección en que se mueven las agujas de un reloj

English	Unit/Topic	Español
coefficient the number multiplied by a variable in an algebraic expression	1/A	**coeficiente** número multiplicado por una variable en una expresión algebraica
common difference the number added to each consecutive term in an arithmetic sequence	2/C 2/I	**diferencia común** número sumado a cada término consecutivo en una secuencia aritmética
common ratio the number that each consecutive term is multiplied by in a geometric sequence	2/C 2/I	**proporción constante** el número que cada término esta multiplicado por en una secuencia geométrica
compass an instrument for creating circles or transferring measurements that consists of two pointed branches joined at the top by a pivot	5/C	**compás** instrumento utilizado para crear círculos o transferir medidas, que consiste en dos brazos terminados en punta y unidos en la parte superior por un pivote
compression a transformation in which a figure becomes smaller; compressions may be horizontal (affecting only horizontal lengths), vertical (affecting only vertical lengths), or both	5/E	**compresión** transformación en la que una figura se hace más pequeña; las compresiones pueden ser horizontales (cuando afectan sólo la longitud horizontal), verticales (cuando afectan sólo la longitud vertical), o en ambos sentidos
conditional relative frequency a joint frequency compared to either the sum of all frequencies or the sum of the frequencies for a category of the given data	4/B	**frecuencia condicional relativa** frecuencia conjunta comparada con la suma de todas las frecuencias o la suma de las frecuencias para una categoría de los datos dados
congruency transformation a transformation in which a geometric figure moves but keeps the same size and shape	5/E	**transformación de congruencia** transformación en la que se mueve una figura geométrica pero se mantiene el mismo tamaño y la misma forma
congruent having the same shape, size, lines, and angles; the symbol for congruent is ≅	5/A 5/C 5/D 5/E 6/A	**congruente** tiene la misma forma, tamaño, líneas y anglos; el símbolo paracongruente es ≅
congruent angles two angles that have the same measure	5/F	**ángulos congruentes** dos ángulos con la misma medida

English	Unit/Topic	Español
congruent sides two sides that have the same length	5/F	**lados congruentes** dos lados con la misma longitud
congruent triangles triangles having the same angle measures and side lengths	5/F	**triángulos congruentes** triángulos con las mismas medidas de ángulos y longitudes de lados
consistent a system of equations with at least one ordered pair that satisfies both equations	3/B	**consistente** sistema de ecuaciones con al menos un par ordenado que satisface ambas ecuaciones
constant a quantity that does not change	1/A	**constante** cantidad que no cambia
constraint a restriction or limitation on any of the variables in an equation or inequality	1/D 2/B	**limitación** una restricción o limitación de cualquiera de las variables en una ecuación o desigualdad
construct to create a precise geometric representation using a straightedge along with either patty paper (tracing paper), a compass, or a reflecting device	5/C	**construir** crear una representación geométrica precisa mediante regla de borde recto y papel encerado (papel para calcar), compás o un dispositivo de reflexión
construction a precise representation of a figure using a straightedge and a compass, patty paper and a straightedge, or a reflecting device and a straightedge	5/C 5/D	**construcción** representación precisa de una figura mediante regla de borde recto y compás, papel encerado y una regla de borde recto, o un dispositivo de reflexión y una regla de borde recto
continuous having no breaks	2/D	**continuo** sin interrupciones
coordinate plane a plane determined by a set of two number lines, called the axes, that intersect at right angles	1/C	**plano de coordenadas** un plano determinado por un conjunto de dos líneas numéricas, llamadas los ejes, que se cruzan en ángulos rectos
correlation a relationship between two events, where a change in one event is related to a change in the second event. A correlation between two events does not imply that the first event is responsible for the change in the second event; the correlation only shows how likely it is that a change also took place in the second event.	4/C	**correlación** relación entre dos eventos en la que el cambio en un evento se relaciona con un cambio en el segundo evento. Una correlación entre dos eventos no implica que el primero sea responsable del cambio en el segundo; la correlación sólo demuestra cuán probable es que también se produzca un cambio en el segundo evento.

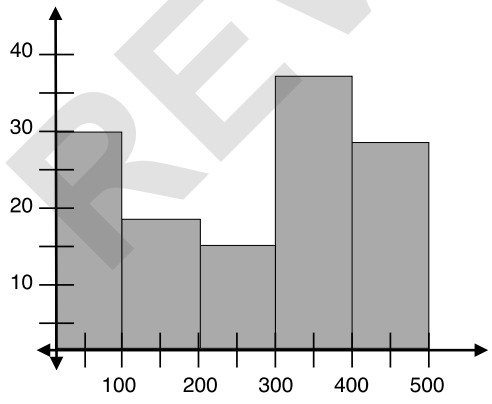
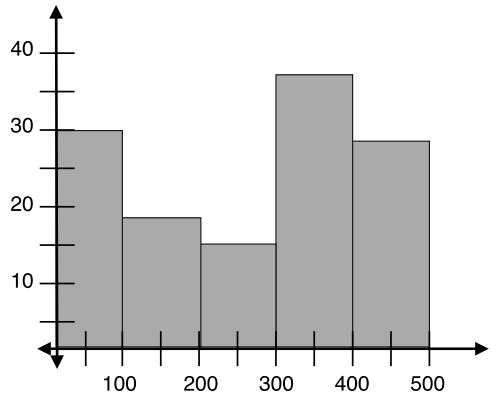
English	Unit/Topic	Español
correlation coefficient a quantity that assesses the strength of a linear relationship between two variables, ranging from –1 to 1; a correlation coefficient of –1 indicates a strong negative correlation, a correlation coefficient of 1 indicates a strong positive correlation, and a correlation coefficient of 0 indicates a very weak or no linear correlation	4/C	**coeficiente de correlación** cantidad que evalúa la fuerza de una relación lineal entre dos variables, que varía de –1 a 1; un coeficiente de correlación de –1 indica una fuerte correlación negativa, un coeficiente de correlación de 1 indica una fuerte correlación positiva, y un coeficiente de correlación de 0 indica una correlación muy débil o no lineal
corresponding angles 1. angles of two figures that lie in the same position relative to the figure. In transformations, the corresponding angles are the preimage and image angles, so $\angle A$ and $\angle A'$ are corresponding angles and so on. 2. a pair of angles in a similar position	5/E 5/F	**ángulos correspondientes** 1. ángulos de dos figuras que se ubican en la misma posición relativa a la figura. En las transformaciones, los vangulos correspondientes son los angulos de preimagen e imagen, de manera que $\angle A$ y $\angle A'$ son los angulos correspondientes, etc. 2. un par de ángulos en una posición similar
Corresponding Parts of Congruent Triangles are Congruent (CPCTC) if two or more triangles are proven congruent, then all of their corresponding parts are congruent as well	5/F	**Las partes correspondientes de triángulos congruentes son congruentes (CPCTC)** si se comprueba que dos o más triángulos son congruentes, entonces todas sus partes correspondientes son también congruentes
corresponding sides sides of two figures that lie in the same position relative to the figure. In transformations, the corresponding sides are the preimage and image sides, so $\overline{AB}$ and $\overline{A'B'}$ are corresponding sides and so on.	5/E 5/F	**lados correspondientes** lados de dos figuras que están en la misma posición relativa a la figura. En las transformaciones, los lados correspondientes son los de preimagen e imagen, entonces $\overline{AB}$ y $\overline{A'B'}$ son los lados correspondientes, etc.
counterclockwise rotating a figure in the opposite direction that the hands on a clock move	5/B 5/E	**en sentido antihorario** rotación de una figura en la dirección opuesta a la que se mueven las agujas de un reloj

English	Unit/Topic	Español
curve the graphical representation of the solution set for $y = f(x)$; in the special case of a linear equation, the curve will be a line	2/A	**curva** representación gráfica del conjunto de soluciones para $y = f(x)$; en el caso especial de una ecuación lineal, la curva será una recta
	D	
dependent a system of equations that has an infinite number of solutions; lines coincide when graphed	3/B	**dependiente** sistema de ecuaciones con una cantidad infinita de soluciones; las rectas coinciden cuando se grafican
dependent variable generally labeled on the *y*-axis**;** the quantity that is based on the input values of the independent variable	1/C	**variable dependiente** generalmente designada en el eje *y*; cantidad que se basa en los valores de entrada de la variable independiente
diameter a line segment passing through the center of a circle connecting two points on the circle; twice the radius	5/D	**diámetro** línea recta que pasa por el centro de un círculo y conecta dos puntos en el círculo; dos veces el radio
dilation a transformation in which a figure is either enlarged or reduced by a scale factor in relation to a center point	5/E	**dilatación** transformación en la que una figura se amplía o se reduce por un factor de escala en relación con un punto central
discrete individually separate and distinct	2/C	**discreto** individualmente aparte y distinto
distance along a line the linear distance between two points on a given line; written as *PQ*	5/A	**distancia a lo largo de una recta** distancia lineal entre dos puntos de una determinada línea; se expresa como *PQ*
distance formula formula that states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	6/A 6/B	**fórmula de distancia** fórmula que establece la distancia entre los puntos (x_1, y_1) y (x_2, y_2) equivale a $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
domain the set of all input values for which a relation or function is defined; the set of *x*-values that are valid for a relation or function	2/A 2/D	**dominio** el conjunto de todos los valores de entrada para los que se define una relación o una función; el conjunto de valores *x* que son válidos para una relación o función

English	Unit/Topic	Español
dot plot a frequency plot that shows the number of times a response occurred in a data set, where each data value is represented by a dot. Example: 	4/A	**diagrama de puntos** diagrama de frecuencia que muestra la cantidad de veces que se produjo una respuesta en un conjunto de datos, en el que cada valor de dato está representado por un punto. Ejemplo: 
drawing a precise representation of a figure, created with measurement tools such as a protractor and a ruler	5/C	**dibujo** representación precisa de una figura, creada con herramientas de medición tales como transportador y regla
E		
elimination method adding or subtracting the equations in the system together so that one of the variables is eliminated; multiplication might be necessary before adding the equations together	3/B	**método de eliminación** suma o sustracción conjunta de ecuaciones en el sistema de manera de eliminar una de las variables; podría requerirse multiplicación antes de la suma conjunta de las ecuaciones
end behavior the behavior of the graph as x approaches positive infinity and as x approaches negative infinity	2/E	**comportamiento final** el comportamiento de la gráfica al aproximarse x a infinito positivo o a infinito negativo
endpoint either of two points that mark the ends of a line segment; a point that marks the end of a ray	5/C	**extremo** uno de los dos puntos que marcan el final de un segmento de recta; punto que marca el final de una semirrecta
equation a mathematical sentence that uses an equal sign (=) to show that two quantities are equal	1/B 2/G	**ecuación** declaración matemática que utiliza el signo igual (=) para demostrar que dos cantidades son equivalentes
equidistant the same distance from a reference point	5/C 5/E	**equidistante** a la misma distancia de un punto de referencia
equilateral triangle a triangle with all three sides equal in length	5/D	**triángulo equilátero** triángulo con sus tres lados de la misma longitud

English	Unit/Topic	Español
explicit formula a formula used to find the nth term of a sequence; if n is the term number, then the explicit formula for the nth term of an arithmetic sequence is $a_n = a_1 + (n - 1)d$, and the explicit formula for the nth term of a geometric sequence is $a_n = a_1 \bullet r^{n-1}$	2/C 2/I	**fórmula explícita** una fórmula usada para encontrar el término n de una secuencia; si n es el número del término, entonces la fórmula explícita para el término n de una secuencia aritmética es $a_n = a_1 + (n - 1)d$, y la fórmula explícita para el término n de una secuencia geométrica es $a_n = a_1 \bullet r^{n-1}$
explicit function a function in which the dependent variable can be written in terms of the independent variable; $f(x) = 2x$ is an explicit function, where x is the independent variable and $f(x)$ is the dependent variable	2/G	**función explícita** una función en la que la variable dependiente se puede escribir en términos de la variable independiente; $f(x) = 2x$ es una función explícita, donde x es la variable independiente y $f(x)$ es la variable dependiente
exponent the number of times a factor is being multiplied together in an exponential expression; in the expression a^b, b is the exponent	1/A	**exponente** cantidad de veces que se multiplica un factor en forma conjunta en una expresión exponencial; en la expresión a^b, b es el exponente
exponential decay an exponential equation with a base, b, that is between 0 and 1 exclusive (that is, $0 < b < 1$); an example is the formula $y = a(1 - r)^t$, where a is the initial value, $(1 - r)$ is the base (with $0 < r < 1$), t is the variable exponent, and y is the final value	1/B 1/C	**decaimiento exponencial** una ecuación exponencial con una base, b, que está entre 0 y 1 exclusivo (es decir, $0 < b < 1$); un ejemplo es la fórmula $y = a(1 - r)^t$, donde a es el valor inicial, $(1 - r)$ es la base (con $0 < r < 1$), t es el exponente variable y y es el final valor
exponential equation an equation whose independent variable is in the exponent; the general form of its equation is $f(x) = ab^x + k$, where a is the initial value, b is the base, x is the input value, k is the vertical shift, and $f(x)$ is the output. Another form is $y = ab^{\frac{x}{t}}$, where t is the interval over which y changes by a factor of b, and x is measured in the same units as t.	1/B 1/C 2/G	**ecuación exponencial** ecuación cuya variable independiente es en el exponente; la forma general de su ecuación es $f(x) = ab^x + k$, donde a es el valor inicial, b es la base, x es el valor de entrada, k es el desplazamiento vertical y $f(x)$ es el valor de salida. Otra forma es $y = ab^{\frac{x}{t}}$, donde t es el intervalo en el que y cambia por un factor de b, y x se mide en las mismas unidades como t.

English	Unit/Topic	Español
exponential function a function whose independent variable is in the exponent; the general form of its equation is $f(x) = ab^x + k$, where a is the initial value, b is the base, x is the input value, k is the vertical shift, and $f(x)$ is the output	2/E 2/F 2/J	**función exponencial** una función cuya variable independiente es en el exponente; la forma general de su ecuación es $f(x) = ab^x + k$, donde a es el valor inicial, b es la base, x es el valor de entrada, k es el desplazamiento vertical y $f(x)$ es el valor de salida
exponential growth an exponential equation with a base, b, greater than 1 ($b > 1$); an example is the formula $y = a(1 + r)^t$, where a is the initial value, $(1 + r)$ is the base (with $r > 0$), t is the variable exponent, and y is the final value	1/B 1/C	**crecimiento exponencial** una ecuación exponencial con una base, b, mayor que 1 ($b > 1$); un ejemplo es la fórmula $y = a(1 + r)^t$, donde a es el valor inicial, $(1 + r)$ es la base (con $r > 0$), t es el exponente variable ey y es el valor final
expression a combination of variables, quantities, and mathematical operations; 4, $8x$, and $b + 10^2$ are all expressions	2/G	**expresión** combinación de variables, cantidades y operaciones matemáticas; 4, $8x$ y $b + 10^2$ son todas expresiones
extrema the minima and maxima of a function	2/D	**extremos** los mínimos y máximos de una función
	F	
factor one of two or more numbers or expressions that are multiplied to produce a product	1/A 2/F	**factor** uno de dos o más números o expresiones que se multiplican para producir un producto
first quartile the value that identifies the lower 25% of the data; the median of the lower half of the data set; written as Q_1	4/A	**primer cuartil** valor que identifica el 25% inferior de los datos; mediana de la mitad inferior del conjunto de datos; se expresa Q_1
formula a literal expression or equation that states a specific rule or relationship among quantities	1/E	**fórmula** expresión literal o ecuación que establece una regla específica o relación entre cantidades
function a relation in which each element in the domain is mapped onto exactly one element in the range; that is, for every value of x, there is exactly one value of y	2/A 2/G 2/H 4/B	**función** relación en la que cada elemento de un dominio se combina con exactamente un elemento del rango; es decir, para cada valor de x, existe exactamente un valor de y
function notation a way to name a function using $f(x)$ to represent the dependent variable instead of y	2/A	**notación de función** forma de nombrar una función con el uso de $f(x)$ como la variable dependiente en lugar de y

English	Unit/Topic	Español
	G	
geometric sequence an exponential function that results in a sequence of numbers separated by a common ratio	2/C 2/I	**secuencia geométrica** una función exponencial que produce como resultado una secuencia de números separados por una relación común
graphing method solving a system by graphing equations on the same coordinate plane and finding the point of intersection	3/B	**método de representación gráfica** resolución de un sistema mediante graficación de ecuaciones en el mismo plano de coordenadas y hallazgo del punto de intersección
growth factor the factor by which a quantity increases or decreases over time	2/F 2/J	**factor de crecimiento** múltiplo por el que aumenta o disminuye una cantidad con el tiempo
	H	
half plane a planar region containing all points that lie on one side of a boundary line; one-half of a plane	2/B	**semiplano** una región plana que contiene todos los puntos que se encuentran en un lado de una línea de límite; la mitad de un avión
histogram a frequency plot that shows the number of times a response or range of responses occurred in a data set. Example: 	4/A	**histograma** una diagrama de frecuencia que muestra la cantidad de veces que se produce una respuesta o rango de respuestas en un conjunto de datos. Ejemplo: 

English	Unit/Topic	Español
	I	
image the new, resulting figure after a transformation	5/A 5/E	**imagen** nueva figura resultante después de una transformación
included angle the angle between two sides	5/F	**ángulo incluido** ángulo entre dos lados
included side the side between two angles of a triangle	5/F	**lado incluido** lado entre dos ángulos de un triángulo
inclusive when the points in a plane that lie along the boundary line of an inequality are included in the solution	2/B	**inclusivo** cuando los puntos en un plano que se encuentran a lo largo de la línea de límite de una desigualdad se incluyen en la solución
inconsistent a system of equations with no solutions; lines are parallel when graphed	3/B	**inconsistente** sistema de ecuaciones sin soluciones; las líneas son paralelas cuando se las grafica
independent a system of equations with exactly one solution	3/B	**independiente** sistema de ecuaciones con una solución exacta
independent variable generally labeled on the *x*-axis; the quantity that changes based on values chosen	1/C	**variable independiente** generalmente designada en el eje *x*; cantidad que cambia según valores seleccionados
inequality a mathematical sentence that shows the relationship between quantities that may or may not be equivalent. An inequality contains one or more of the following symbols: $<$, $>$, $\leq$, $\geq$, or $\neq$.	1/B 1/D	**desigualdad** enunciado matemático que demuestra la relación entre cantidades que pueden ser o no equivalentes. Una desigualdad contiene uno o más de los siguientes símbolos: $<$, $>$, $\leq$, $\geq$ o $\neq$.
inscribe to draw one figure within another figure so that every vertex of the enclosed figure touches the outside figure	5/D	**inscribir** dibujar una figura dentro de otra de manera que cada vértice de la figura interior toque la exterior
integer the set of positive and negative whole numbers and 0; the set {... –3, –2, –1, 0, 1, 2, 3, ...}	2/D	**entero** el conjunto de números enteros positivos y negativos y 0; el conjunto {... –3, –2, –1, 0, 1, 2, 3, ...}
intercept the value of the *x*- or *y*-coordinate where a line or curve intersects the *x*- or *y*-axis, respectively	2/B 2/D	**intersección** valor de la coordenada *x* o *y* donde una línea o curva interseca el eje *x* o *y*, respectivamente
interquartile range the difference between the third and first quartiles; 50% of the data is contained within this range	4/A	**rango intercuartílico** diferencia entre el tercer y primer cuartil; el 50% de los datos está contenido dentro de este rango

English	Unit/Topic	Español
interval the continuous set of real numbers between two given numbers	2/D 2/F	**intervalo** conjunto continuo de números reales entre dos números dados
inverse operation an operation that reverses the effect of another operation. Addition and subtraction are inverse operations, and multiplication and division are inverse operations.	1/E	**operación inversa** operación que revierte el efecto de otra. La adición y la sustracción son operaciones inversas, y la multiplicación y división son operaciones inversas.
irrational number a real number that cannot be written as $\frac{m}{n}$, where m and n are integers and $n \neq 0$; a non-terminating or non-repeating decimal	2/D	**número irracional** un número real que no puede ser escrito como $\frac{m}{n}$, donde m y n son números enteros y $n \neq 0$; un no-terminación o no repetitivo decimal
isometry a transformation in which the preimage and image are congruent	5/A 5/E	**isometría** transformación en la que la preimagen y la imagen son congruentes
	J	
joint frequency the number of times a pair of characteristics appear together	4/B	**frecuencia conjunta** cantidad de veces que un par de características aparecen juntas
	L	
laws of exponents rules that must be followed when working with exponents	3/A	**leyes de los exponentes** normas que deben cumplirse cuando se trabaja con exponentes
like terms terms that contain the same variables raised to the same power	1/A	**términos semejantes** términos que contienen las mismas variables elevadas a la misma potencia
line the straight path connecting two points and extending beyond the points in both directions; written as $\overleftrightarrow{PQ}$ or ℓ	5/A 5/C	**línea recta** trayectoria recta que conecta dos puntos y que se extiende más allá de los puntos en ambas direcciones; se expresa como $\overleftrightarrow{PQ}$ o ℓ
line of reflection the perpendicular bisector of the segments that connect the corresponding vertices of the preimage and the image	5/E	**línea de reflexión** bisectriz perpendicular de los segmentos que conectan los vértices correspondientes de la preimagen y la imagen
line of symmetry a line separating a figure into two halves that are mirror images	5/A	**línea de simetría** línea que separa una figura en dos mitades que son imágenes en espejo

English	Unit/Topic	Español
line segment a part of a line that is between two endpoints and that includes the endpoints; written as $\overline{PQ}$	5/A 5/C	**segmento de recta** parte de una línea que se encuentra entre dos puntos finales y que incluye los puntos finales; escrito como $\overline{PQ}$
line symmetry e exists for a figure if for every point on one side of the line of symmetry, there is a corresponding point the same distance from the line of symmetry on the other side	5/A	**simetría lineal** la que existe en una figura si para cada punto a un lado de la línea de simetría, hay un punto correspondiente a la misma distancia de la línea de simetría en el otro lado
linear equation a first-degree equation that can be written in the form $ax + by = c$, where a, b, and c are rational numbers; when written as $y = mx + b$, m is the slope of the line, and b is its y-intercept. The graph of a linear equation is a straight line.	1/B 1/C 2/A 2/G	**ecuación lineal** ecuación de primer grado que puede expresarse en la forma $ax + by = c$, donde a, b y c son números racionales; cuando se expresa como $y = mx + b$, m es la pendiente de la recta y b es el intercepto de y. La representación gráfica de una ecuación lineal es una línea recta.
linear fit (or linear model) an approximation of data using a linear function	4/C	**ajuste lineal (o modelo lineal)** aproximación de datos con el uso de una función lineal
linear function a first-degree equation that can be written in the form $f(x) = mx + b$, in which m is the slope of the line and b is the y-intercept. The graph of a linear function is a straight line.	2/E 2/F 2/J	**función lineal** una ecuación de primer grado que puede expresarse en la forma $f(x) = mx + b$, en la que m es la pendiente de la recta y b es el intercepto de y. El gráfico de una función lineal es una línea recta.
literal equation an equation that involves two or more variables	1/E	**ecuación literal** ecuación que incluye dos o más variables
	M	
marginal frequency the total number of times a characteristic occurs	4/B	**frecuencia marginal** cantidad total de veces que aparece una característica
mean the average value of a data set, found by summing all values and dividing by the number of data points	4/A	**media** valor promedio de un conjunto de datos, que se determina al sumar todos los valores y dividirlos por la cantidad de puntos de datos

English	Unit/Topic	Español
mean absolute deviation the average distance between each data point and the mean; found by summing the absolute values of the difference between each data point and the mean, then dividing this sum by the total number of data points	4/A	**desviación media absoluta** distancia promedio entre cada punto de datos y la media; se determina al sumar los valores absolutos de la diferencia entre cada punto de datos y la media y luego dividir esta suma por la cantidad total de puntos de datos
measures of center values that describe expected and repeated data values in a data set; the mean and median are two measures of center	4/A	**medidas de centro** valores que describen los valores de datos esperados y repetidos de un conjunto de datos; la media y la mediana son dos medidas de centro
measures of spread a measure that describes the variance of data values, and identifies the diversity of values in a data set	4/A	**medidas de dispersión** medidas que describen la varianza de los valores de datos e identifican la diversidad de valores en un conjunto de datos
median the middle-most value of a data set; 50% of the data is less than this value, and 50% is greater than it	4/A	**mediana** valor medio exacto de un conjunto de datos; el 50% de los datos es menor que ese valor, y el otro 50% es mayor
median of a triangle the segment joining the vertex to the midpoint of the opposite side	5/C	**mediana de un triángulo** segmento que une el vértice con el punto medio del lado opuesto
midpoint a point on a line segment that divides the segment into two equal parts	5/C	**punto medio** punto en un segmento de recta que lo divide en dos partes iguales
midsegment a line segment joining the midpoints of two sides of a triangle	5/C	**segmento medio** segmento de recta que une los puntos medios de dos lados de una figura
multiplicative inverse a number or algebraic expression that when multiplied by the original number or algebraic expression has a product of 1; also called the *reciprocal*	1/E	**inverso multiplicativo** número o expresión algebraica que, cuando se multiplica por el número o la expresión algebraica original, tiene un producto de 1; también se llama *recíproco*

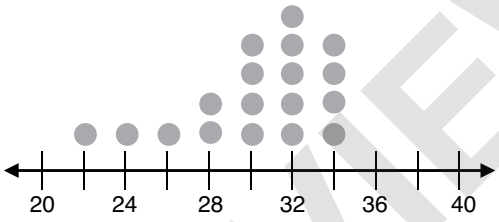
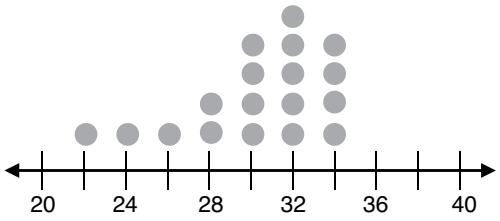
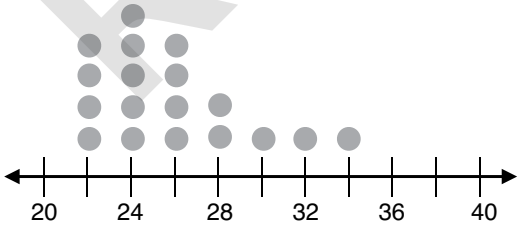
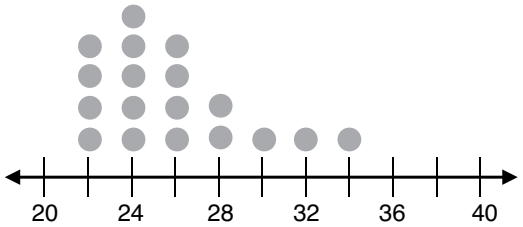
English	Unit/Topic	Español
	N	
natural numbers the set of positive integers {1, 2, 3, ...}	2/C 2/D	**números naturales** conjunto de enteros positivos {1, 2, 3, ...}
negative function a function or a portion of a function where the *y*-values are less than 0 for all *x*-values	2/D	**función negativa** función o porción de una función en la que los valores *y* son menores que 0 para todos los valores *x*
non-inclusive when the points in a plane that lie along the boundary line of an inequality are not included in the solution	2/B	**no inclusivo** cuando los puntos en un plano que se encuentran a lo largo de la línea de límite de una desigualdad no están incluidos en la solución
non-rigid motion a transformation done to a figure that changes the figure's shape and/or size	5/E	**movimiento no rígido** transformación hecha a una figura que cambia su forma o tamaño
	O	
obtuse angle an angle measuring greater than 90° but less than 180°	5/A	**ángulo obtuso** ángulo que mide más de 90° pero menos de 180°
one-to-one a relationship wherein each input is mapped to exactly one output, and each output is associated with exactly one input	5/A	**unívoca** relación en la que cada entrada se asigna a exactamente una salida, y cada salida está asociada con exactamente una entrada
order of operations the order in which expressions are evaluated from left to right (grouping symbols, evaluating exponents, completing multiplication and division, completing addition and subtraction)	1/A	**orden de las operaciones** orden en el que se evalúan las expresiones de izquierda a derecha (con agrupación de símbolos, evaluación de exponentes, realización de multiplicaciones y divisiones, sumas y sustracciones)
ordered pair the coordinates of a point in a coordinate plane, (x, y) where the order is significant	2/A	**par ordenado** coordenadas de un punto en un plano de coordenadas, (x, y), en los que el orden es significativo
outlier a data value that is much greater than or much less than the rest of the data in a data set; mathematically, any data less than $Q_1 - 1.5(IQR)$ or greater than $Q_3 + 1.5(IQR)$ is an outlier	4/A	**valor atípico** valor de datos que es mucho mayor o mucho menor que el resto de los datos de un conjunto de datos; en matemática, cualquier dato menor que $Q_1 - 1{,}5(IQR)$ o mayor que $Q_3 + 1{,}5(IQR)$ es un valor atípico

English	Unit/Topic	Español
	P	
parallel lines lines in a plane that do not share any points and never intersect; written as $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$; line segments and rays can also be parallel	5/A 5/C 6/A	**líneas paralelas** líneas en un plano que no comparten ningún punto y nunca se cortan; se expresan como $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$; segmentos de línea y los rayos también pueden ser paralelos
parallelogram a special type of quadrilateral with two pairs of opposite sides that are paralle	6/A	**paralelogramo** un tipo especial de cuadrilátero con dos pares de lados opuestos paralelos
parameter a constant in a function that determines the specific graph of the function but not the type of the function	2/J	**parámetro** una constante en una función que determina el gráfico específico de la función pero no el tipo de la función
perimeter the distance around a two-dimensional figure	6/B	**perímetro** distancia alrededor de una figura bidimensional
perpendicular bisector a line that intersects a segment at its midpoint at a right angle	5/C	**bisectriz perpendicular** línea que corta un segmento en su punto medio en ángulo recto
perpendicular lines two lines that intersect at a right angle (90˚); written as $\overleftrightarrow{AB} \perp \overleftrightarrow{PQ}$; line segments and rays can also be perpendicular	5/A 5/C 6/A	**líneas perpendiculares** dos líneas que se cortan en ángulo recto (90˚); se expresan como $\overleftrightarrow{AB} \perp \overleftrightarrow{PQ}$; segmentos de línea y los rayos también pueden ser perpendicular
point an exact position or location in a given plane	5/A	**punto** posición o ubicación exacta en un plano determinado
point of intersection the point at which two lines cross or meet	3/B	**punto de intersección** punto en que se cruzan o encuentran dos líneas
point of rotation the fixed location that an object is turned around; the point can lie on, inside, or outside the figure	5/E	**punto de rotación** ubicación fija en torno a la que gira un objeto; el punto puede estar encima, dentro o fuera de la figura
polygon two-dimensional figure with at least three sides	6/B	**polígono** figura bidimensional con al menos tres lados
positive function a function or a portion of a function where the *y*-values are greater than 0 for all *x*-values	2/D	**función positiva** una función o porción de una función en la que los valores *y* son mayores que 0 para todos los valores *x*
postulate a true statement that does not require a proof	5/F	**postulado** afirmación verdadera que no requiere prueba

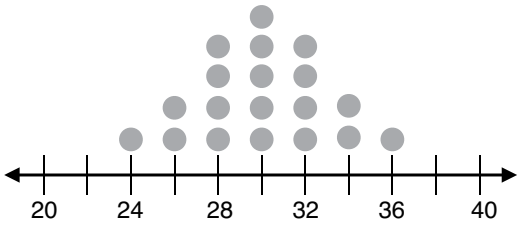
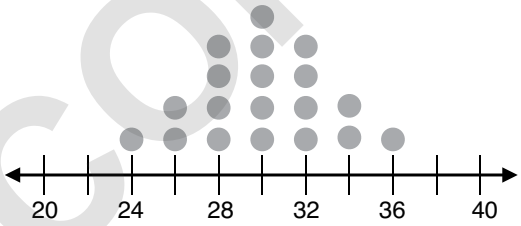
English	Unit/Topic	Español
preimage the original figure before undergoing a transformation	5/A 5/E	**preimagen** figura original antes de sufrir una transformación
properties of equality rules that allow you to balance, manipulate, and solve equations	3/A	**propiedades de igualdad** normas que permiten equilibrar, manipular y resolver ecuaciones
properties of inequality rules that allow you to balance, manipulate, and solve inequalities	3/A	**propiedades de desigualdad** normas que permiten equilibrar, manipular y resolver desigualdades
Q		
quadrant the coordinate plane is separated into four sections: • In Quadrant I, x and y are positive. • In Quadrant II, x is negative and y is positive. • In Quadrant III, x and y are negative. • In Quadrant IV, x is positive and y is negative.	5/B	**cuadrante** plano de coordenadas que se divide en cuatro secciones: • En el cuadrante I, x y y son positivos. • En el cuadrante II, x es negativo y y es positivo. • En el cuadrante III, x y y son negativos. • En el cuadrante IV, x es positivo y y es negativo.
quadrilateral a polygon with four sides	6/A	**cuadrilátero** polígono con cuatro lados
quantity a value or expression that may be expressed in numbers	1/B	**cantidad** valor o expresión que puede expresarse en números
R		
radius a line segment that extends from the center of a circle to a point on the circle. Its length is half the diameter.	5/D	**radio** segmento de línea que se extiende desde el centro de un círculo hasta un punto de la circunferencia del círculo. Su longitud es la mitad del diámetro.
range the set of all outputs of a relation or function; the set of y-values for which a function is defined	2/A	**rango** conjunto de todas las salidas de una función; conjunto de valores de y para el que se define una función
rate a ratio that compares different kinds of units	1/B	**tasa** proporción en que se comparan distintos tipos de unidades
rate of change a ratio that describes how much one quantity changes with respect to the change in another quantity; also known as the slope of a line	2/D 2/F	**tasa de cambio** proporción que describe cuánto cambia una cantidad con respecto al cambio de otra cantidad; también se la conoce como pendiente de una recta

English	Unit/Topic	Español
ratio the relation between two quantities; can be expressed in words, or as a fraction, decimal, or percent	2/D	**proporción** relación entre dos cantidades; puede expresarse en palabras, fracciones, decimales o como porcentaje
rational number a real number that can be written as $\frac{m}{n}$, where both m and n are integers and $n \neq 0$; a terminating or repeating decimal	2/D	**número racional** en los que m y n son enteros y $n \neq 0$; cualquier número que puede escribirse como decimal finito o periódico
ray a part of a line that starts at a point and extends infinitely in one direction; written as $\overrightarrow{PQ}$	5/A 5/C	**semirrecta** parte de una línea que comienza en un punto y se extiende infinitamente en una dirección línea con un solo extremo; se expresa como $\overrightarrow{PQ}$
real numbers the set of all rational and irrational numbers	2/D	**números reales** conjunto de todos los números racionales e irracionales
reciprocal a number that when multiplied by the original number or algebraic expression has a product of 1; also called the *multiplicative inverse*	1/E	**recíproco** número que cuando se multiplica por el número original o la expresión algebraica tiene un producto de 1; también llamada la *inversa multiplicativa*
rectangle a special parallelogram with four right angles	6/A	**rectángulo** paralelogramo especial con cuatro ángulos rectos
recursive formula a formula used to find the next term of a sequence when the previous term or terms are known; the recursive formula for an arithmetic sequence is $a_n = a_{n-1} + d$; the recursive formula for a geometric sequence is $a_n = a_{n-1} \cdot r$	2/C 2/I	**fórmula recursiva** fórmula que se utiliza para encontrar el término siguiente de una secuencia cuando se conoce el o los términos anteriores; la fórmula recursiva de una secuencia aritmética es $a_n = a_{n-1} + d$; la fórmula recursiva para una secuencia geométrica es $a_n = a_{n-1} \cdot r$
reflection a transformation where a mirror image is created; also called a flip	5/A 5/B	**reflexión** transformación por la cual se crea una imagen en espejo
regular hexagon a six-sided polygon with all sides congruent and all angles measuring 120°	5/D	**hexágono regular** polígono de seis lados con todos los lados iguales y en el que todos los ángulos miden 120°
regular polygon a closed two-dimensional figure with all sides and all angles congruent	5/A 5/D	**polígono regular** figura bidimensional cerrada con todos los lados y todos los ángulos congruentes

English	Unit/Topic	Español
relation a set of ordered pairs	2/A	**relación** un conjunto de pares ordenados
relative maximum the greatest value of a function for a particular interval of the function	2/D	**máximo relativo** el mayor valor de una función para un intervalo particular de la función
relative minimum the least value of a function for a particular interval of the function	2/D	**mínimo relativo** el menor valor de una función para un intervalo particular de la función
residual the vertical distance between an observed data value and an estimated data value on a line of best fit	4/B	**residual** distancia vertical entre un valor de datos observado y un valor de datos estimado sobre una línea de ajuste óptimo
residual plot provides a visual representation of the residuals for a set of data; contains the points (x, residual for x)	4/B	**diagrama residual** brinda una representación visual de los residuales para un conjunto de datos; contiene los puntos (x, residual de x)
rhombus a special parallelogram with four congruent sides	6/A	**rombo** paralelogramo especial con sus cuatro lados congruente
right angle an angle measuring 90˚	5/A	**ángulo recto** ángulo que mide 90˚
rigid motion a transformation done to a figure that maintains the figure's shape and size or its segment lengths and angle measures	5/E 5/F	**movimiento rígido** transformación que se realiza a una figura que mantiene su forma y tamaño o las longitudes de sus segmentos y las medidas de ángulos
rotation a transformation that turns a figure around a fixed center point; also called a turn	5/A 5/B	**rotación** transformación que convierte una figura alrededor de un punto central fijo; también llamado un giro
S		
scale factor a multiple of the lengths of the sides from one figure to the transformed figure. If the scale factor is larger than 1, then the figure is enlarged. If the scale factor is between 0 and 1, then the figure is reduced.	5/E	**factor de escala** múltiplo de las longitudes de los lados de una figura a la figura transformada. Si el factor de escala es mayor que 1, entonces la figura se agranda. Si el factor de escala se encuentra entre 0 y 1, entonces la figura se reduce.
scatter plot a graph of data in two variables on a coordinate plane, where each data pair is represented by a point	4/B	**diagrama de dispersión** gráfica de datos en dos variables en un plano de coordenadas, en la que cada par de datos está representado por un punto

English	Unit/Topic	Español
sequence an ordered list of numbers	2/C	**secuencia** lista ordenada de números
side-angle-side (SAS) if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent	5/F	**lado-ángulo-lado (SAS)** si dos lados y el ángulo incluido de un triángulo son congruentes con dos lados y el ángulo incluido de otro triángulo, entonces los dos triángulos son congruentes
side-side-side (SSS) if three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent	5/F	**lado-lado-lado (SSS)** si los tres lados de un triángulo son congruentes con los tres lados de otro triángulo, entonces los dos triángulos son congruentes
sketch a quickly done representation of a figure; a rough approximation of a figure	5/C	**bosquejo** representación de una figura realizada con rapidez; aproximación imprecisa de una figura
skewed to the left data concentrated on the higher values in the data set, which has a tail to the left. Example: 	4/A	**desviados hacia la izquierda** datos concentrados en los valores más altos del conjunto de datos, que tiene una cola hacia la izquierda. Ejemplo: 
skewed to the right data concentrated on the lower values in the data set, which has a tail to the right. Example: 	4/A	**desviados hacia la derecha** datos concentrados en los valores más bajos del conjunto de datos, que tiene una cola hacia la derecha. Ejemplo: 

English	Unit/Topic	Español
slope the measure of the rate of change of one variable with respect to another variable; slope = $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$; the slope in the equation $y = mx + b$ is m	1/C 2/D 2/F 2/G 4/C 6/A	**pendiente** medida de la tasa de cambio de una variable con respecto a otra; pendiente = $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$; la pendiente en la ecuación $y = mx + b$ es m
slope-intercept form of a linear equation the form $y = mx + b$, where m is the slope of the line and b is the y-intercept	2/D	**forma pendiente-intersección de una ecuación lineal** la forma $y = mx + b$, donde m es la pendiente y b es el punto de intersección con el eje y
slope-intercept method a way to use the slope-intercept form of a linear equation, $y = mx + b$, to graph the line, by first plotting the y-intercept, b, and then applying the slope, m	2/D	**método de pendiente-intersección** una manera de utilizar la forma pendiente-intersección de una ecuación lineal, $y = mx + b$, para graficar la línea, trazando primero la intersección y, b, y luego aplicando la pendiente, m
solution a value that makes an equation true	1/B	**solución** valor que hace verdadera la ecuación
solution set the value or values that make a sentence or statement true; the set of ordered pairs that represent all of the solutions to an equation or a system of equations	1/B 1/D 2/A	**conjunto de soluciones** valor o valores que hacen verdadera una afirmación o declaración; conjunto de pares ordenados que representa todas las soluciones para una ecuación o sistema de ecuaciones
solution to a system of linear inequalities the set of points that lie in the intersection of the half planes of the inequalities and which may also lie on the boundary lines; the solution set is the set of all points that satisfy the inequalities in the system	2/B	**solución a un sistema de desigualdades lineales** el conjunto de puntos que se encuentran en la intersección de los planos de la mitad de las desigualdades y que también pueden situarse en las líneas de contorno; el conjunto solución es el conjunto de todos los puntos que satisfacen las desigualdades en el sistema
square a regular polygon with four congruent sides and four right angles	5/D 6/A	**cuadrado** polígono regular con cuatro lados congruentes y cuatro ángulos rectos
straightedge a bar or strip of wood, plastic, or metal having at least one long edge of reliable straightness	5/C	**regla de borde recto** barra o franja de madera, plástico o metal que tiene, al menos, un borde largo de rectitud confiable

English	Unit/Topic	Español
substitution method solving one of a pair of equations for one of the variables and substituting that into the other equation	3/B	**método de sustitución** solución de un par de ecuaciones para una de las variables y sustitución de eso en la otra ecuación
symmetric situation in which data is concentrated toward the middle of the range of data; data values are distributed in the same way above and below the middle of the sample. Example: 	4/A	**simétrico** situación en la que los datos se concentran hacia el medio del rango de datos; los valores de datos se distribuyen de la misma manera por encima y por debajo del medio de la muestra. Ejemplo: 
system a set of two or more equations	2/A	**sistema** un conjunto de dos o más ecuaciones
system of equations a set of equations with the same unknowns	1/D 3/B	**sistema de ecuaciones** un conjunto de ecuaciones con las mismas incógnitas
system of inequalities a set of two or more inequalities with the same unknowns	1/D 2/B	**sistema de desigualdades** un conjunto de dos o más desigualdades con las mismas incógnitas
	T	
term a number, a variable, or the product of a number and variable(s)	1/A	**término** número, variable o producto de un número y una o más variables
third quartile value that identifies the upper 25% of the data; the median of the upper half of the data set; 75% of all data is less than this value; written as Q_3	4/A	**tercer cuartil** valor que identifica el 25% superior de los datos; mediana de la mitad superior del conjunto de datos; el 75% de los datos es menor que este valor; se expresa como Q_3
transformation a change in a geometric figure's position, shape, or size	2/H 5/A	**transformación** cambio en la posición, la forma o el tamaño de una figura geométrica

English	Unit/Topic	Español
translation 1. moving a graph either vertically, horizontally, or both, without changing its shape; a slide 2. a transformation that moves each point of a figure the same distance in the same direction; also called a slide	2/H 5/A 5/B	**traslación** 1. mover un gráfico verticalmente, horizontalmente o ambos, sin cambiar su forma; una diapositiva 2. transformación que mueve cada punto de una figura a la misma distancia en la misma dirección; también llamado una diapositiva
trend a pattern of behavior, usually observed over time or over multiple iterations	4/B	**tendencia** patrón de comportamiento, que se observa por lo general en el tiempo o en múltiples repeticiones
triangle a three-sided polygon with three angles	5/D	**triángulo** polígono de tres lados con tres ángulos
two-way frequency table a table that divides responses into categories, showing both a characteristic in the table rows and a characteristic in the table columns; values in cells are a count of the number of times each response was given by a respondent with a certain characteristic	4/B	**tabla de frecuencia de doble entrada** tabla que divide las respuestas en dos categorías, y muestra una característica en las filas y una en las columnas; los valores de las celdas son un conteo de la cantidad de veces que un respondedor da una respuesta con una determinada característica
	U	
undefined slope occurs when the denominator of the slope formula is equal to 0; the corresponding line is a vertical line	2/D	**pendiente indefinida** se produce cuando el denominador de la fórmula de la pendiente es igual a 0; la correspondiente línea es una línea vertical
unit rate a ratio of two measurements, the second of which is 1	1/B	**tasa unitaria** una proporción de dos medidas, de las que la segunda es 1
	V	
variable a letter used to represent an unknown value or a value that changes	1/A 1/B 2/G	**variable** una letra utilizada para representar un valor desconocido o un valor que cambia
vertical shift the number of units the graph of the function is moved up or down in a plane; a type of translation	2/H 2/J	**desplazamiento vertical** cantidad de unidades que el gráfico de la función se desplaza hacia arriba o hacia abajo en un plano; un tipo de traslación

English	Unit/Topic	Español
	W	
whole numbers the set of positive integers and 0: {0, 1, 2, 3, ...}	2/D	**números enteros** conjunto de enteros positivos que incluye el 0: {0, 1, 2, 3, ...}
	X	
***x*-intercept** the *x*-coordinate of the point where a line or a curve intersects the *x*-axis	1/C 2/B 2/D 2/E 2/F	**intersección *x*** la coordenada *x* del punto en que una recta o curva corta el eje *x*
	Y	
***y*-intercept** the *y*-coordinate of the point where a line or a curve intersects the *y*-axis	1/C 2/B 2/D 2/E 2/F 2/G 4/C	**intersección *y*** la coordenada *y* del punto en que una recta o curva corta el eje *y*