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# Improving Mathematics Teaching and Learning through Formative Assessment: The Case of OGAP

Marge Petit, OGAPMath LLC

Caroline Ebby, CPRE, University of Pennsylvania

Janine Remillard, University of Pennsylvania

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# Overview

1. Development of the Ongoing Assessment Project (OGAP) and early research (Marge)
2. Large-scale impact study (Caroline)
3. Design-based implementation study (Janine)
4. Questions
5. Small discussion groups



# OGAP is...

An **intentional** and **systematic mathematics formative assessment system** based on the math education research..

- ✓ Additive Reasoning
- ✓ Multiplicative reasoning
- ✓ Fractions
- ✓ Proportionality

.... on how students develop understanding of concepts and fluency...

... as well as errors student make or pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems.

# Supported by...

- Systems, tools, and routines designed to elicit and act on evidence
  - Item banks
  - Learning trajectories at a grain size that is usable at the classroom level
- Extensive professional development and ongoing support

# OGAP Development and Implementation

Phase 1: 2003 – 2008

Phase 2: 2008 – 2014

Phase 3: 2014 – 2017

Phase 4: 2016– 2019



# Phase 1: 2003 - The context ...

- Teachers overused results of both external and classroom based summative assessment for instructional decisions.
- Math education research not accessible to classroom teachers or teachers did not realize the importance of the ME research.
- “first generation of practice based on NCTM Standards”  
(Stein et al., 2008)

# Celebrating...

One tricycle has three wheels. How many wheels do 29 tricycles have?

Sam's Response



Samir's Response

$$29 \times 3 = 87$$
$$(20 \times 3) + (9 \times 3) = 87$$

Wow .. Look at all the ways students can solve the same problems

Ava's Response

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42  
45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, 78,  
81, 84, 87



Ongoing Assessment Project

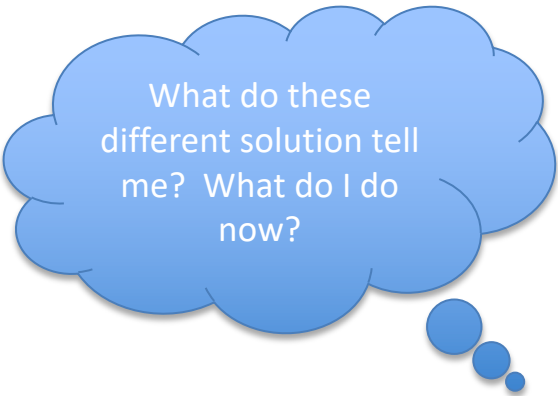


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**Samir's Response**

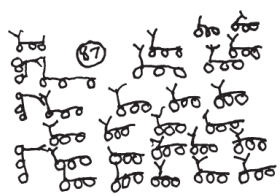
$$29 \times 3 = 87$$

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**Ava's Response**

$$3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42$$

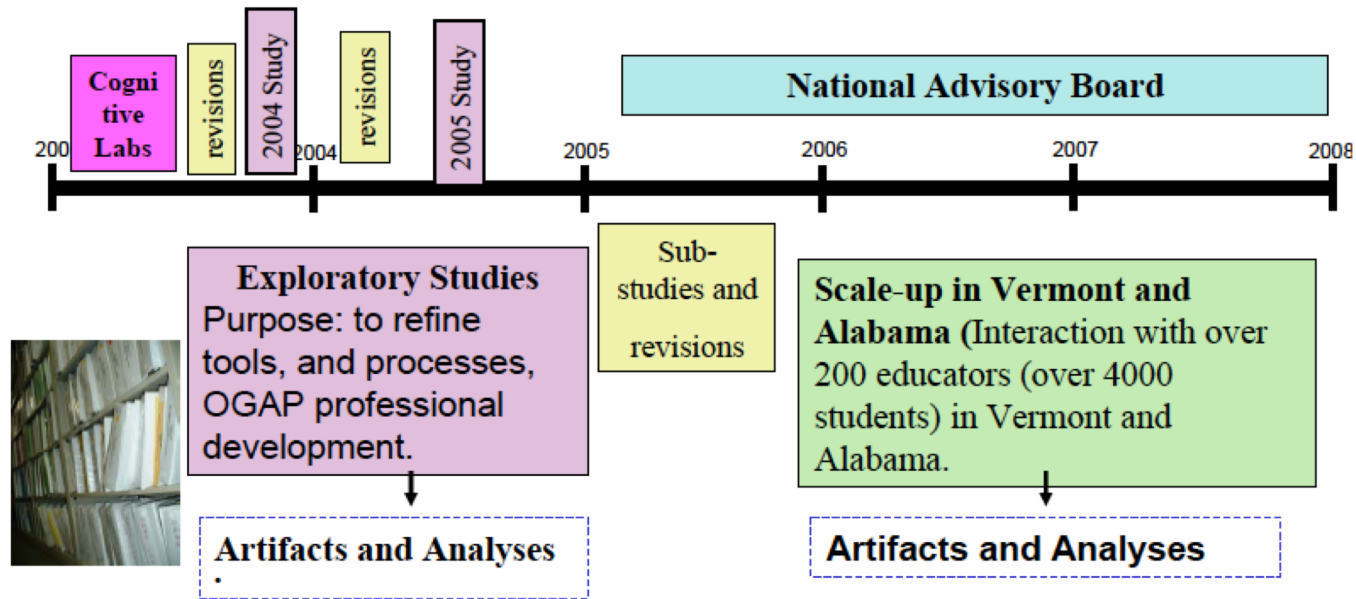
$$45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, 78, 81, 84, 87$$

**Sam's Response**


Multiplicative Strategies	Multiplicative			
Algorithms	Distributive Property	Associative Property	Doubling & Halving	
Partial Products	Traditional	$4 \times 16 = 4(10 + 6)$ $= 4(10) + 4(6)$ $= 40 + 24$ $= 64$	$(8 \times 2) \times 5 = 8(2 \times 5)$ $= 8 \times 10$ $= 80$	$16 \times 4 = 8 \times 8$ $= 64$
240 400 672	Known or Derived Fact	Commutative Property	Powers of Ten	
	$4 \times 6 = 24$	$16 \times 4 = 4 \times 16$	$5 \times 400 = 5 \times 4 \times 10 \times 10$	
<b>Transitional Strategies</b>				
Open Area Model	Considers both dimensions of an array or area model		Area Model	
			Considers BOTH dimensions of an array or area model, moving away from needing to see every square unit	
<b>Early Transitional Strategies</b>				
Skip Counting	Skip Counting with a Model	Equal groups in an array	Area Model	
3, 6, 9, 12, 15			$6 \times 4 = 24$	
Building up	Considers only ONE dimension of an array or area model		Considers BOTH dimensions of an array or area model	
$3 + 3 + 3 + 3 = 12$				
<b>Additive Strategies</b>				
Repeated addition with or without a model – $3 \times 4 = 12$				
$3 + 3 + 3 + 3 = 12$	Subitizing in small groups		$3 + 3 + 3 + 3$	
<b>Early Additive Strategies</b>				
Modeling, counting by ones	Inconsistent Grouping	Modeling, counting by subgroups		
<b>Non-Multiplicative Strategies</b>				
Adds or subtracts factors. Uses incorrect operation. Models factors incorrectly. Not enough information. Guesses. Uses procedures incorrectly.				
<b>Underlying Issues/Errors</b>				
- Doesn't consider reasonableness of solution		- Error in calculation, place value, vocabulary, property or relationship, equation, or model	- Misinterprets the remainders - Units inconsistent or missing	

This is a derivative product of the Vermont Mathematics Partnership Ongoing Assessment Project (OGAP) which was funded by NSF (DHR-0227037) and the US DOE (S366R20002). © 2012 Marge Peck Consulting, M.P.C.E. Hulbert, R. Laird. Version 27 January 2013.

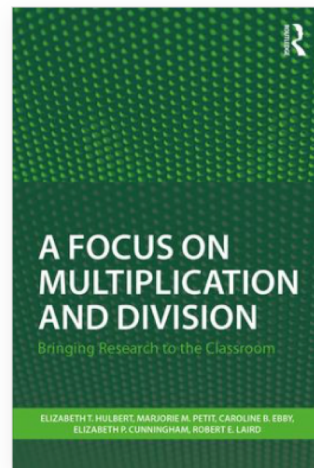
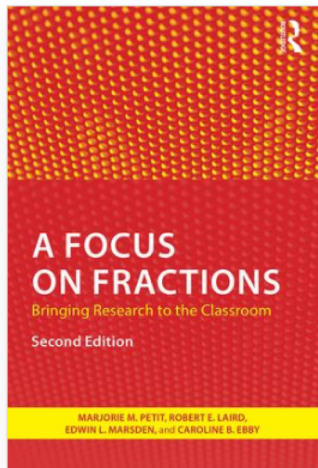
# Phase 1 – Designed Based Research and Development



Funded by NSF and DOE

# Teachers – Why weren't we ever told about this research?

The context has changed!



# Phase 2: 2008 - 2014

- Partnerships with Teachers College and CPRE at UPenn develops
- Materials refined with use with thousands of educators and students
- Project scales up (implementation & impact research)
- Publications



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# Phase 3

## Impacts on Teacher and Student Learning

Caroline Ebby, Jonathan Supovitz, Janine Remillard, & Robert Nathenson



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# 2-year Study of OGAP

- Randomized Control Trial (2014-2016)
  - 47 public schools, 13 charters
  - 31 treatment and 30 comparison schools
- All teachers in grades 3-5 (n=678)
- All students in grades 3-5 (n=13,988)



# Theory of Action

## Intensive Summer PD

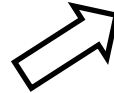
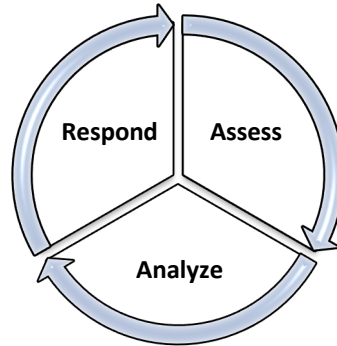
- Content
- Learning Trajectory
- Formative assessment



## Implementation

Regular FA Cycles

- In Classroom
- In PLCs



## Increase in Teacher Knowledge

- Analysis of student thinking
- Learning trajectory orientation
- Informed instructional responses
- Pedagogical Content Knowledge

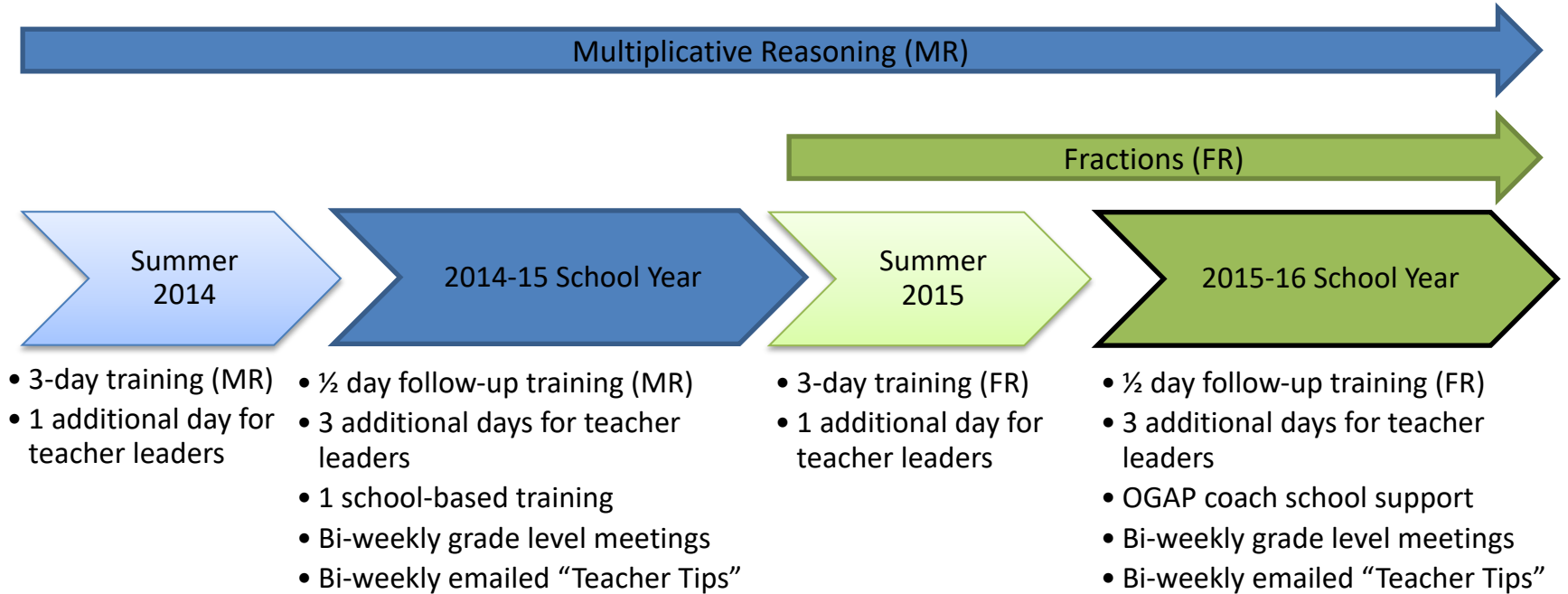


## Improvements in Student Learning

- Performance
- Sophistication of solution strategy



# Intervention Timeline



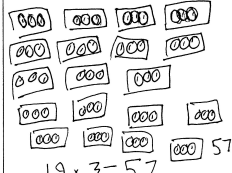
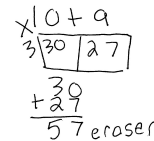
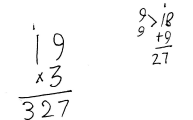
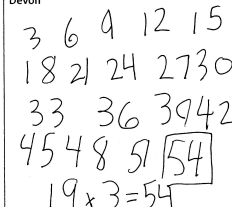
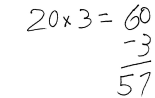
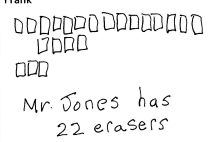
# Measuring Impact

	Aligned Measure	Additional Dimensions
Teacher Knowledge	TASK	Formative Assessment Capacity: <ul style="list-style-type: none"><li>• Ability to make sense of and respond to student work in the content area</li></ul>
Student Learning	LTA-MR	Multiplicative Reasoning: <ul style="list-style-type: none"><li>• Problem Solving Accuracy</li><li>• Sophistication of Strategy</li></ul> Fraction Concepts

# Teacher Analysis of Student Knowledge (TASK)

Mr. Jones has 19 packages of erasers. There are 3 erasers in each package. How many erasers does Mr. Jones have in all?

Show your work and write an equation.

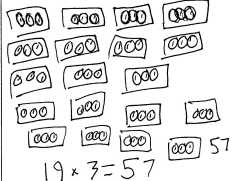
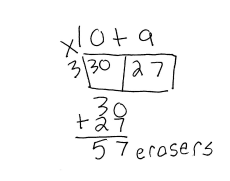
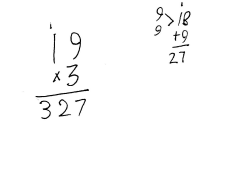
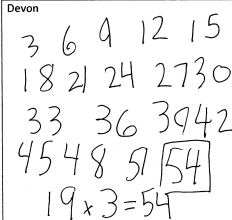
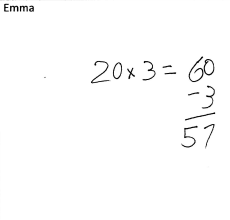
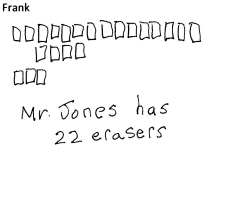
<p>Abby</p>  <p><math>19 \times 3 = 57</math></p>	<p>Brad</p>  <p><math>30 + 27 = 57</math> erasers</p>	<p>Carla</p>  <p><math>19 \times 3 = 57</math></p>
<p>Devon</p>  <p><math>19 \times 3 = 54</math></p>	<p>Emma</p>  <p><math>20 \times 3 = 60</math> <math>60 - 3 = 57</math></p>	<p>Frank</p>  <p>Mr. Jones has 22 erasers</p>

<http://www.cpre.org/task>

# Teacher Analysis of Student Knowledge (TASK)

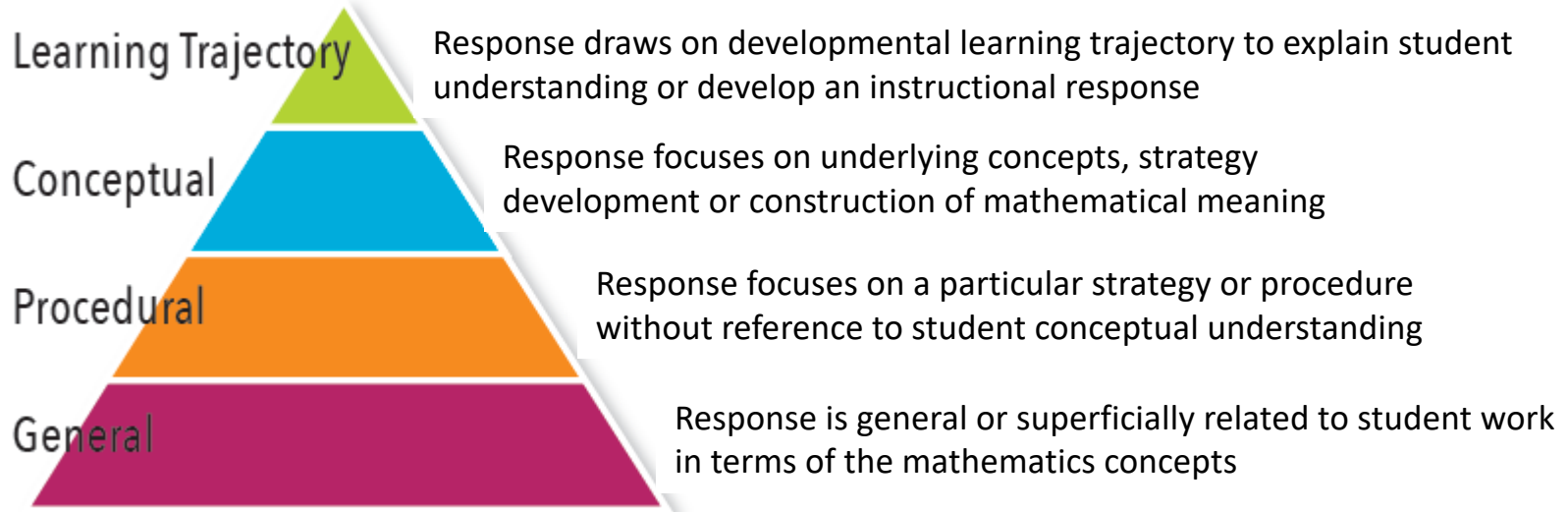
- Analysis of Student Thinking (AST)
- Learning Trajectory Orientation (LTO)
- Instructional Decision Making (IDM)

Mr. Jones has 19 packages of erasers. There are 3 erasers in each package. How many erasers does Mr. Jones have in all?  
Show your work and write an equation.

Abby	Brad	Carla
 <p><math>19 \times 3 = 57</math></p>	 <p><math>30 + 27 = 57</math> erasers</p>	 <p><math>19 \times 3 = 57</math></p>
Devon	Emma	Frank
 <p><math>19 \times 3 = 54</math></p>	 <p><math>20 \times 3 = 60</math> <math>60 - 3 = 57</math></p>	 <p>Mr. Jones has 22 erasers</p>

[www.cpre.org/task](http://www.cpre.org/task)

# TASK Scoring Rubric



<http://www.cpre.org/task>

# Study Design

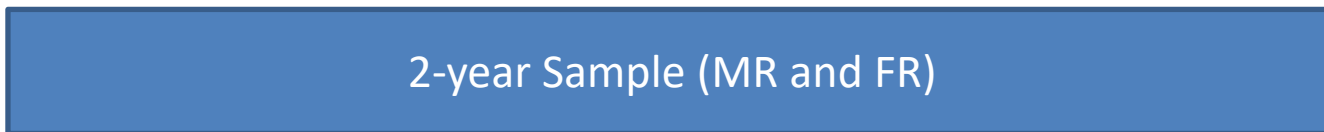
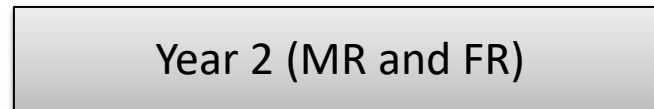
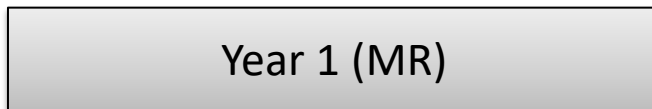


TASK MR-A

TASK MR-B

TASK FR-A

TASK MR-C

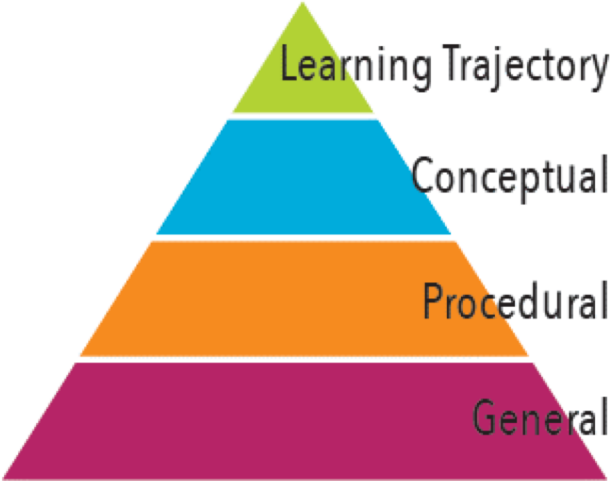
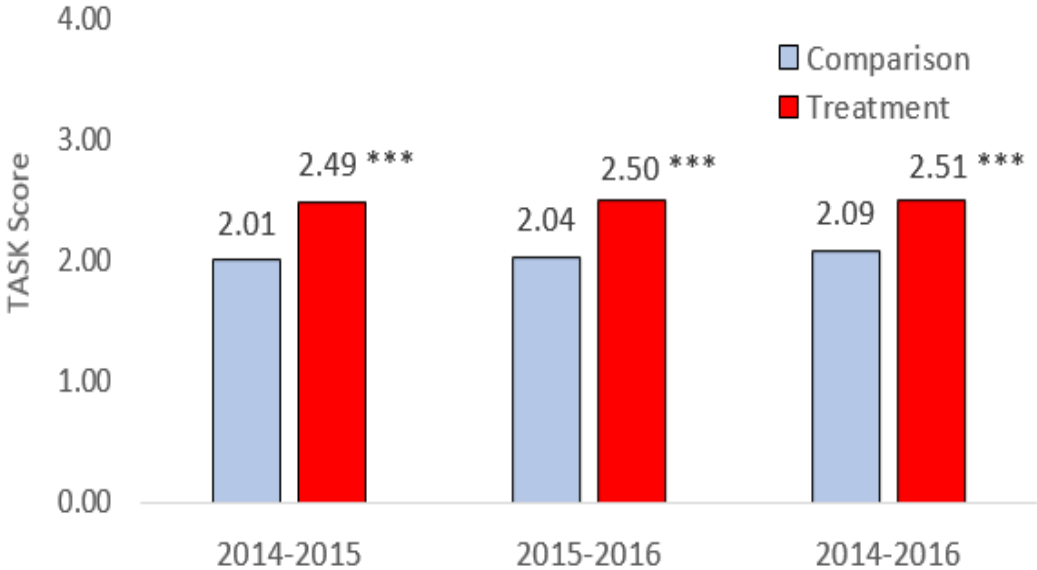


# Results: Teacher Knowledge

	Year 1 (n=435)	Year 2 (n=291)	2 yr Sample (n=246)
Constant	1.07*** (.195)	1.114*** (.258)	1.284*** (.322)
Task Baseline	.472*** (.062)	.453*** (.049)	.424*** (.065)
Treatment (OGAP)	.338*** (.046)	.192** (.066)	.318*** (.083)
Multiplicative Reasoning		.183*** (.050)	.194*** (0.54)
Experience	.0001 (.004)	-.007+ (.004)	-.005 (.005)
ELL Teacher	-.004 (.062)	.029 (.111)	.105 (.155)
SPED Teacher	.021 (.063)	-.192** (.064)	-.1 (.120)
School size	-.001 (.008)	-.010 (.016)	-.008 (.017)
% Free/reduced lunch	.086 (.173)	.263 (.403)	.046 (.425)

\*\* p<.01, \*\*\* p<.001

Figure 1. OGAP Impacts on Teacher Knowledge  
Teacher Knowledge of Student Thinking (TASK)



# Learning Trajectory Assessment in Multiplicative Reasoning (LTA-MR)

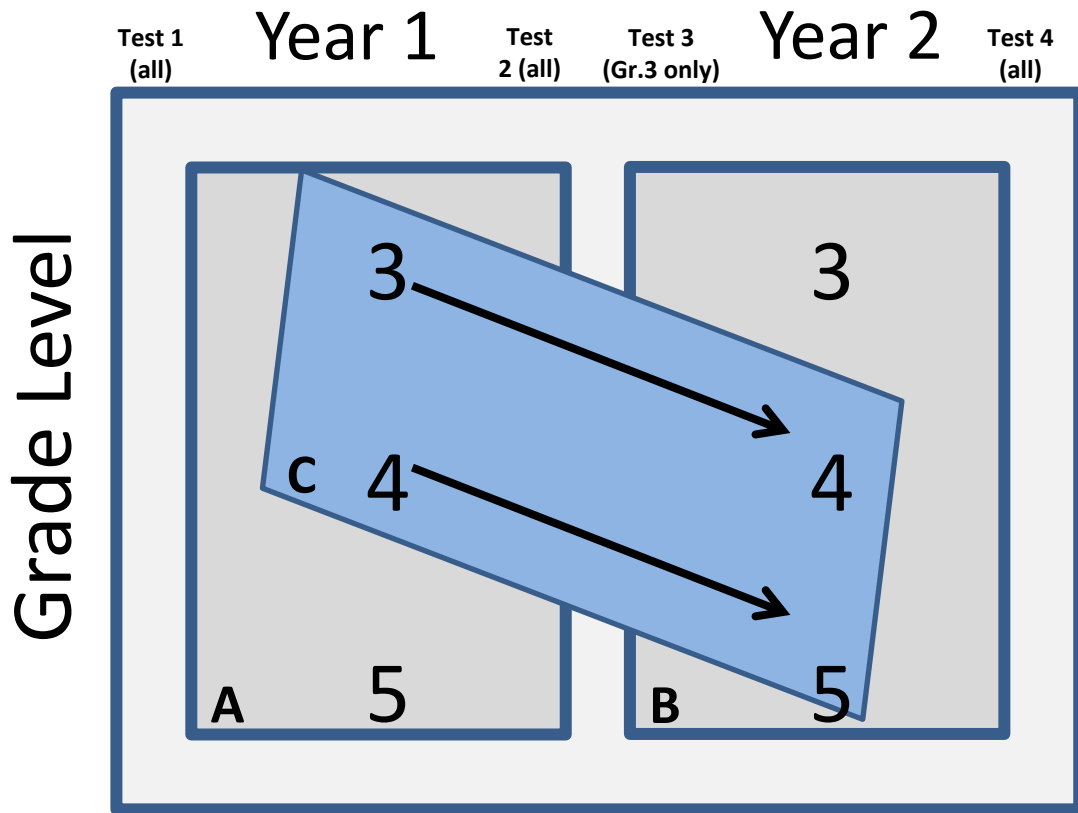
- 7 questions on each  
(Grades 3 - 5)
- Reflects CCSSM expectations  
problems structures and  
number complexity
- Vertically equated  
some overlapping items

*A piece of elastic that is 15 inches long can be stretched to 450 inches.*

*How many times its original length has the elastic been stretched?*

*Show your work.*





# Results: Student Learning

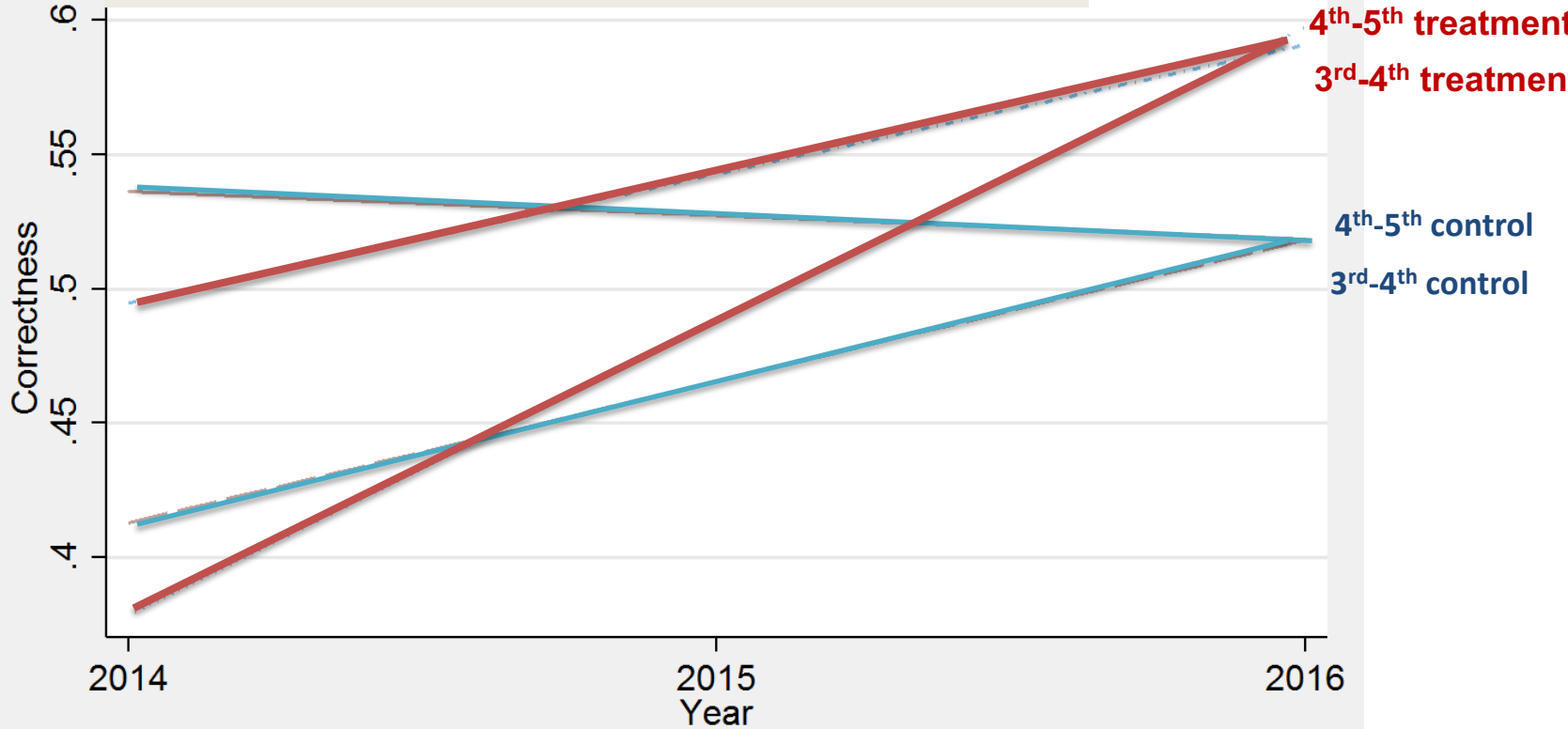
## Problem Solving Accuracy

## Sophistication

Variable	Year 1 (n=9,099)	Year 2 (n=7,162)	Two Year (n=4,508)	Year 1 (n=9,099)	Year 2 (n=7,162)	Two Year (n=4,508)
Constant	.780*** (.096)	.608*** (.067)	.755*** (.079)	2.44*** (.191)	2.031*** (.206)	3.000*** (.267)
Pre-Test	.366*** (.013)	.443*** (.024)	.329*** (.016)	.462*** (.017)	.579*** (.027)	.472*** (.025)
Treatment (OGAP)	.110*** (.032)	.043* (.022)	.095*** (.029)	.245*** (.068)	.131* (.065)	.259** (.099)
Free/Reduced Lunch	-.036** (.012)	-.041*** (.010)	-.053*** (.019)	-.122*** (.030)	-.048+ (.031)	-.108* (.046)
Students with Disability	-.194*** (.013)	-.124*** (.017)	-.174*** (.022)	-.584*** (.036)	-.389*** (.055)	-.588*** (.079)
English language learners	-.126*** (.019)	-.091*** (.013)	-.093*** (.018)	-.327*** (.045)	-.212*** (.039)	-.311*** (.065)

\* P < .05 \*\* p<.01, \*\*\* p<.001

# Problem Solving Accuracy Gains 2014-2016



Ongoing Assessment Project

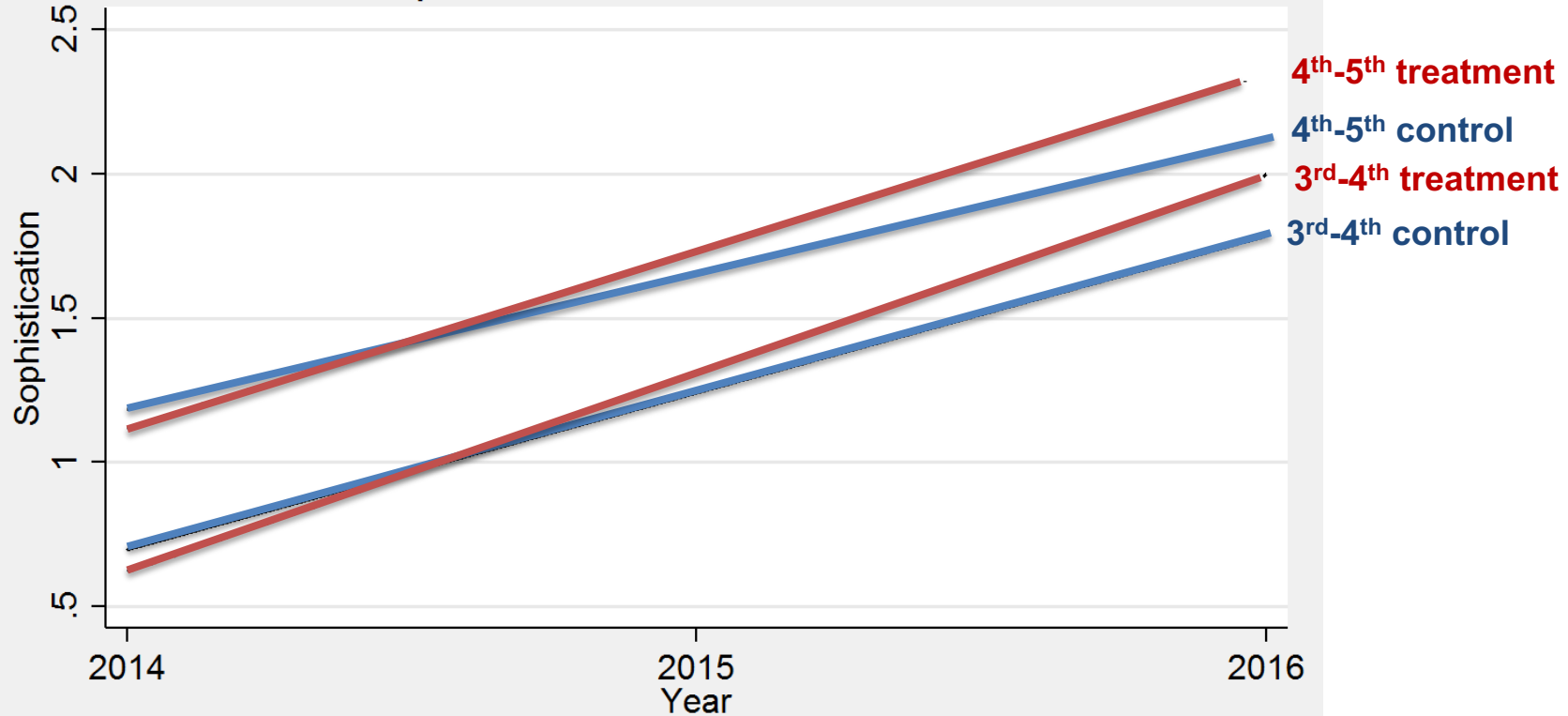


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## Sophistication Gains, 2014-2016



Ongoing Assessment Project

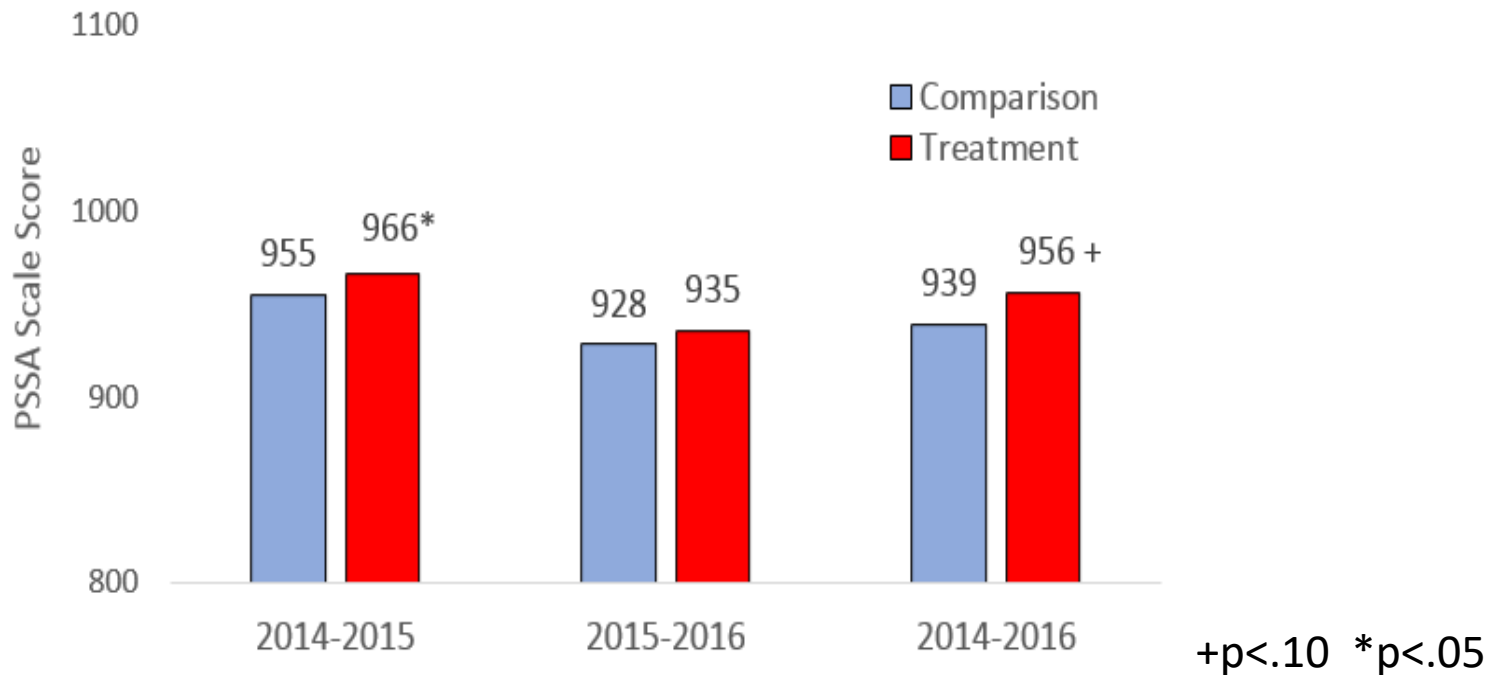


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# Impact on State Test Scores



# Summary of Findings

- Impacts on both teacher and student learning
- Strongest in first year, but persisted through 2 year,
- OGAP resulted in increases in both accuracy and sophistication of strategy
- Multiple challenges to implementation

# Implications

- Evidence for a model that can help teachers gain access to knowledge from research
- LTA-MR provides evidence of performance that includes both accuracy and sophistication

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# Phase 4

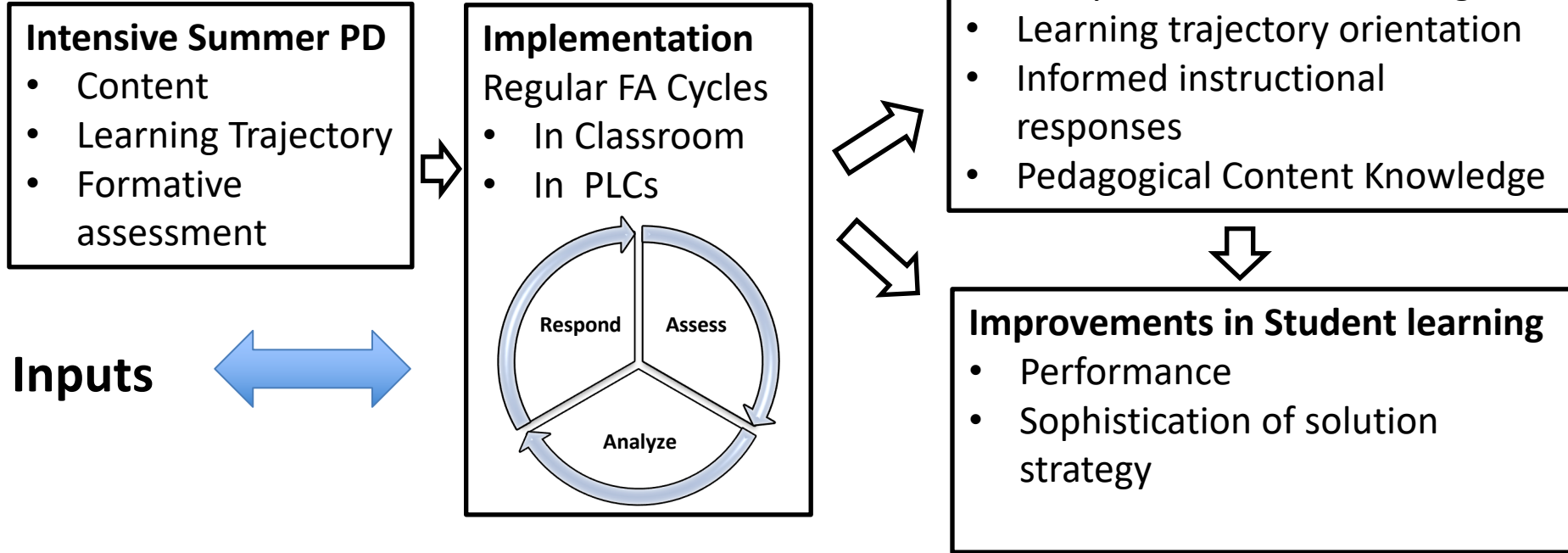
## A Design-Based Implementation Project: Philadelphia Mathematics Leadership Collaborative (PMLC)

PMLC Team at CPRE

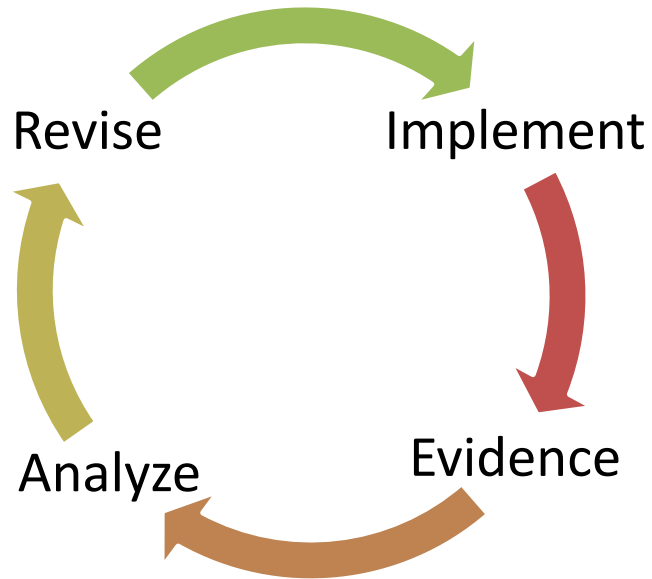
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# Theory of Action



# Design-Based Implementation Research



- Form of *design research* that supports productive adaptation of programs as they go to scale
- Focus on the system in context
- Systematic inquiry on process
- Collaboration across boundaries

(Penuel, Fishman, Cheng, Sabelli, 2011)

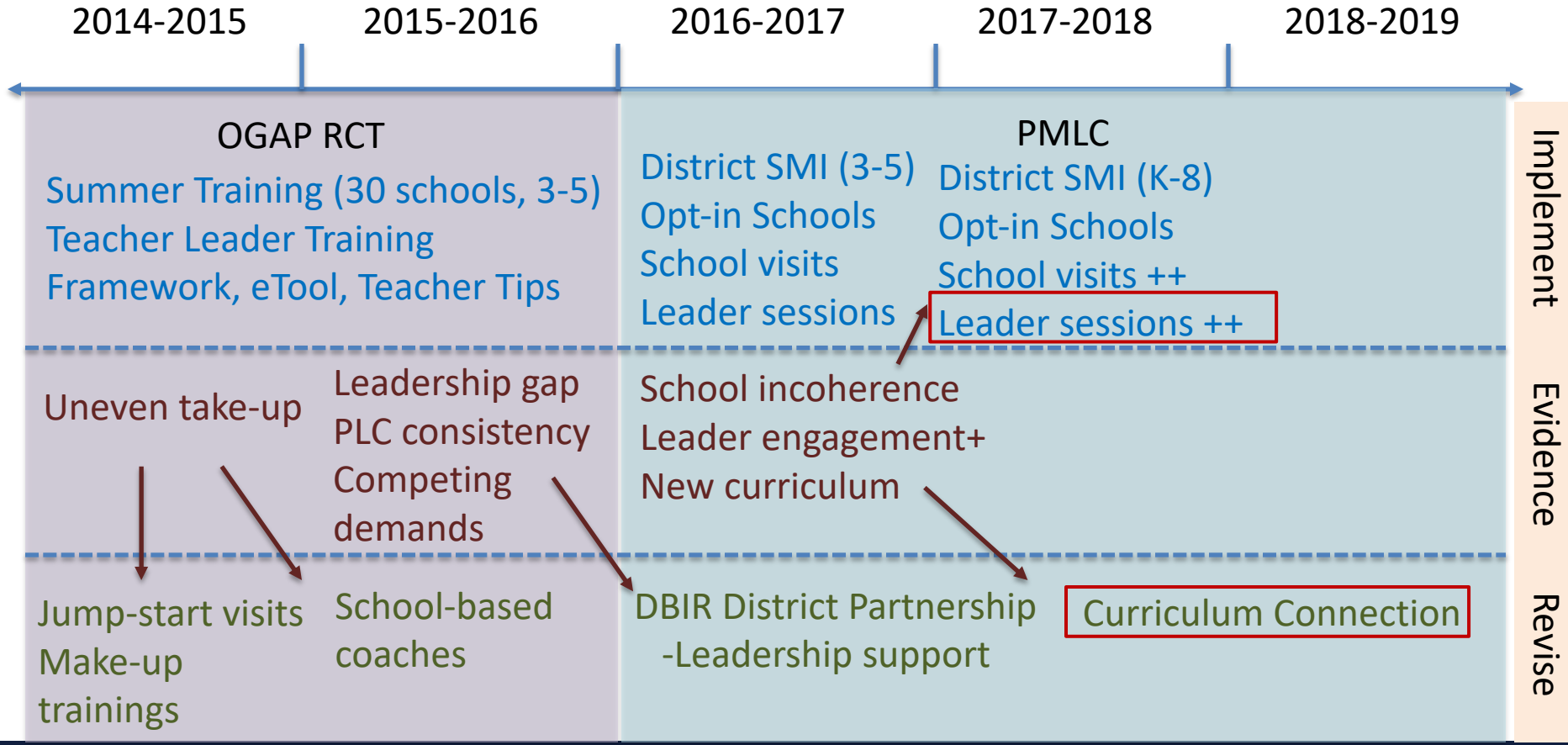
# “Scaling up” as a Site for Research

Phase 1: Can we design tools and routines to support teachers to use research to inform their practice?

Phase 2: Can these tools gain traction in more settings?

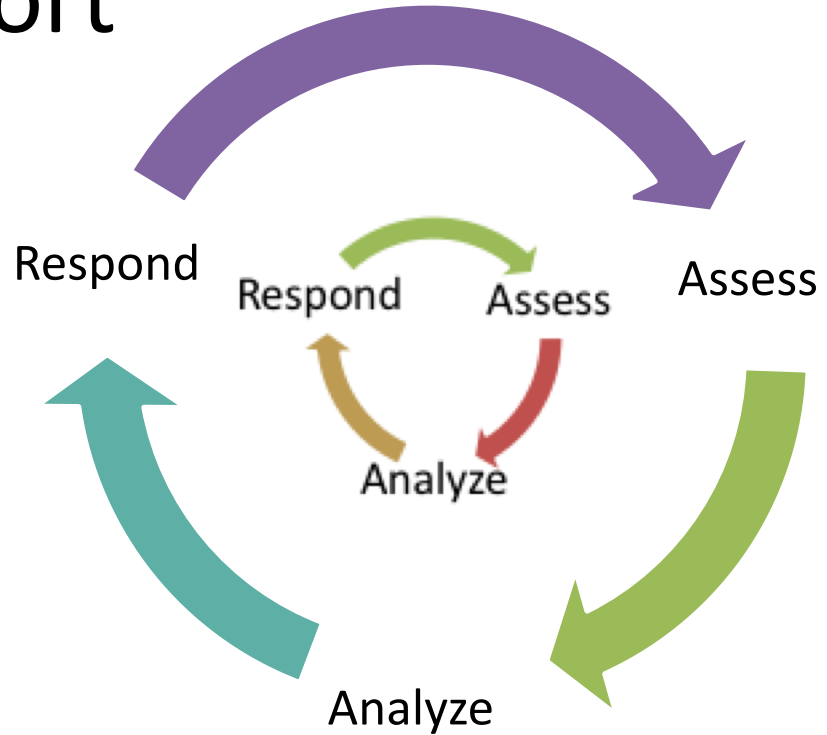
Phase 3: Does using these tools and routines support teacher and student learning?

Phase 4: What types of supports enhance the implementation of OGAP in a large urban system?



# Leadership Support

- Taking a formative approach to supporting teachers
- Understanding components of OGAP
- Understanding teacher development/change
- Strategies to enact formative approach to leadership



# Curriculum Connection



The screenshot shows the 'enVisionmath 2.0' interface. At the top, it says 'OGAP connections'. Below that, the page is titled 'Potential Response Strategy'. On the left, there are filters for 'Grade 3 Lesson 4-5' and 'Your Sort' (A, ET, T, NM) and 'Our Sort' (A, M, ET, T, NM). A 'Download Sample Work' button is at the bottom left. The main content area is titled 'Response Strategies' and lists: 'Do Nothing' (0 mins), 'Warm Up' (5-10 mins), 'Mini-Lesson' (15 mins), 'Re-teach' (45-60 mins), and 'Stations/Centers' (45-60 mins). The 'Warm Up' strategy is selected, showing a 'Student Work Comparison' of two student solutions for 'Lesson 1-6: Division of Fractions'. Below the work is a 'Trend Assessment' box with text: 'What is the evidence of developing understanding that can be built upon? All of the students are able to find a strategy that gets them to the correct answer! Most students are modeling with an array. What are the issues or concerns evidenced in the student work? None of the students are using labels. This should be addressed instructionally with the whole group at some point.' A link 'Click here for further ENVISION CURRICULUM CONNECTIONS in this chapter.' and a 'SELECT THE NEXT ITEM' button are at the bottom.



- 1. Analyze:** Sort 5 examples of student work from Solve & Share
- 2. Analyze:** Examine sorting rationale
- 3. Respond:** Review instructional response possibilities
- 4. Assess:** Select and download related OGAP items.

# Questions?

Break-Out Discussion Groups:

1. OGAP tools and routines (Marge)
2. Our measures (TASK and LTA-MR) (Caroline)
3. Studying implementation (Janine)

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# Contact Us

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- cbe@upenn.edu
- janiner@upenn.edu

