Revealing Base Ten Understanding Through Written Formative Assessment Tasks

Nicole Fletcher
CPRE,
University of Pennsylvania

Caroline Brayer Ebby
CPRE,
University of Pennsylvania

Beth Hulbert
OGAP Math

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Today

- Introduction
- Theoretical background on base-ten
- Modes of inquiry for the research
- Examining student work
- Instructional implications
- Discussion

Introduction

- Larger study:
  - Early-stage design and development study
  - Collaborative effort with the Ongoing Assessment Project (OGAP)
  - Development and piloting of formative assessment tools and routines for addition, subtraction, and number for grades K-3
- Current Inquiry:
  - Examine student work samples collected through piloting to illustrate what can be learned about children's base ten understanding
  - Discuss how this information can be used to inform instruction

Base Ten in Early Mathematics

Base ten and place value understanding is a central topic of early elementary mathematics

- Common Core State Standards of Mathematics
- NCTM's *Principles and Standards for School Mathematics*

Development of Children's Conception of Ten (Steffe)

Ten as a numerical composite

- Focus on the individual units that make up ten

Ten as an abstract composite unit

- Begin to view one ten as made up of ten ones
- Can often count by tens but do not increment by tens
- Counting by tens "signifies another ten that is experienced as being next to the ten just counted" rather than ten more (Cobb & Wheatley, 1988, p. 6)

Ten as an iterable unit

- Can compose and decompose numbers by incrementing or decrementing by tens and ones while maintaining a concept of the number as a single entity

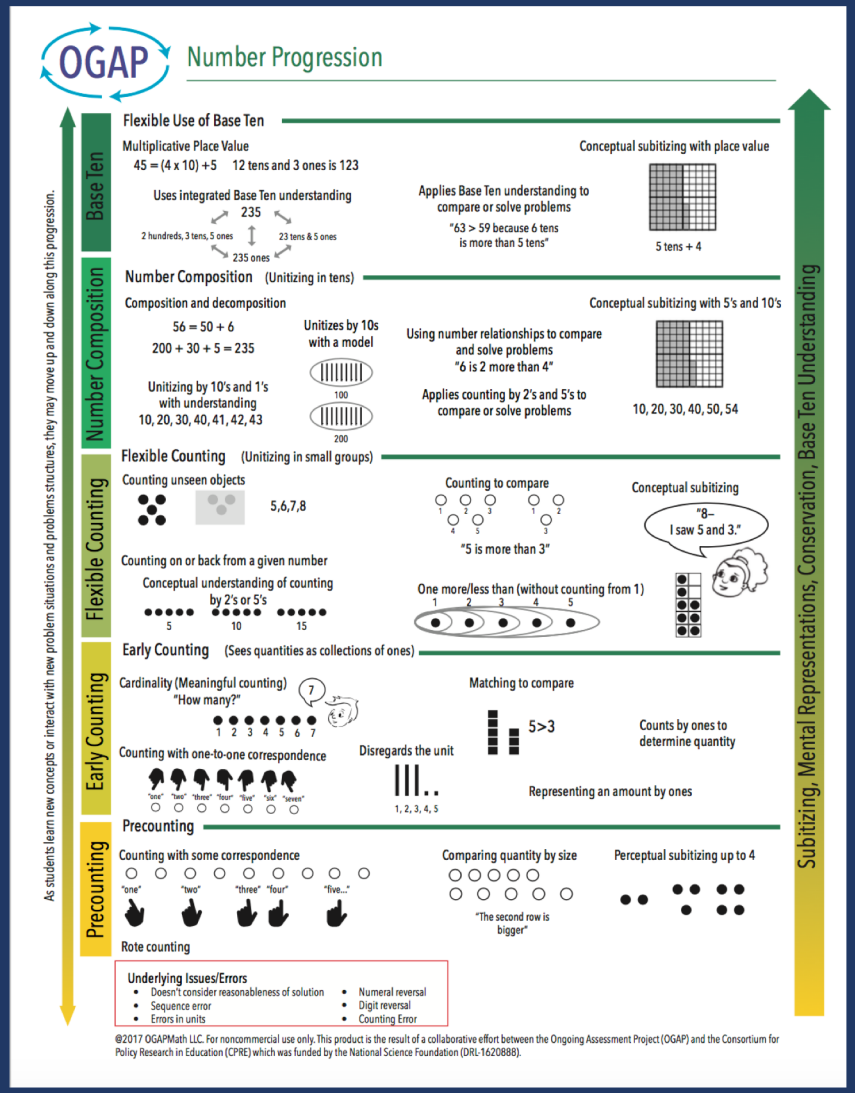
(in Cobb & Wheatley, 1988)

Levels of Sophistication in Student Reasoning about Place Value (Battista)

1. operating on numbers as collections of ones (without skip counting by place value)
2. operating on numbers by skip counting by place value
3. operating on numbers by combining and separating place value parts
4. understanding place value in expanded algorithms
5. understanding place value in traditional algorithms
6. generalizing place value understanding to larger numbers, numbers less than 1, and exponential notation

(Battista, 2012)

OGAP Number Progression



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Modes of Inquiry for the Research

For the larger project:

- Written formative assessment items on addition, subtraction, and number were developed and piloted
- Items were piloted in 21 kindergarten through third grade classrooms in five public schools located in a large urban school district in the Mid-Atlantic region of the US during the 2016-2017 school year
- 13 of the piloted items targeted base ten concepts and strategies
- Student work collected during piloting was sorted according to strategies illustrated by the number progression

Modes of Inquiry for the Research

For the current inquiry:

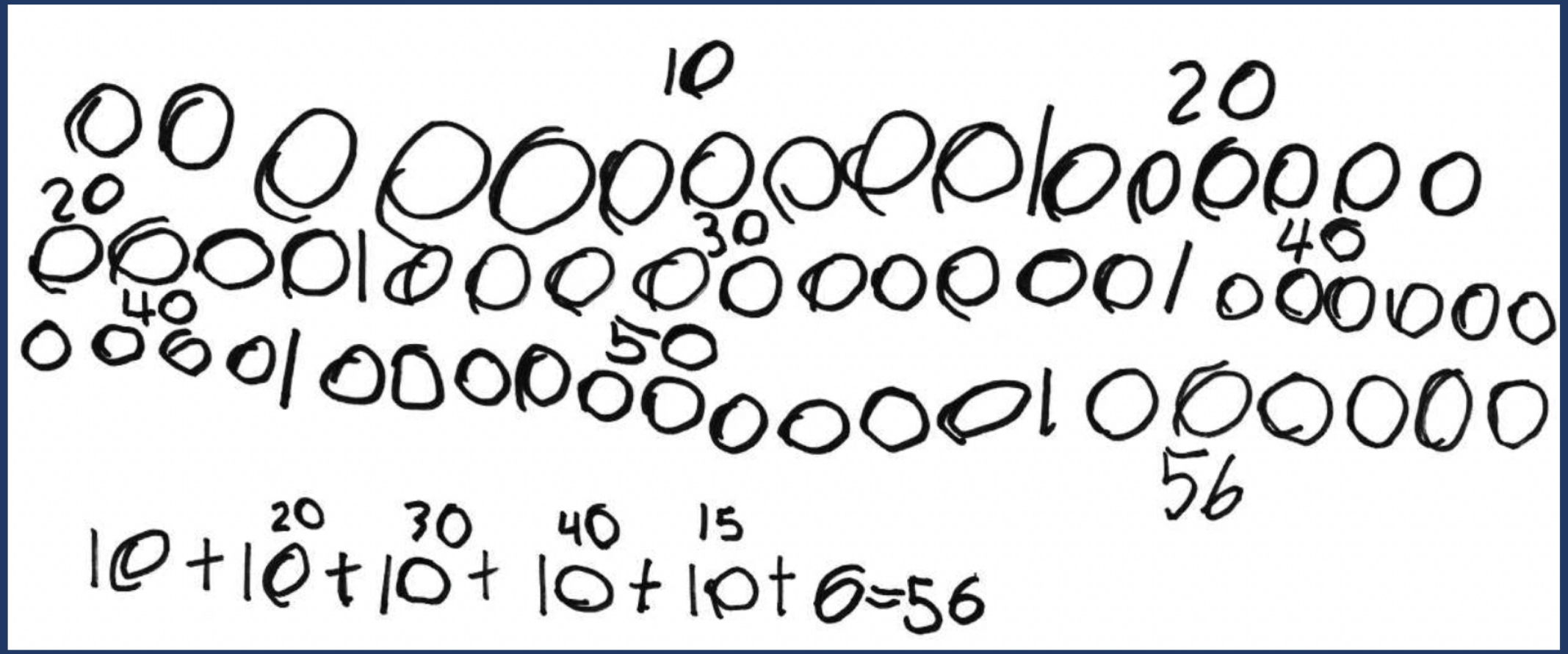
- One problem, piloted in second and third grade, that successfully elicited a range of student strategies in relation to base ten understanding was selected for this analysis
- Student work examples representative of each level of the number progression were selected and analyzed

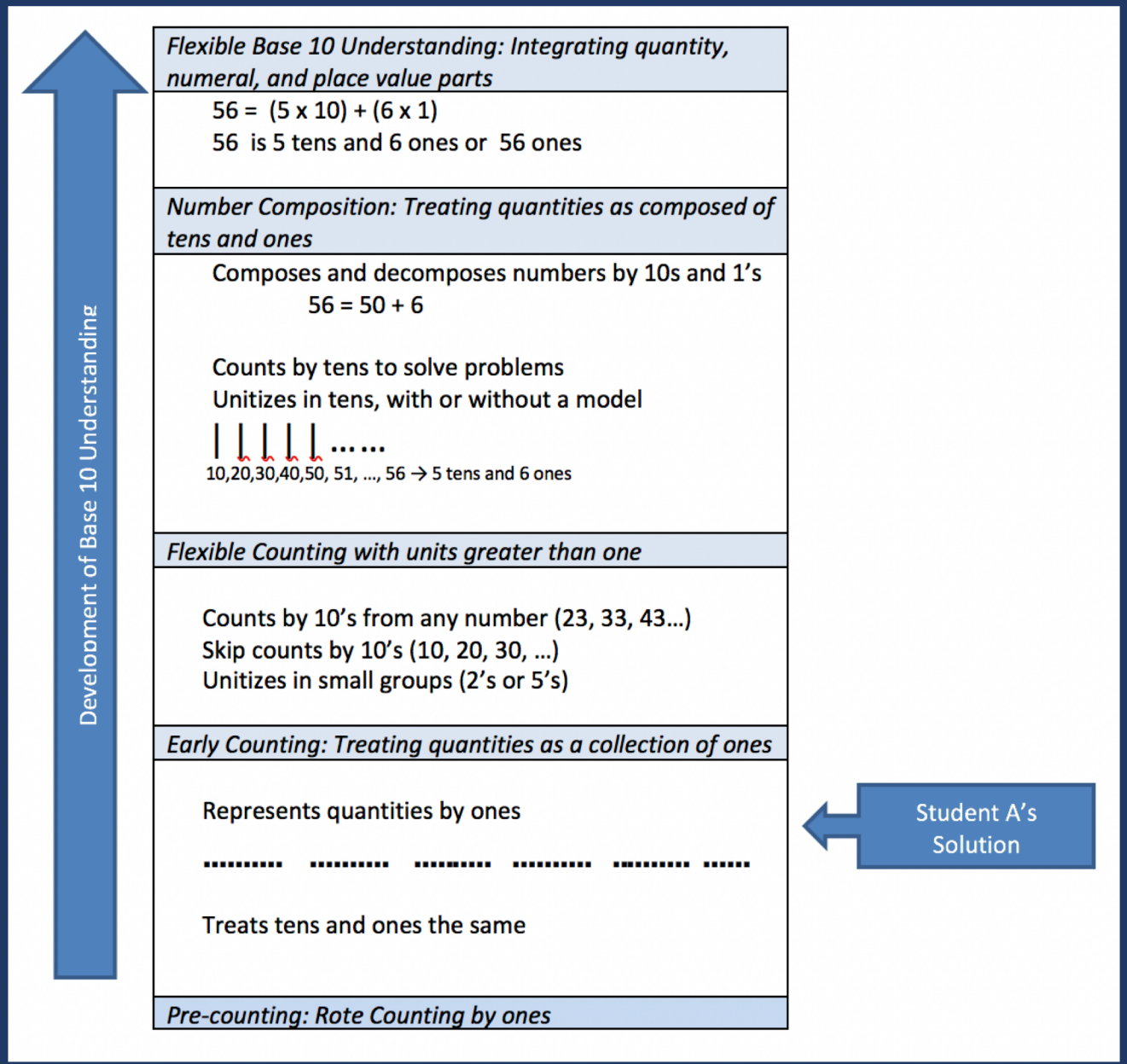
Research Questions

1. What understandings are revealed by students' responses to written formative assessment items designed to elicit base ten reasoning?
2. How can this information be used to inform instructional responses that move students towards increasingly sophisticated base ten understanding?

Shonda bought packs of bracelets to give to the kids in her grade. Each pack has 10 bracelets in it. There are 56 kids in Shonda's grade. How many packs of bracelets did she need to buy?

Student A's Solution




Development of Base 10 Understanding
Flexible Base 10 Understanding: Integrating quantity, numeral, and place value parts
56 = (5 x 10) + (6 x 1)
56 is 5 tens and 6 ones or 56 ones
Number Composition: Treating quantities as composed of tens and ones
Composes and decomposes numbers by 10s and 1's
56 = 50 + 6
Counts by tens to solve problems
Unitizes in tens, with or without a model
10,20,30,40,50, 51, ..., 56 → 5 tens and 6 ones
Flexible Counting with units greater than one
Counts by 10's from any number (23, 33, 43...)
Skip counts by 10's (10, 20, 30, ...)
Unitizes in small groups (2's or 5's)
Early Counting: Treating quantities as a collection of ones
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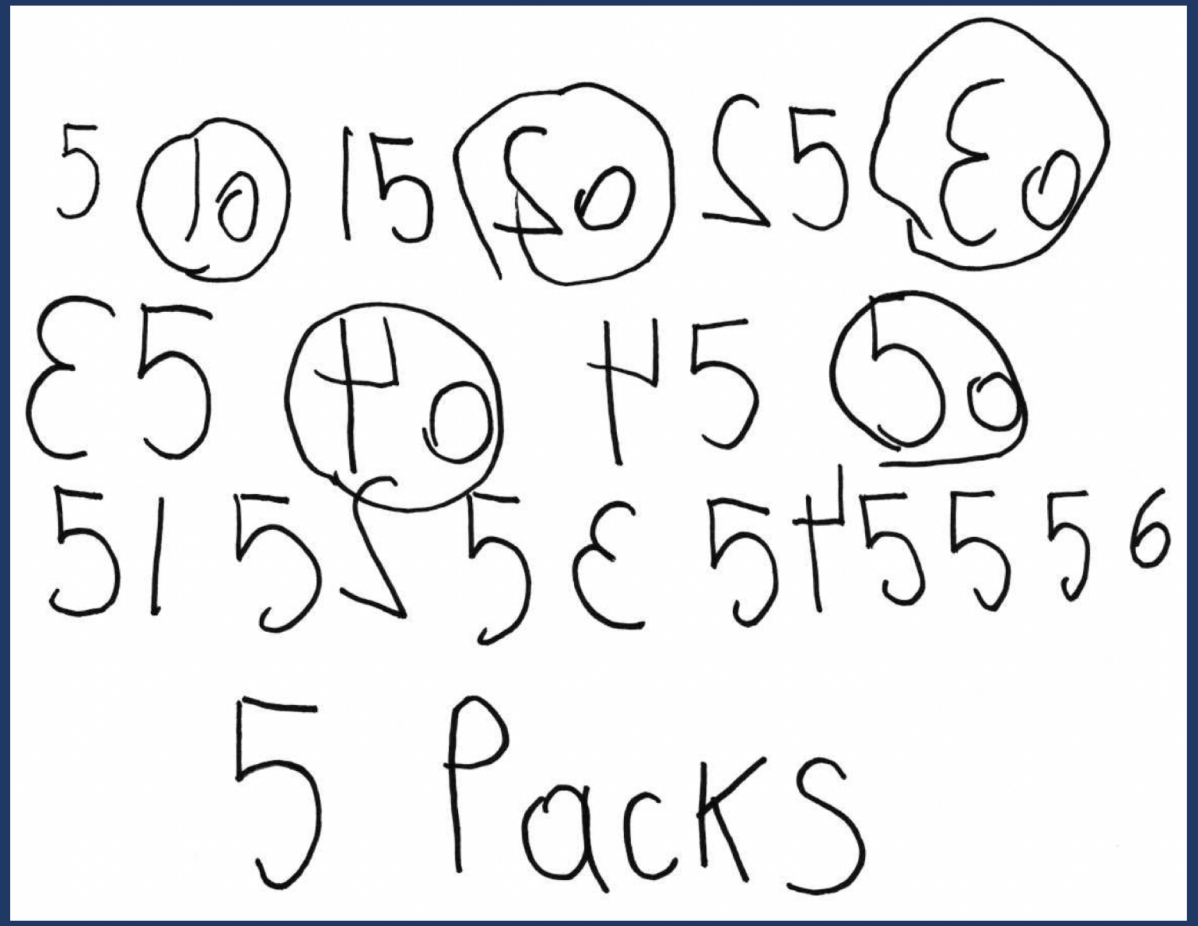
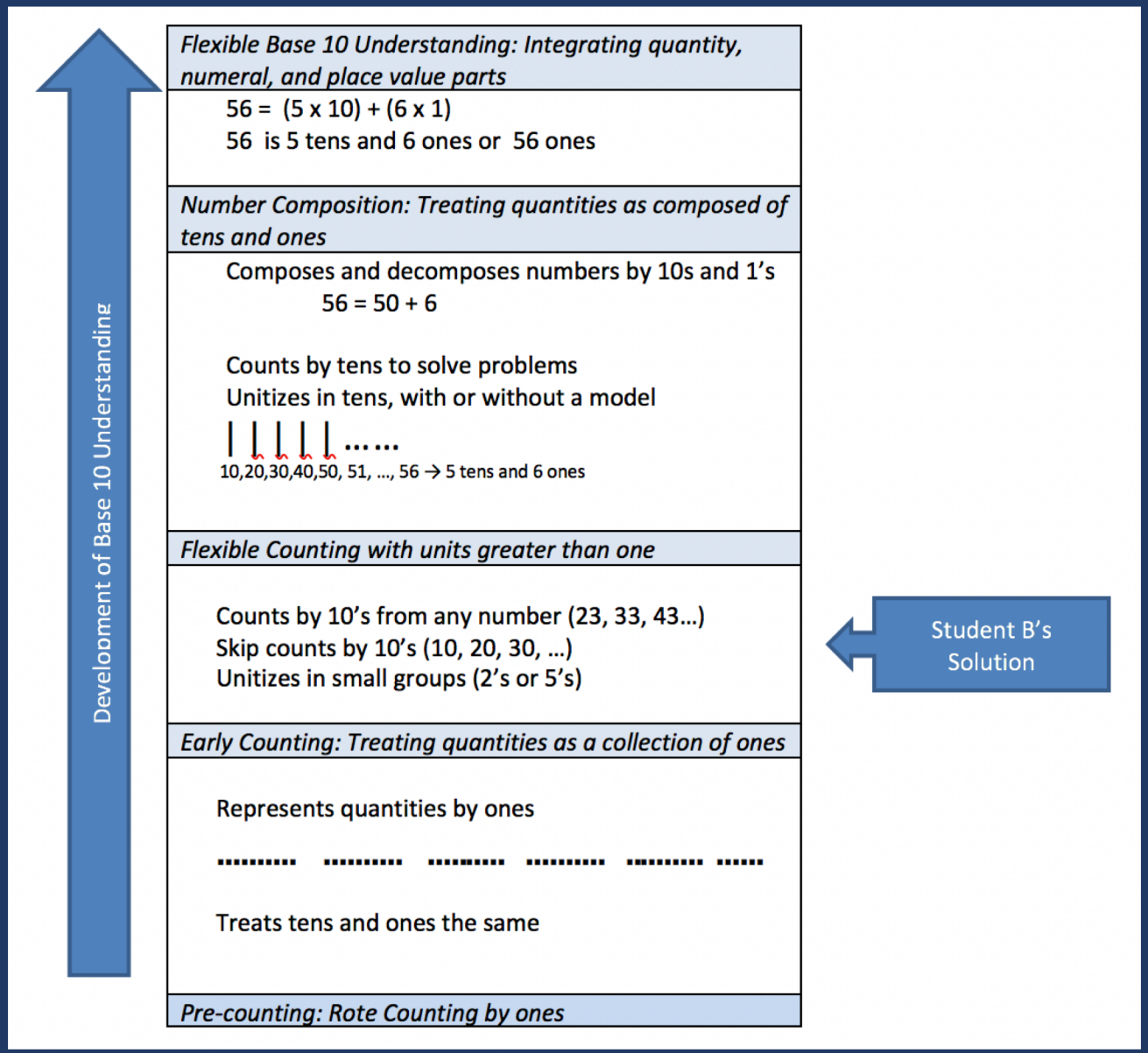
Pre-counting: Rote Counting by ones
Student A's Solution

Student B's Solution

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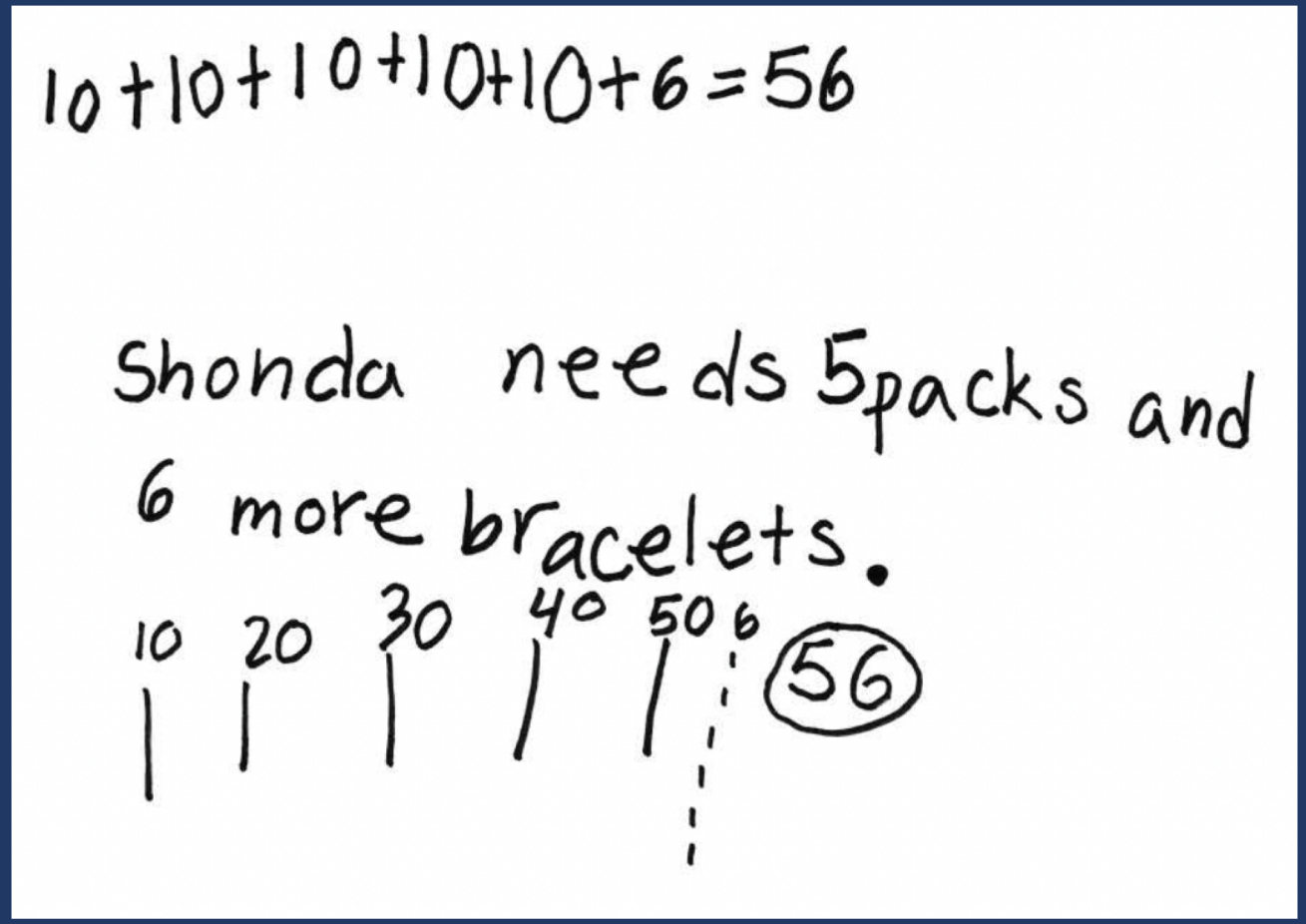
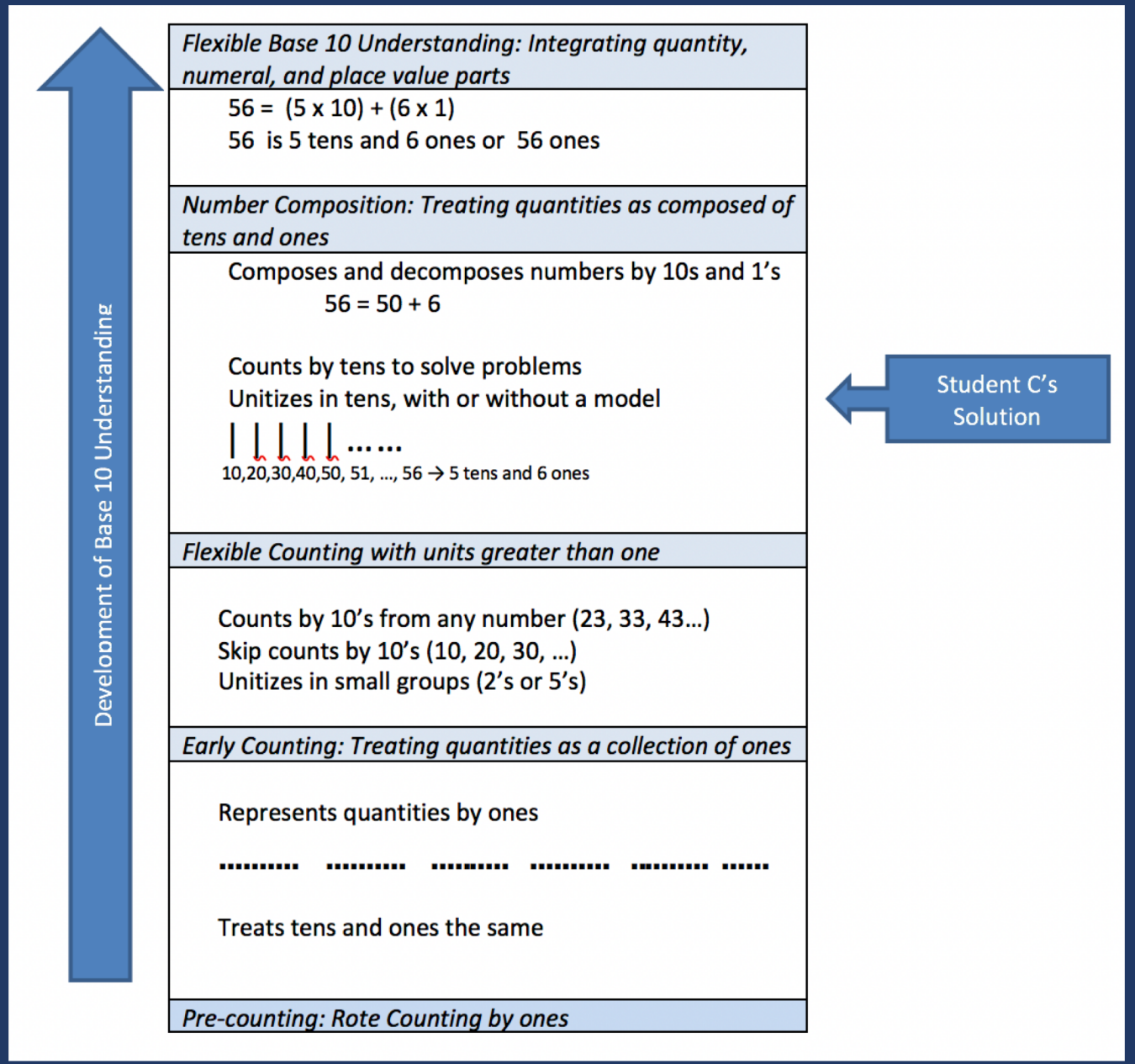
Student C's Solution

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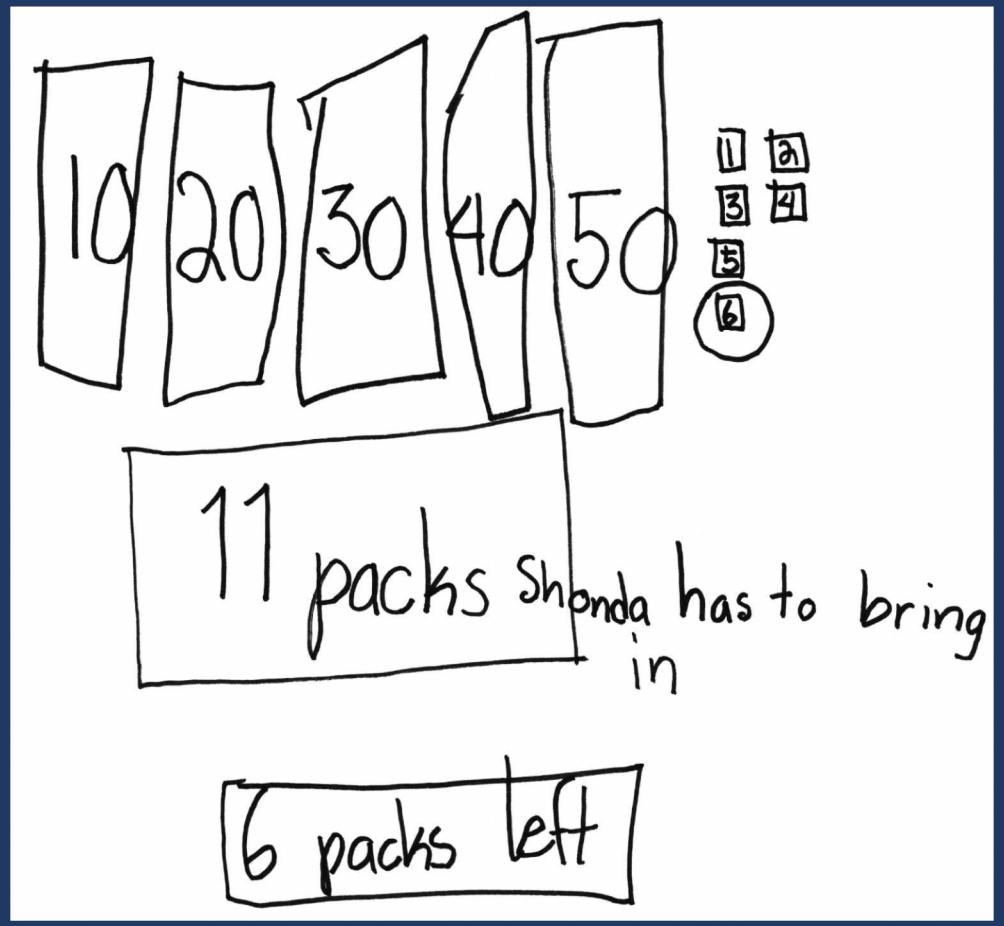

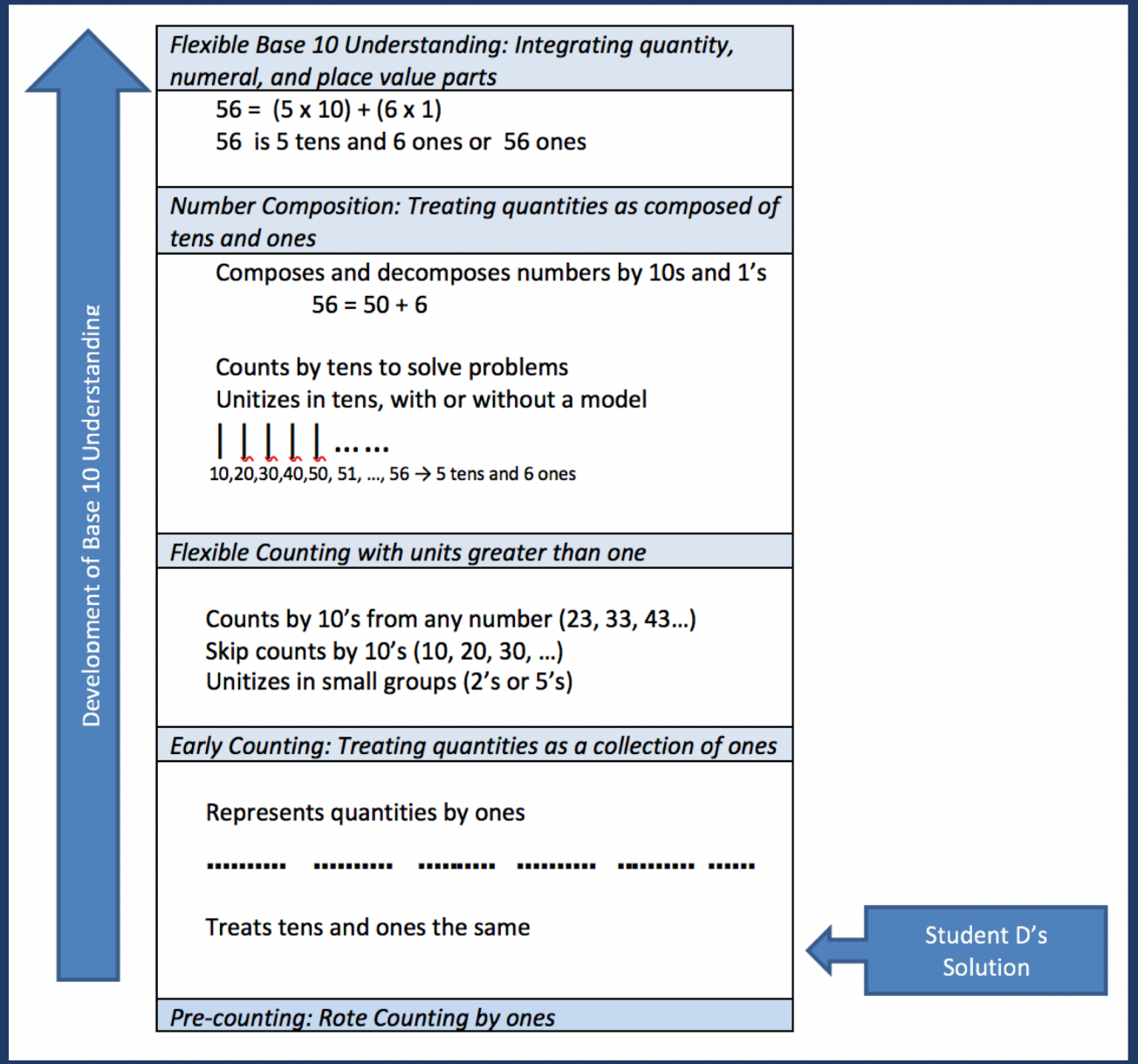
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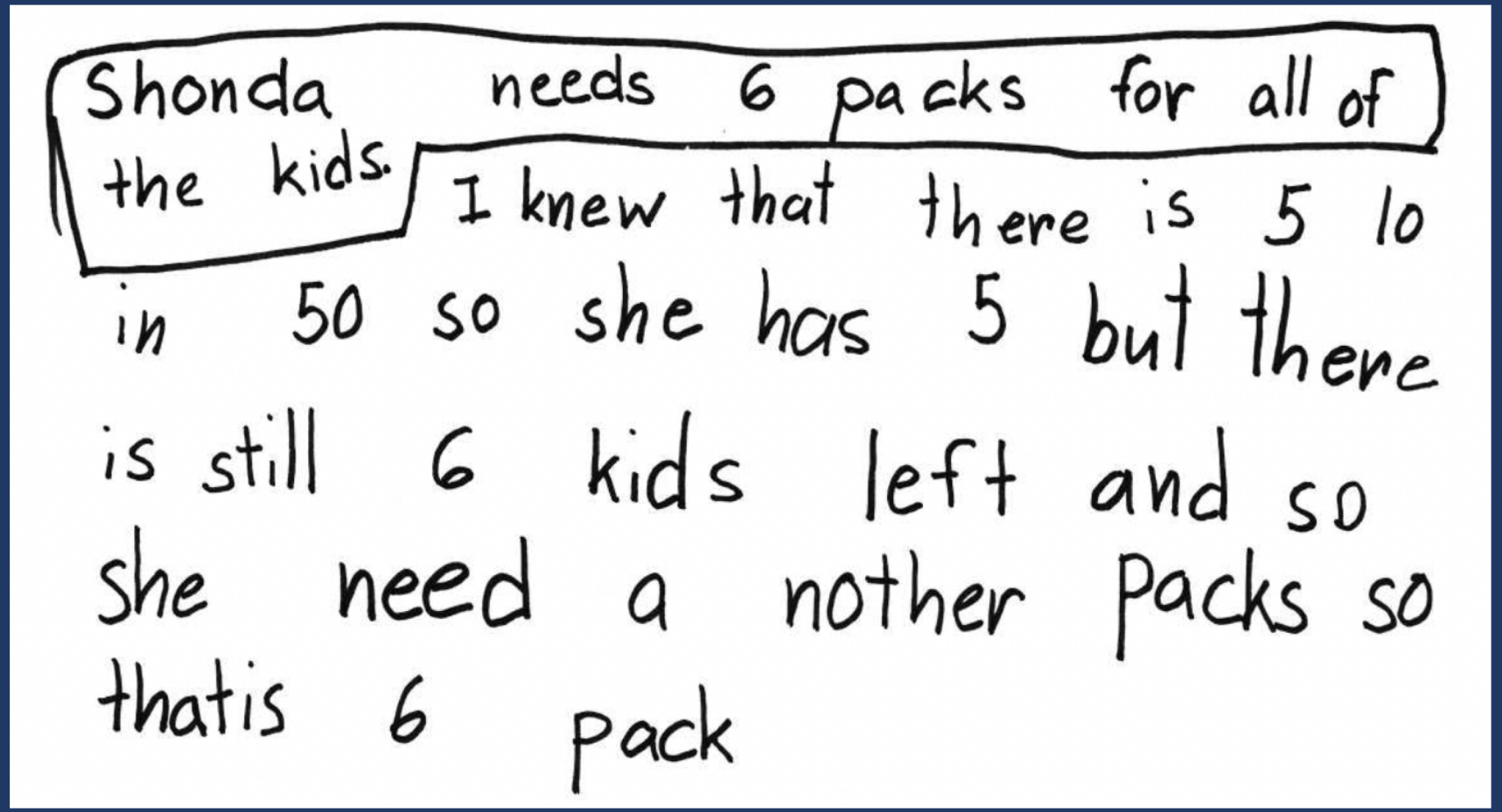
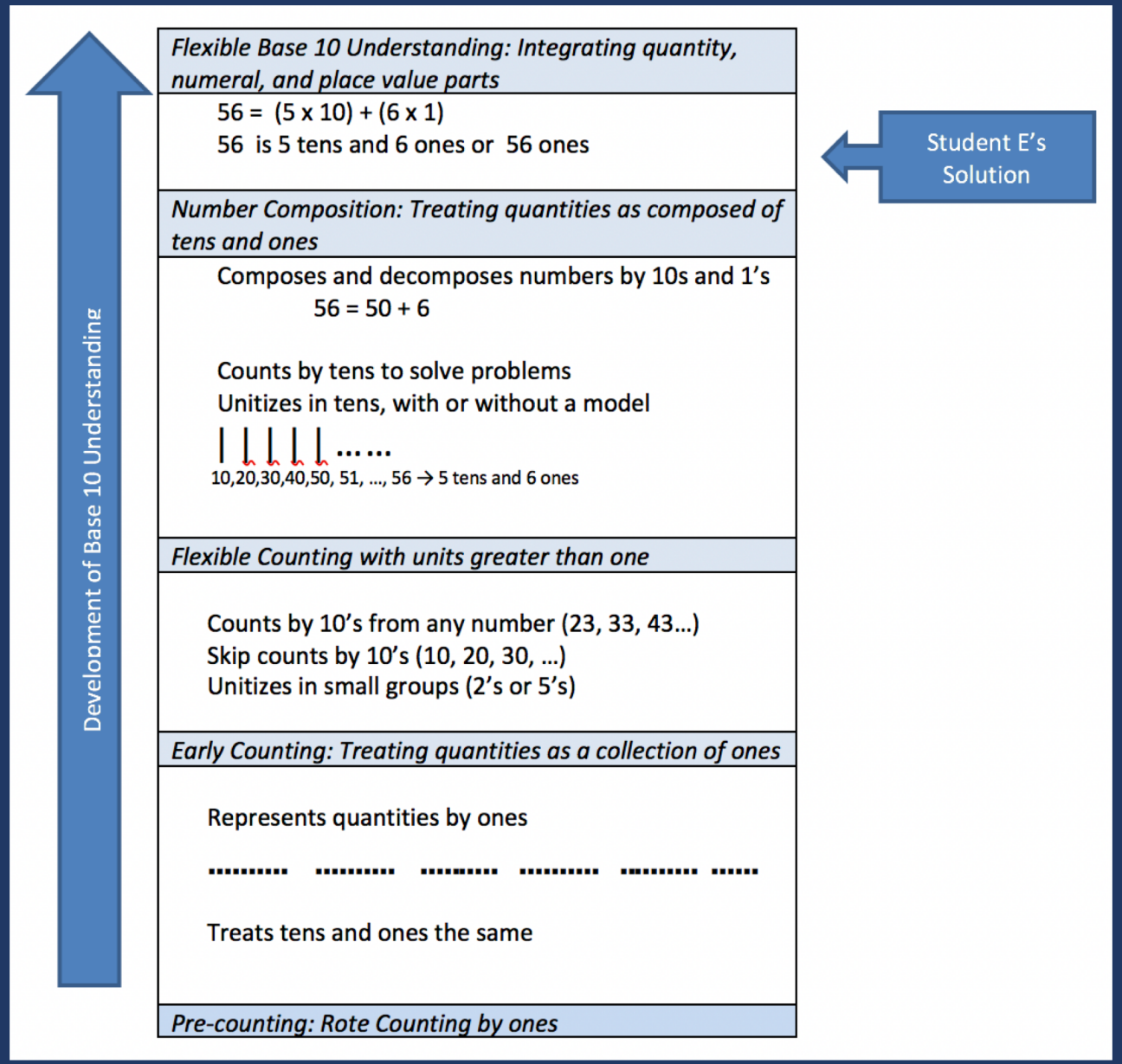
Student E's Solution

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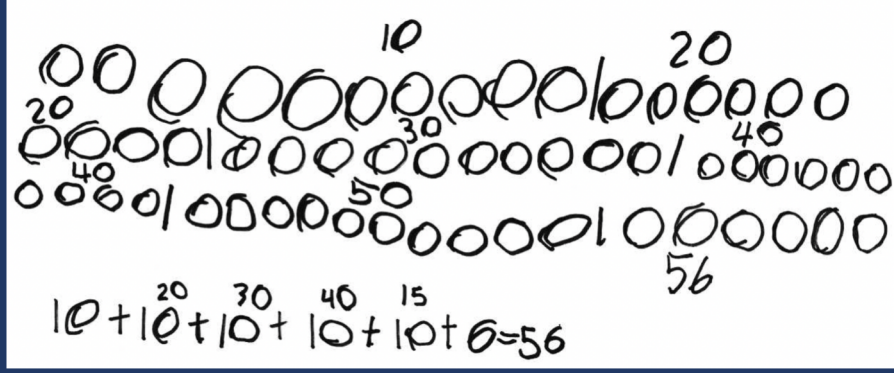
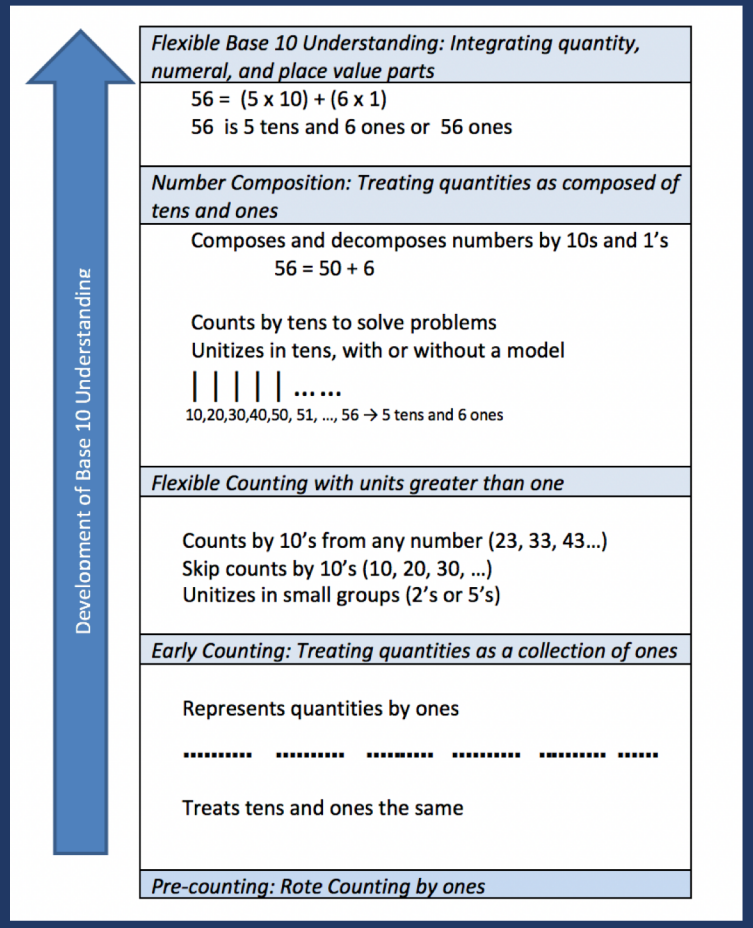
Instructional Implications

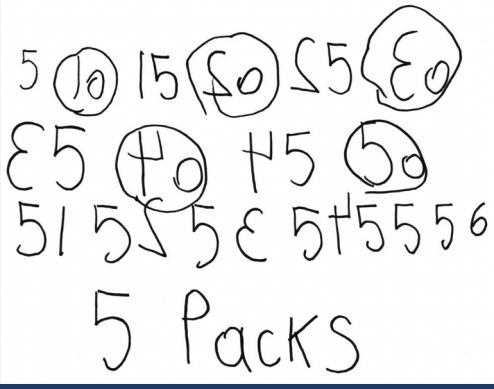


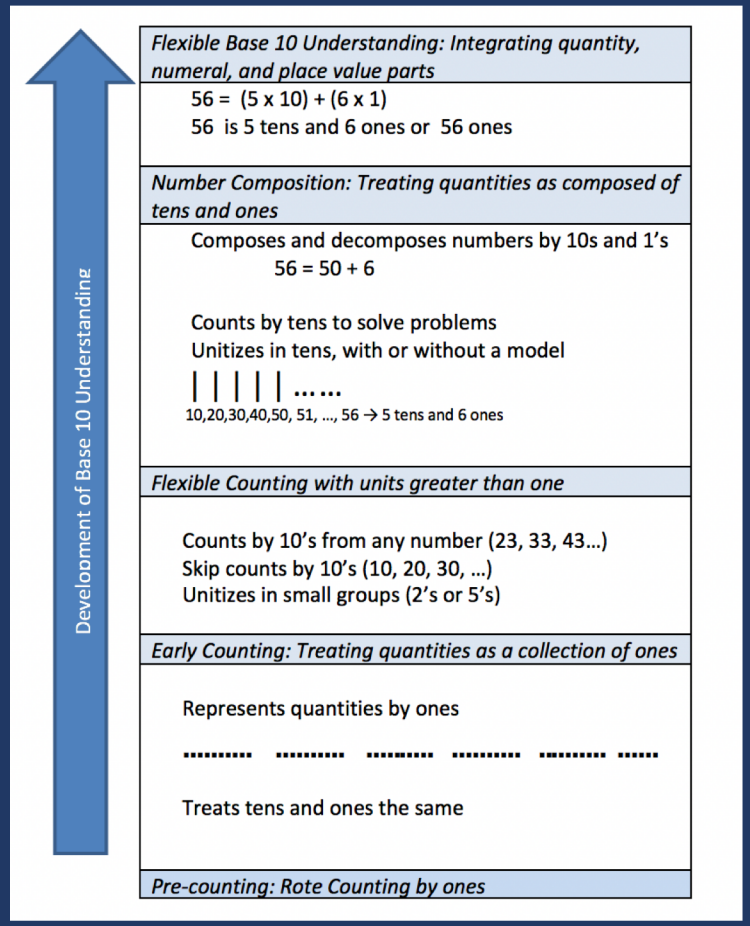


Student A

- Can represent a number by 1s, make groups of 10, and skip count by 10s (early counting)
- Would benefit from making groups of ten with models that show both discrete units and groups of tens so she can construct the tens herself, see both the ones and the *ten-ness* of each group, and connect the number of tens to skip counting by ten

Instructional Implications

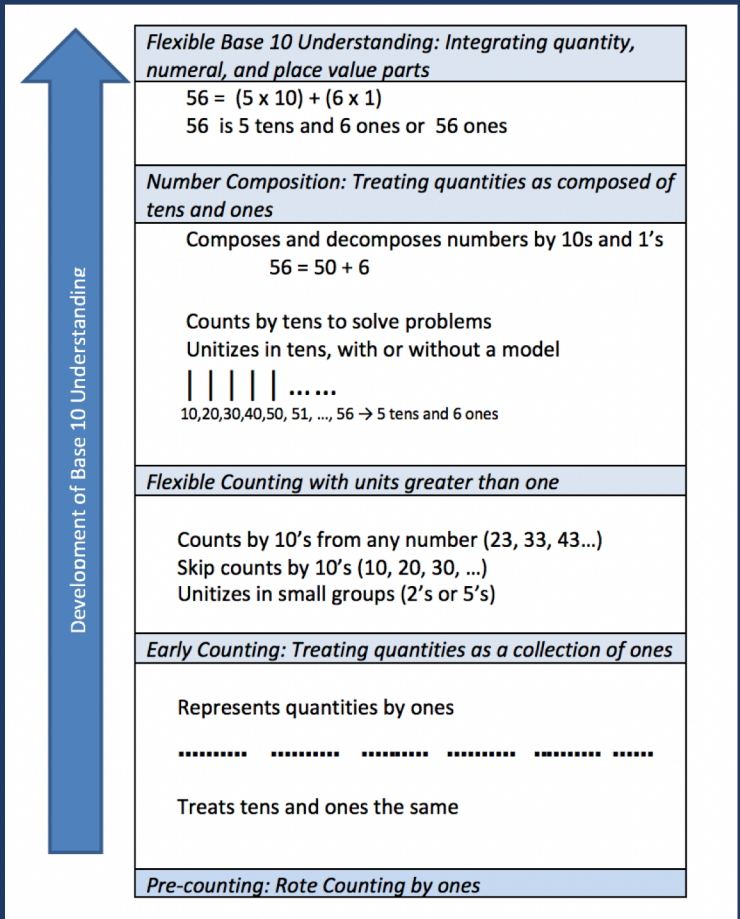


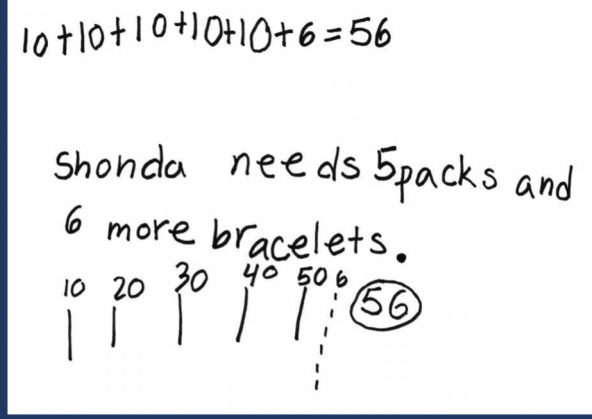


Student B

- Can count by 5s and use this to count by 10s (flexible counting)
- Would benefit from working with concrete representations that support unitizing by tens, such as base-ten blocks

Instructional Implications





Student C

- Can use a visual model to unitize by tens (number composition)
- Would benefit from creating two-digit numbers with visual models, counting the number of tens, and exploring the connection between the two-digit numbers and the number of tens they contain
- May benefit from instruction in recognizing that the ones in a two-digit number are part of the next ten

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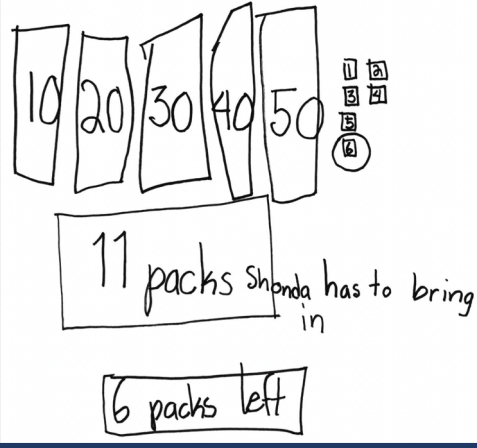
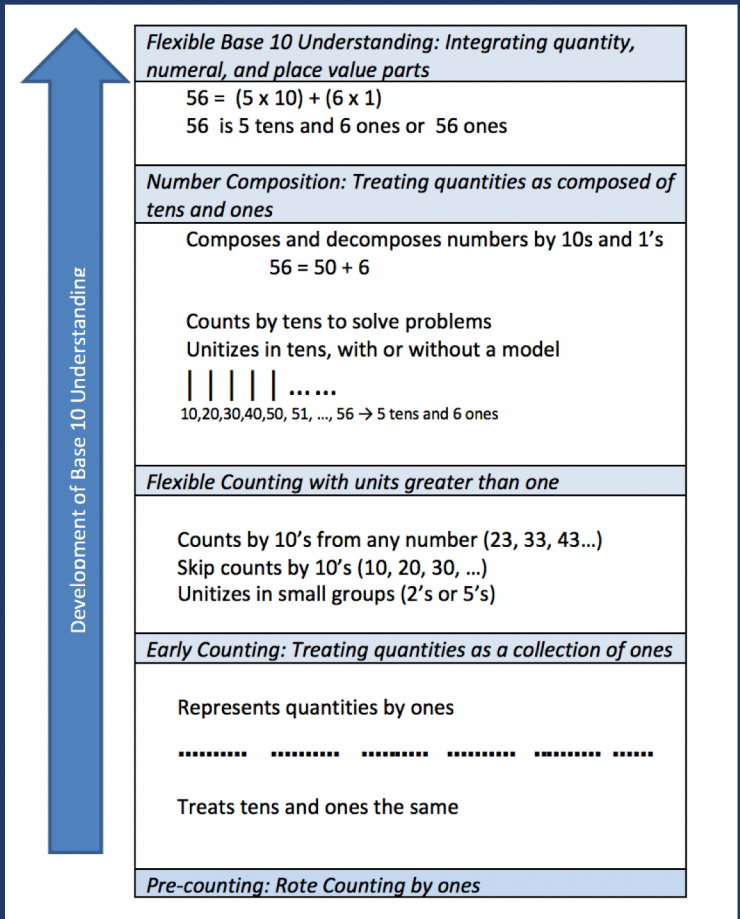
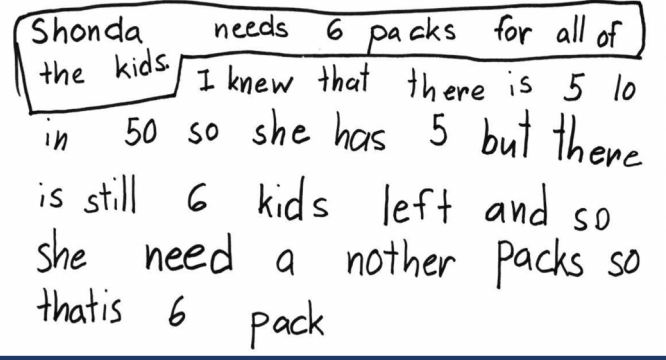


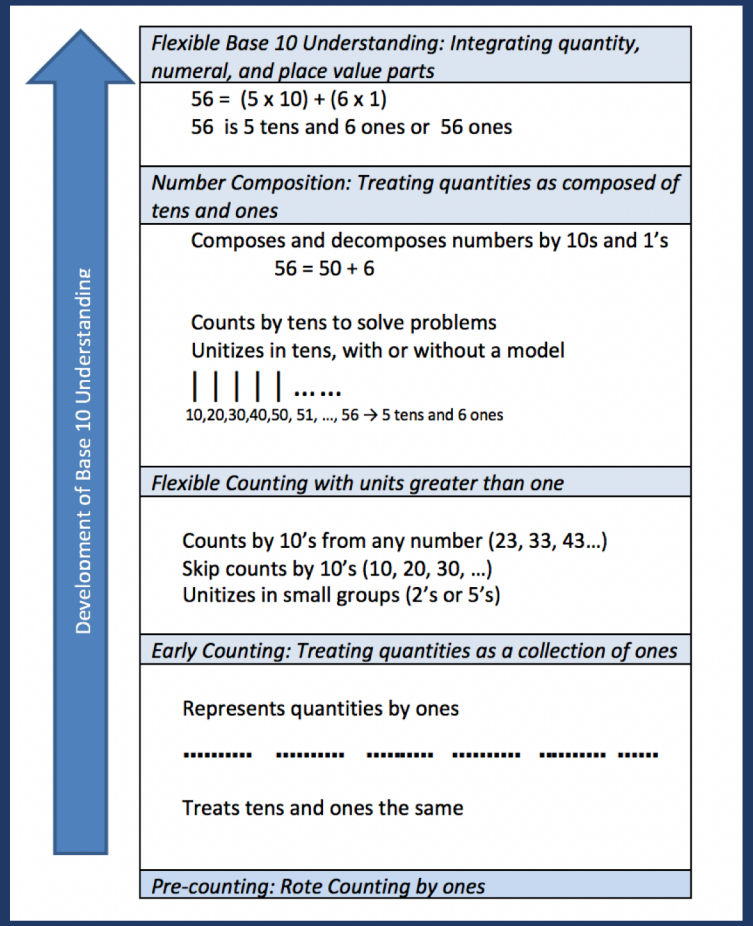


Student D

- Can draw a base-ten block representation of a 2 digit number
- Treats all blocks as having the same value (early counting)
- Would benefit from practice creating and counting tens with visual models such as unifix cube towers or ten frames that show both ones and tens to move towards seeing tens as both "one ten" and "ten ones"

Instructional Implications





Student E

- Can use base-ten understanding and reasoning skills to determine the answer without drawing a model

  (flexible base-ten understanding)
- Would benefit from working with bigger numbers, writing equations to represent the decomposition of quantities, and moving towards a multiplicative understanding of place value

Discussion

- Moving beyond routine base-ten tasks helps to reveal a range of levels of understanding
- Incorporating transitional strategies into instruction can help students move towards flexible use of base-ten strategies
- Administering formative assessment items and utilizing a research-based number progression to assess students' strategies can help teachers connect research to practice and differentiate instruction for all students

Future Research Directions

- Development and piloting of additional base-ten formative assessment tasks
- Further refinement of the number progression as additional student work is collected and analyzed
- Exploring ways to support teachers in implementing formative assessment and in using the number progression to analyze student work and make instructional decisions

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nsf2109@tc.columbia.edu
cbe@upenn.edu
beth@ogapmathllc.com
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