

# NCSM

What's the Good News: Division Understanding Can Be Developed Through  
the Analysis of Students' Preexisting Knowledge

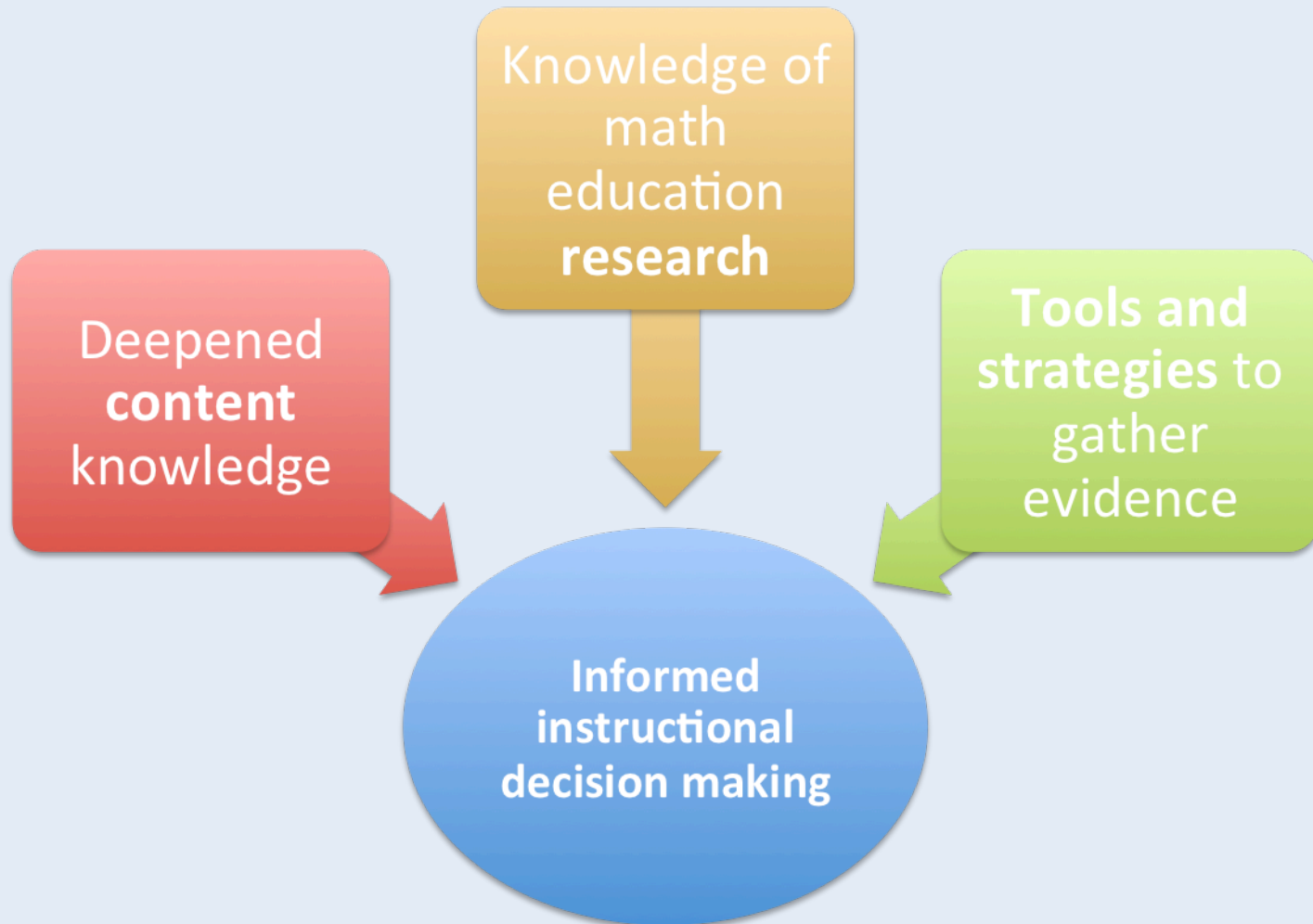
Karen Reinhardt  
[karrein65@gmail.com](mailto:karrein65@gmail.com)

Mary Abele-Austin  
[mabeleaustin@gmail.com](mailto:mabeleaustin@gmail.com)

# The Ongoing Assessment Project (OGAP)

- OGAP is an intentional, systematic, and ongoing **formative assessment** system designed to gather evidence as students learn specific mathematics concepts (Additive Reasoning, Multiplicative Reasoning, Fractions, Proportions).
- OGAP is grounded in the math education research on how students learn math concepts – **Learning Progressions**

# OGAP – Knowledge-based Decision Making...



# The **Essence** of *formative assessment* is...


- ...“the relentless attention to evidence of student thinking” (p. ix) and the systematic and intentional use of this information to inform instruction and student learning (Popham, 2012).
- To be considered formative assessment the evidence must be ‘elicited, interpreted, and used by teachers and learners’ (William 2011, 43).

# OGAP Cycle



# OGAP Multiplicative Reasoning Framework

**OGAP Multiplication Progression**



Multiplicative Strategies				
Algorithms	Distributive Property	Associative Property	Doubling & Halving	
<b>Partial Products</b> $\begin{array}{r} 16 \\ \times 42 \\ \hline 12 \\ 20 \\ 240 \\ \hline 672 \end{array}$	$4 \times 16 = 4(10 + 6)$ $= 4(10) + 4(6)$ $= 40 + 24$ $= 64$	$(8 \times 2) \times 5 = 8(2 \times 5)$ $= 8 \times 10$ $= 80$	$16 \times 4 = 8 \times 8$ $= 64$	
	<b>Known or Derived Fact</b> $4 \times 6 = 24$	<b>Commutative Property</b> $16 \times 4 = 4 \times 16$	<b>Powers of Ten</b> $5 \times 400 = 5 \times 4 \times 10 \times 10$	

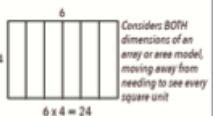
  

**Transitional Strategies**

**Open Area Model** *Considers both dimensions of an array or area model*

$\begin{array}{r} 30 \\ + \\ 20 \\ \hline 50 \\ \times 6 \\ \hline 300 \\ 180 \\ \hline 988 \end{array}$	$\begin{array}{c} 10 \\ + \\ 5 \\ \hline 15 \\ \times 9 \\ \hline 90 \\ 45 \\ \hline 135 \end{array}$	$\begin{array}{c} 6 \\ \times 4 \\ \hline 24 \end{array}$
--	---	---

**Area Model** *Considers BOTH dimensions of an array or area model, moving away from needing to see every square unit*




**Early Transitional Strategies**

**Skip Counting** 3, 6, 9, 12, 15


**Building up**

$$\begin{array}{r} 3 + 3 + 3 + 3 \\ 6 + 6 \\ \hline 12 \end{array}$$

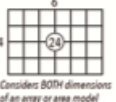
**Skip Counting with a Model**



**Equal groups in an array**




**Area Model -  $6 \times 4 = 24$**




  

**Additive Strategies**

Repeated addition with or without a model -  $3 \times 4 = 12$

$$3 + 3 + 3 + 3 = 12$$



Subitizing in small groups



$3 + 3 + 3 + 3$


**Early Additive Strategies**

**Modeling, counting by ones**



**Inconsistent Grouping**

**Modeling, counting by subgroups**

 $3 \times 4 = 12$ 


**Non-Multiplicative Strategies**

Adds or subtracts factors  
Models factors incorrectly  
Guesses

Uses incorrect operation  
Not enough information  
Uses procedures incorrectly

**Underlying Issues and Errors**

- Doesn't consider reasonableness of solution

- Error in calculation, place value, vocabulary, property or relationship, equation, or model

This is a derivative product of the Vermont Mathematics Partnership Diagnostic Assessment Project (DGAP) which was funded by NSF (EHR-0227767) and the USDOE (S366A020002). © 2017 OGAPMath LLC.

- What do you notice?
- What are the key elements of the MR progression?
- What is unfamiliar?
- What questions do you have?

# Important Ideas About the Multiplication and Division Progressions

- 1) Movement along the Progressions is not linear
- 2) Students' strategies will be at different levels
- 3) The Progressions provide instructional guidance
- 4) The Progressions are not evaluative
- 5) Collection of underlying issues and errors is important

# Recommended OGAP PLC Components

Part I: Content (Division)

Part II: Analysis of student work and evidence for instructional decision making

Part III: Selecting items from the OGAP item bank.

# EG 63 – Bake Sale

Solve this problem two different ways

The 4<sup>th</sup> grade class earned \$132 from a bake sale.

They decided to donate an equal amount of money to four different organizations.

How much money will the class donate to each organization?

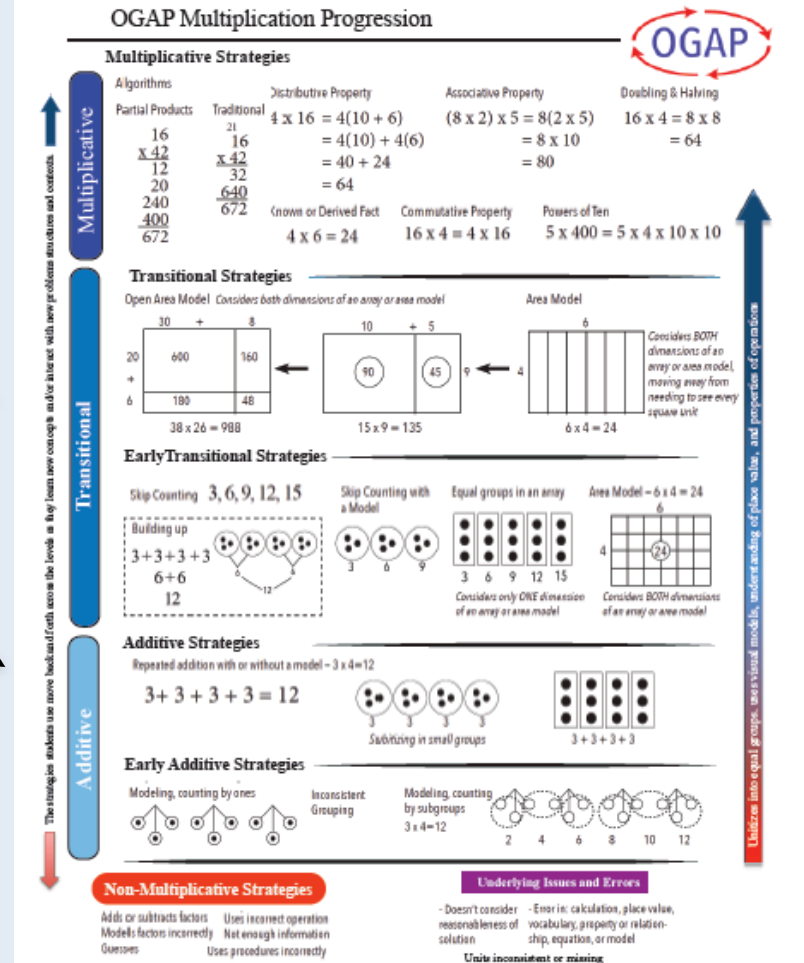
Show your thinking.

# Examining Student Work

1. Sort the student solutions by strategy
  - Not by right and wrong
2. How could you describe the ‘piles’ you have created? What do they all have in common?
  - Put a sticky note on your piles that describes the student strategy

# Examining Student Work

Place each pile of student work on the learning progression



# An OGAP PLC



## Assess

- Why did you give this problem?
- Solve yourself multiple ways.
- Sort student work by similar strategies.

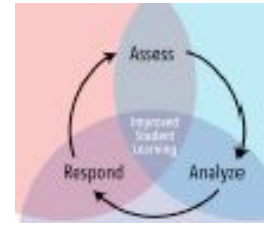
## Analyze

- What's the good news?  
(evidence of developing understanding)
- What issues are evidenced in the student work?

## Respond

- What are possible next instructional steps?
- What is the next item you will give and why?

# OGAP PLC Examining Student Work



<b>Date:</b>	<b>Grade:</b>
<b>Attending:</b>	



## ASSESS

- What task was given?
- Why was this task chosen? What assessment information was needed?
- Solve the given problem multiple ways.
- What strategies might students use (list below)? What errors might be evidenced in the student work?







## The Ongoing Assessment Project (OGAP)

### OGAP Overview

Started in 2003 in Vermont, OGAP is a systematic and intentional formative assessment system in mathematics. It is based on mathematics education research on how students learn specific concepts, common errors students make, pre-conceptions and misconceptions that impact the learning and application of new concepts.

OGAP provides formative assessment training in:

- Additive Reasoning,
- Multiplicative Reasoning,
- Fractions,
- Proportionality

### OGAP Formative Assessment System

This system involves using specific knowledge described below and the OGAP Learning Progressions/Frameworks to:

- Gather and analyze evidence about students' pre-existing knowledge to guide unit planning
- Implement a continuous and intentional system of instruction based on evidence in student solutions. This includes:
  - Regularly using instructionally embedded OGAP formative items.
  - Using the OGAP Framework/Learning Progression to analyze evidence in student work
  - Making timely and effective instructional modifications based on evidence of student understanding

### OGAP Training

Central to all OGAP trainings is a focus on teacher knowledge. Specifically, this knowledge includes:

- mathematical content specific to the related mathematical content
- mathematics education research on how students learn specific mathematics concepts
- deep understanding of the related research-based OGAP Framework/Learning Progression
- formative assessment practices specific to mathematics.

In addition, OGAP training includes personalized school or district follow-up support focused on helping teachers effectively embed OGAP formative assessment into their practice.

### OGAP Experience

OGAP has worked with a variety of partners including the School District of Philadelphia, Charleston County, SC School District, Maryland Department of Education, AMSTI statewide initiative in Alabama, and Omaha, Nebraska Public Schools to name a few.

### To Learn More About OGAP

- Visit [ogapmathllc.com](http://ogapmathllc.com)
- Contact Beth Hulbert [beth@ogapmathllc.com](mailto:beth@ogapmathllc.com)