Learning Trajectories:
A research lens for enhancing formative assessment.

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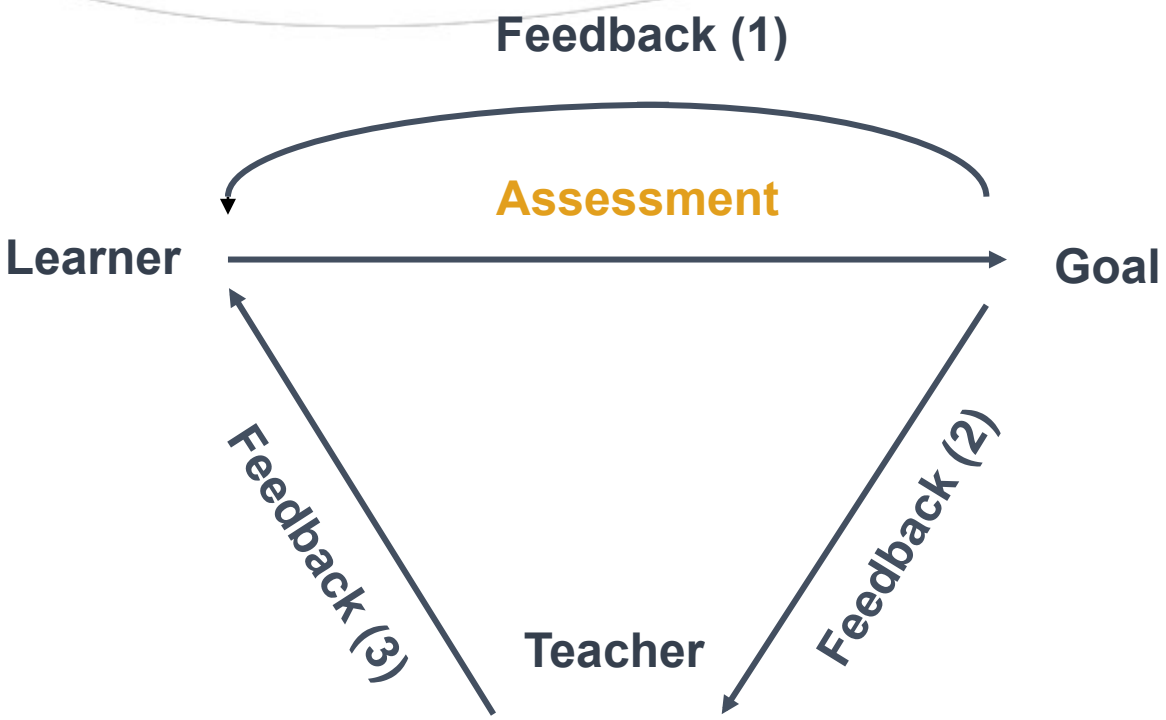
Take Aways

- Effective math instruction and planning for instruction requires careful attention to evidence of student thinking.
- Using learning trajectories/progressions to understand evidence of student thinking provides actionable information based on research on how students learn specific mathematics concepts
- The "essence" of formative assessment is the relentless attention to evidence of student understanding and intentional and systematic use of the evidence for planning and instruction. (Popham, 2012)

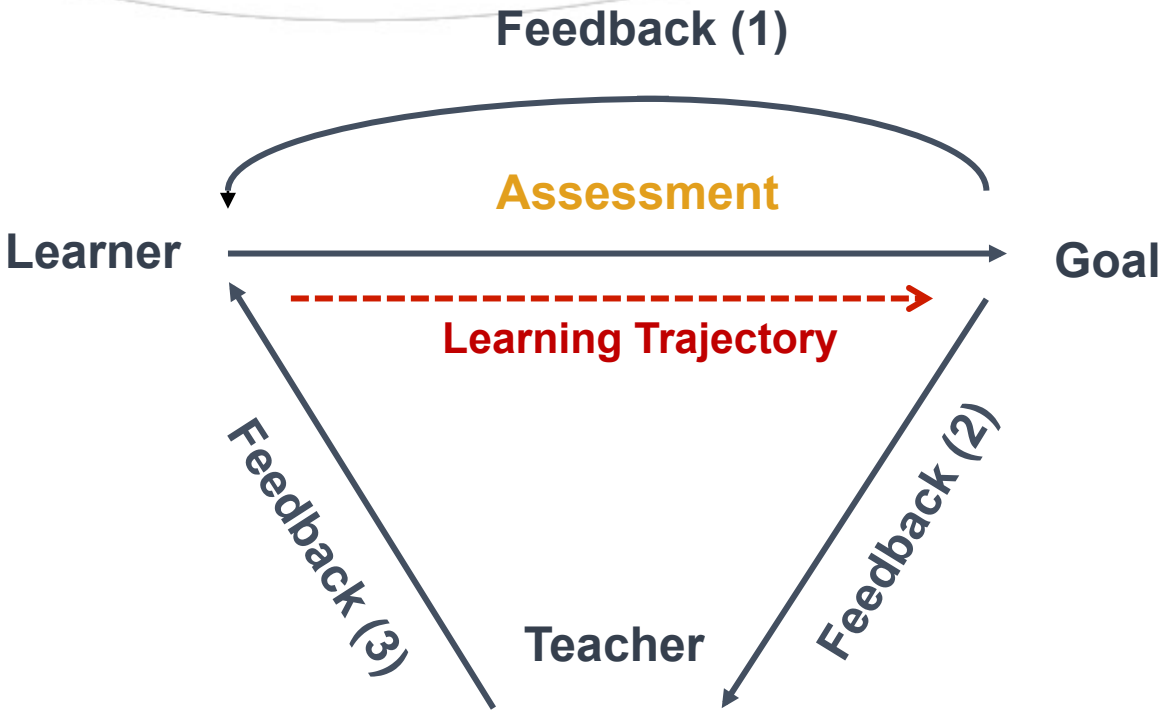
Formative Assessment

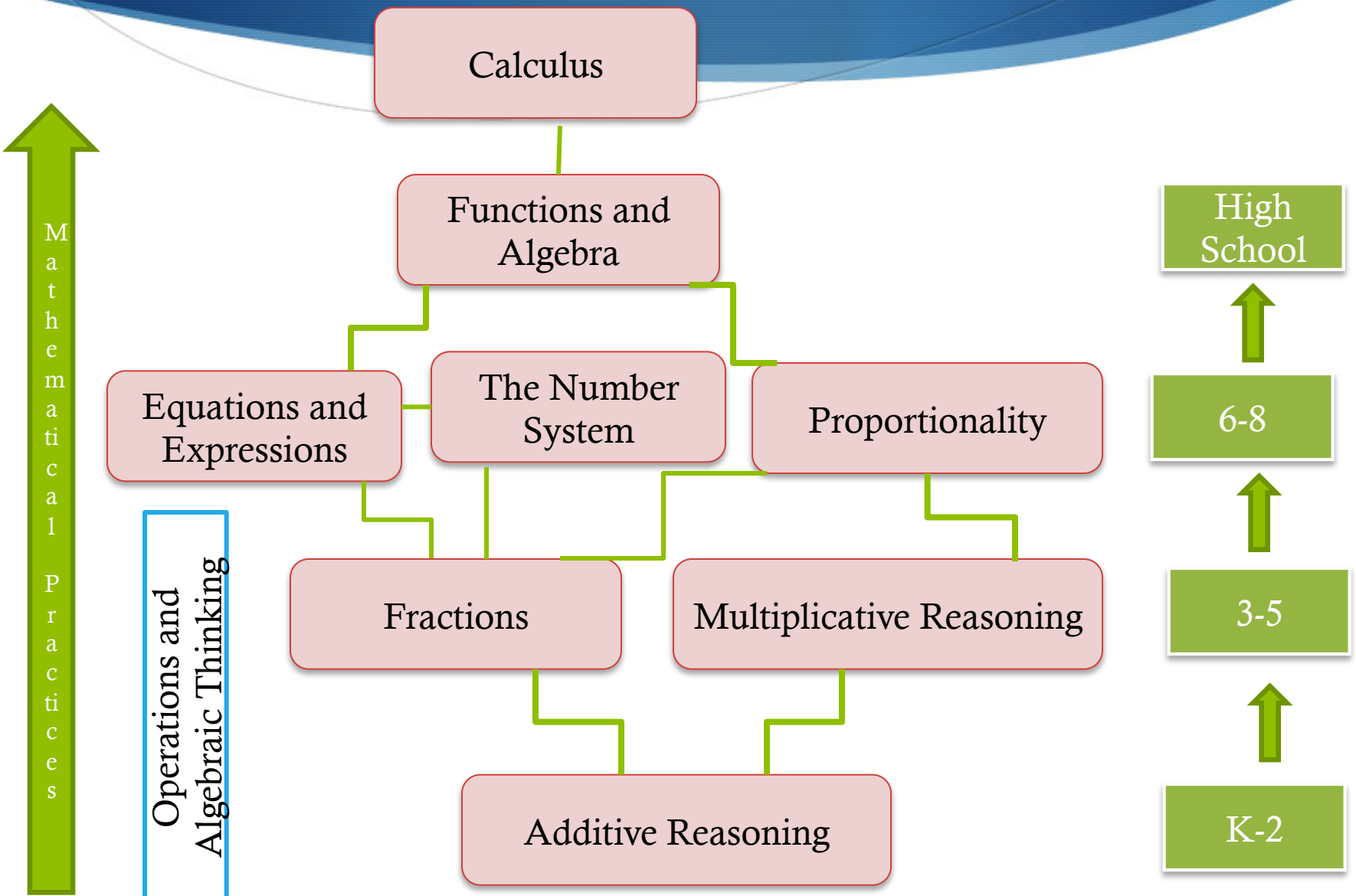


Learning Trajectories

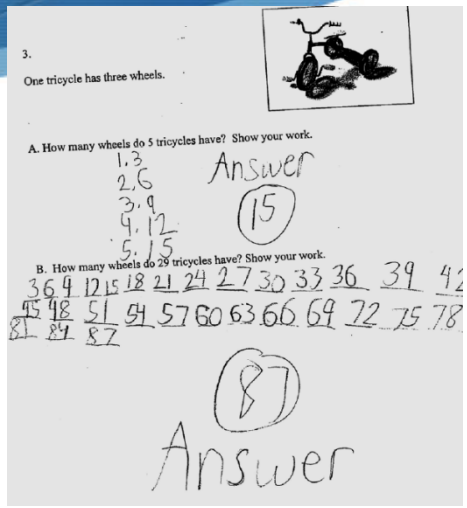
- "Learning progressions are descriptions of the successively more sophisticated ways of thinking about a topic that can follow one another as children learn about and investigate a topic" (NRC, 2007, p. 214)
- Developmental progressions of strategies, concepts and levels of student thinking in particular mathematical domains
- Links research on learning and instructional practice

Formative Assessment



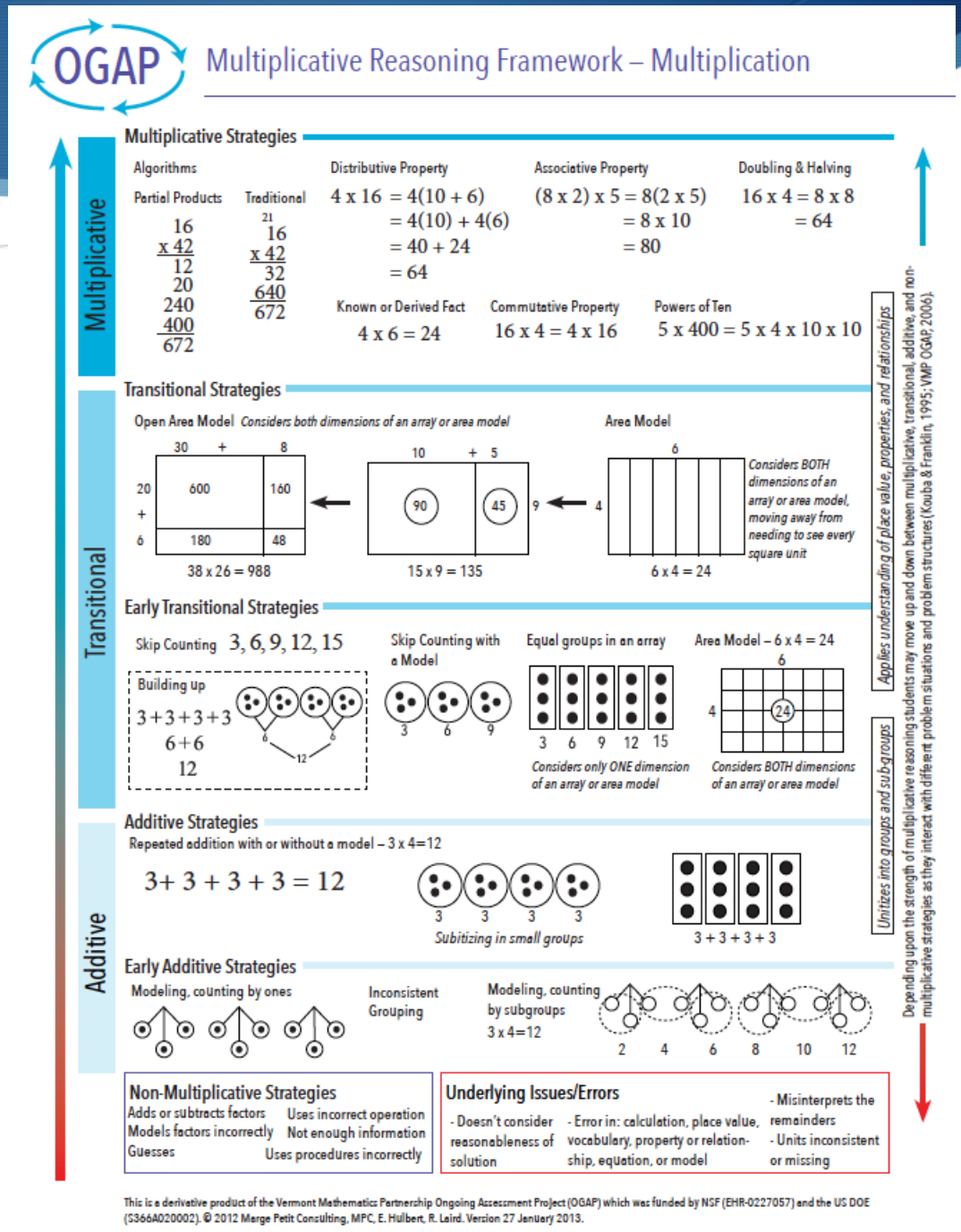

Calculus
Functions and Algebra
Equations and Expressions
The Number System
Proportionality
Fractions
Multiplicative Reasoning
Additive Reasoning
Operations and Algebraic Thinking
Mathematical Practices
High School
6-8
3-5
K-2

Looking at Student Work



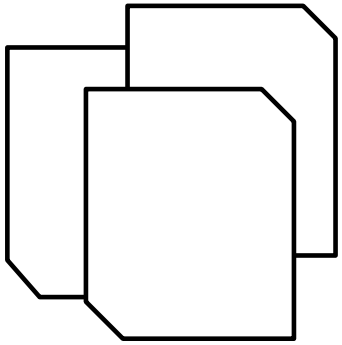
- Review the samples of student work
- What do you notice? Make some observations

An Example: The OGAP Multiplicative Reasoning Progression

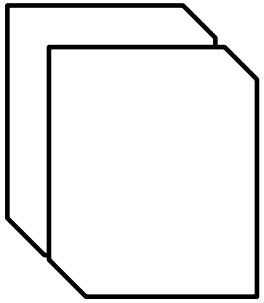


The OGAP Sort

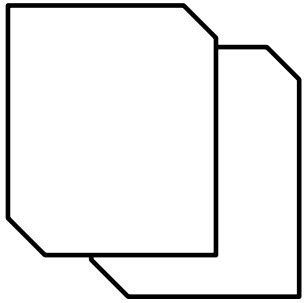
Multiplicative Strategies



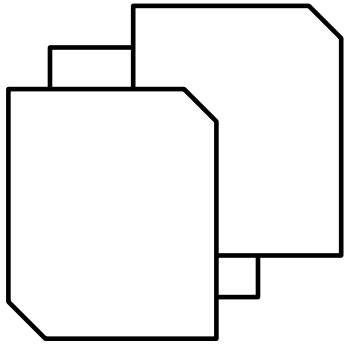
Transitional Strategies



Additive Strategies



Non-multiplicative



Instructional Implications

Student A

One tricycle has three wheels.

A. How many wheels do 5 tricycles have? Show your work.

3+3+3+3+3=
6+6+3=15

B. How many wheels do 29 tricycles have? Show your work. 77

9+9+9+9+9+9+9+9+9+6=77
18+18+18+18+15
36+36+15=77

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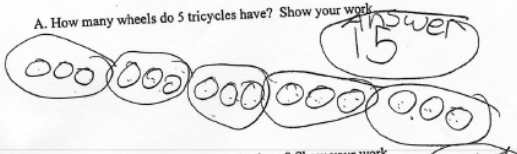
Student C

3.

One tricycle has three wheels.

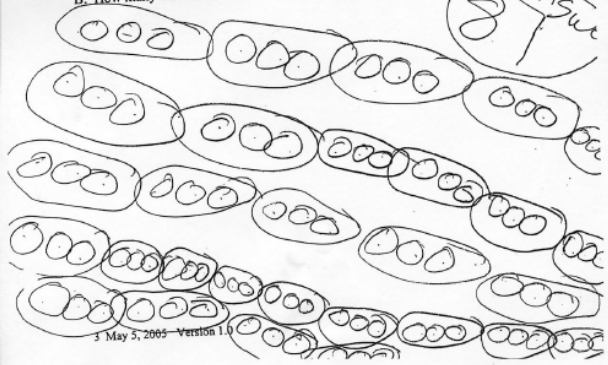
A. How many wheels do 5 tricycles have? Show your work.

Answer 15



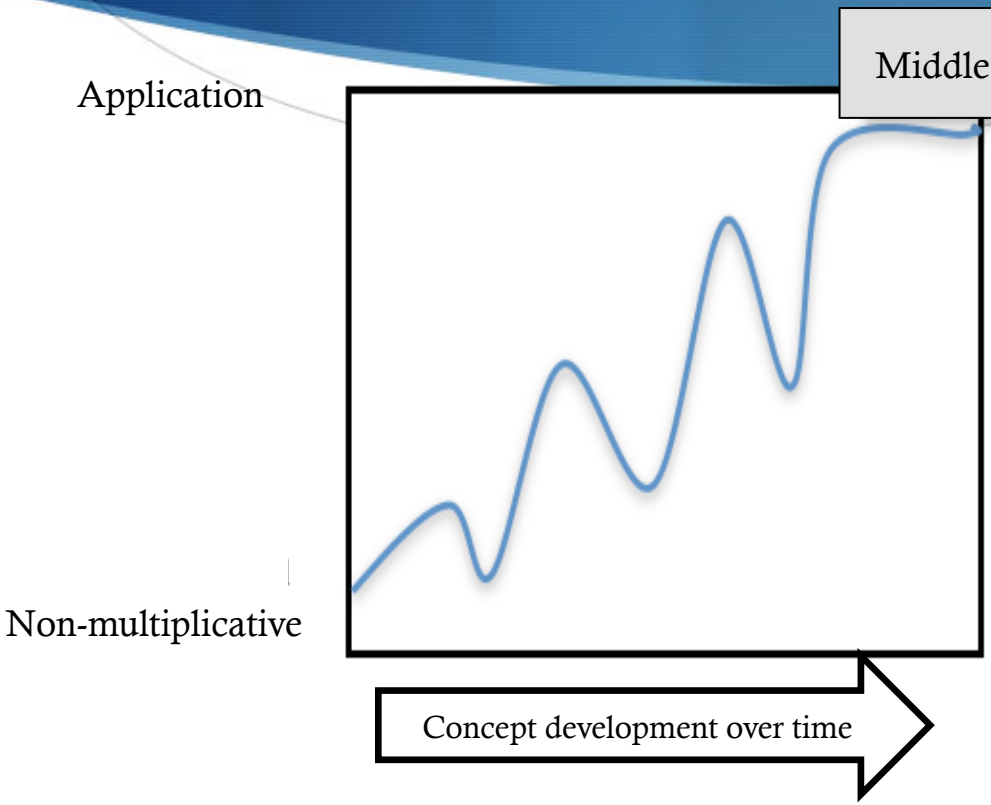
B. How many wheels do 29 tricycles have? Show your work.

Answer 87



3 May 5, 2005 Version 1.0

Students move back and forth…



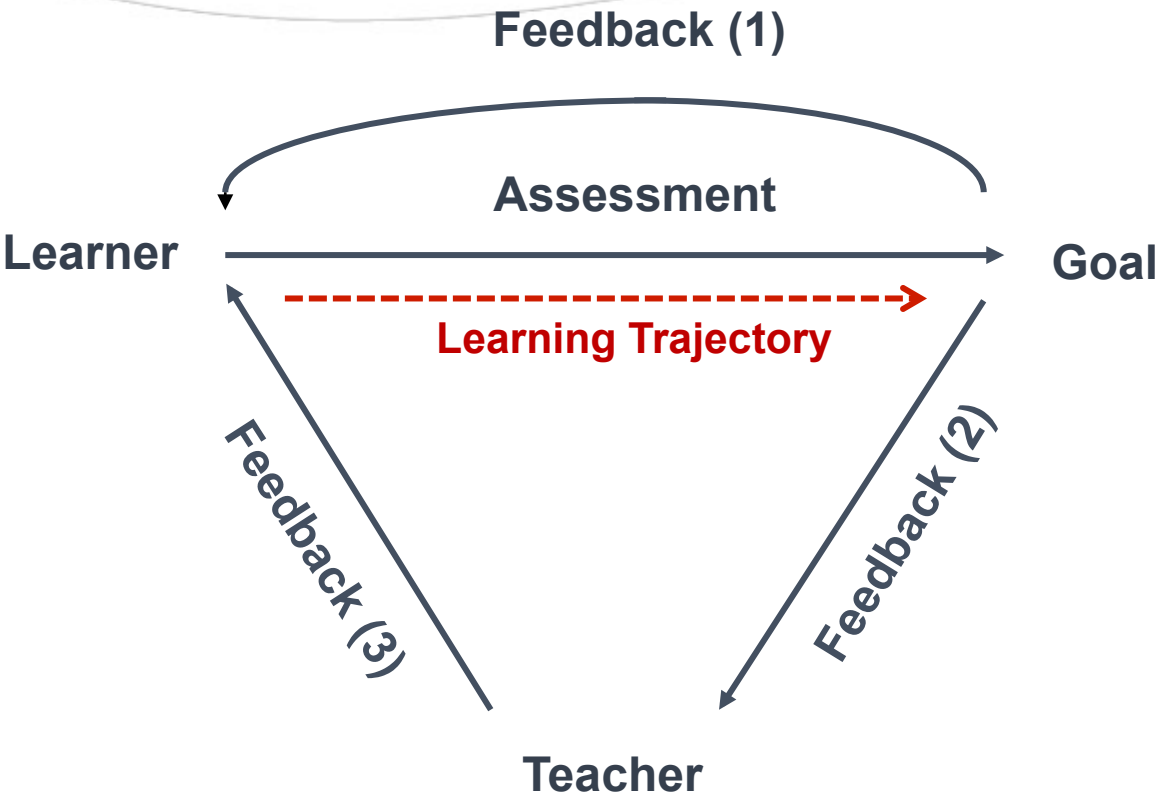
… as they interact with new contexts, different magnitude numbers, number of factors, and more.

Petit, Laird, Marsden, & Ebby, in press 2015

CCSSM

Grade	CCSSM Multiplicative Problem Situations (BOLD = new for grade level)	CCSSM Multiplicative Strategies
2	**Equal groups**	Repeated addition with an array
3	Equal groups, **arrays, equal measures, beginning area** Multiplication and division within 100 and 1 digit x multiples of 10 (e.g., 5 x 50)	Solve multiplication and division problems using *strategies based on place value and properties of operations.*
4	Equal groups, equal measures, **multiplicative comparisons, measurement conversions within systems, area** Multiply 1 digit x up to 4 digits, and 2 digits x 2 digits. Divide up to 4 digits by 1 digit numbers.	Solve multiplication and division problems using *strategies based on place value* (e.g., partial products, area models) and the *properties of operations* and *relationships* (e.g., commutative, associative, and distributive, inverse relationship between multiplication and division).
5	Equal groups, equal measures, multiplicative comparisons, **measurement conversions between systems, area, scaling (multiplicative change)** Fluently multiply whole numbers. Divide up to 4 digits by 2 digits.	Solve multiplication problems *using efficient strategies* (e.g., partial products, traditional algorithms). Solve division problems using *strategies based on place value* (e.g., partial quotients, menus, area models) and the *properties of operations* and *relationships* (e.g., commutative, associative, distributive, and the relationship between multiplication and division).

Learning Trajectory-Oriented Formative Assessment



Citations

- Hulbert, E. Petit, & Laird, R. (2013). *Ongoing assessment project: Professional development materials.* Moretown, VT: Ongoing Assessment Project.
- Popham, W. J. (2012). *Forward*. In E. Wylie, A. Gullickson, K. Cummings, L. Noakes, K. Norman, & S. Veeder (Eds.), *Improving formative assessment to empower student learning* (pp. ix-xii). Thousand Oaks, CA: Corwin Press.
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- Petit, Hulbert, & Laird (2013). *OGAP Multiplicative Reasoning Framework*. Moretown, Vermont.