Formative Assessment Tools and Routines for Additive Reasoning

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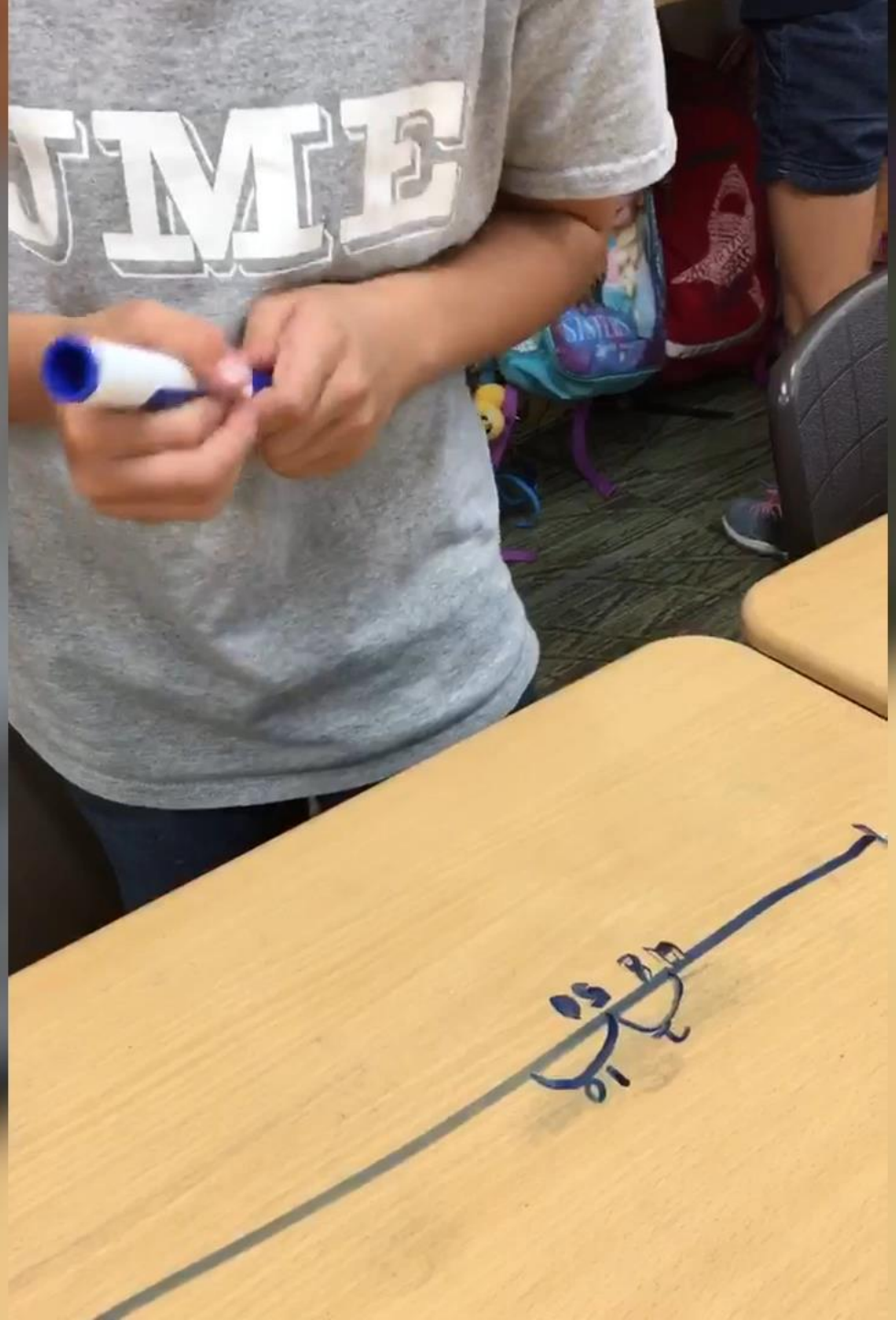
#OGAPMath

Formative Assessment Tools and Routines for Additive Reasoning

Background on the Ongoing Assessment Project (OGAP)

Using the Additive Reasoning progression to analyze and respond to student work

Developing fluency with number lines



OGAP is…

- An intentional and systematic mathematics formative assessment system
- Based on math education research
- Focused on:
  - how students develop understanding of concepts and procedural fluency,
  - errors or pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems.

OGAP

2003-2015

- ✓Multiplicative Reasoning (3-5)
- ✓Fractions (3-5)
- ✓Proportionality (6-8)

2016

Additive Reasoning (K-2)

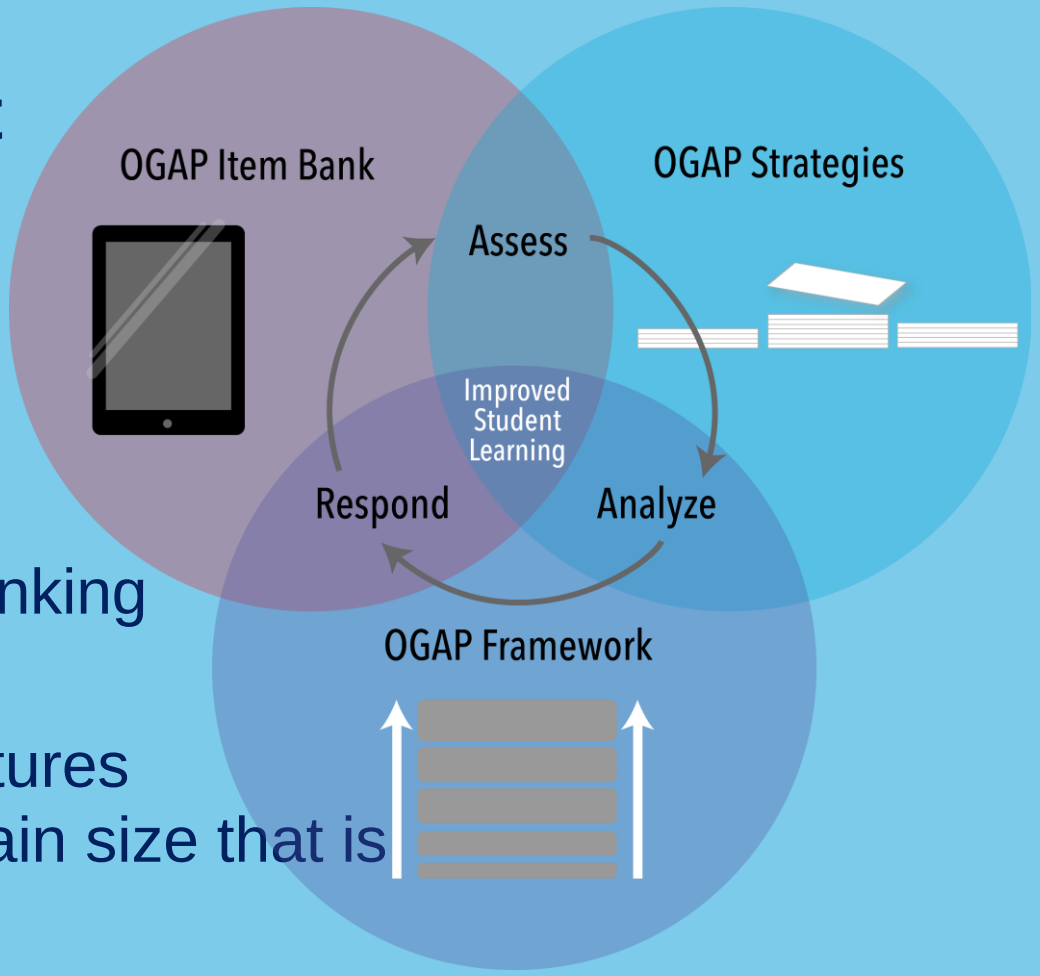
- Alabama
- Vermont
- Pennsylvania
- Maryland
- South Carolina

NSF

CPRE
CONSORTIUM *for* POLICY
RESEARCH *in* EDUCATION

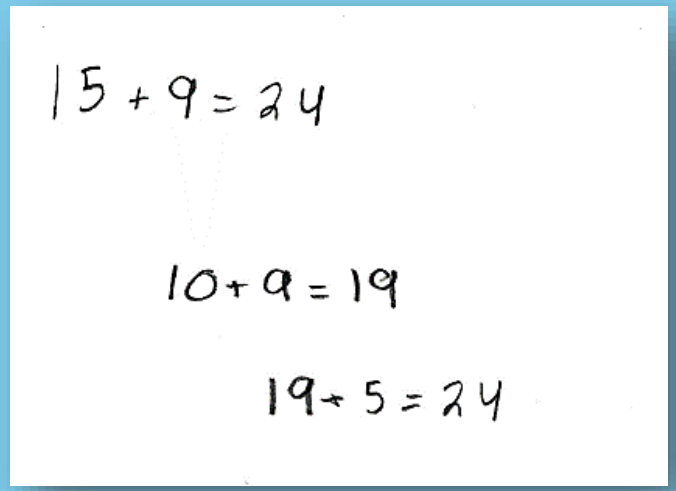
Formative Assessment System

- **Extensive Professional Development**
  - Content
  - Research on student learning
  - Formative assessment routines
- **Formative Assessment Items**
  - Aligned to CCSSM
  - Engineered to elicit evidence of student thinking
- **Frameworks**
  - Information on problem contexts and structures
  - Learning Progressions/Trajectories at a grain size that is usable by teachers

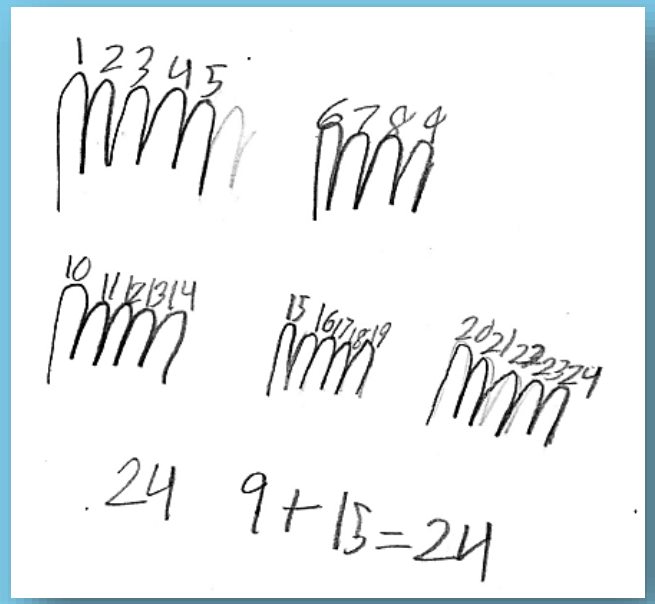


Coral and Sean are counting crayons.
Coral counted 15 blue crayons.
Sean counted 9 red crayons.
How many crayons did they count together?

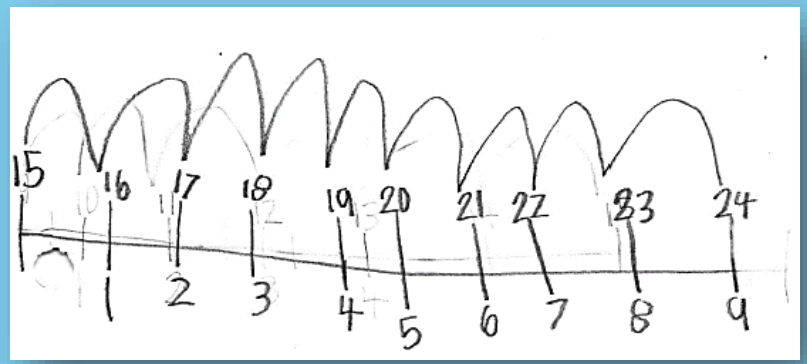
Shantelle



Rachel



Declan

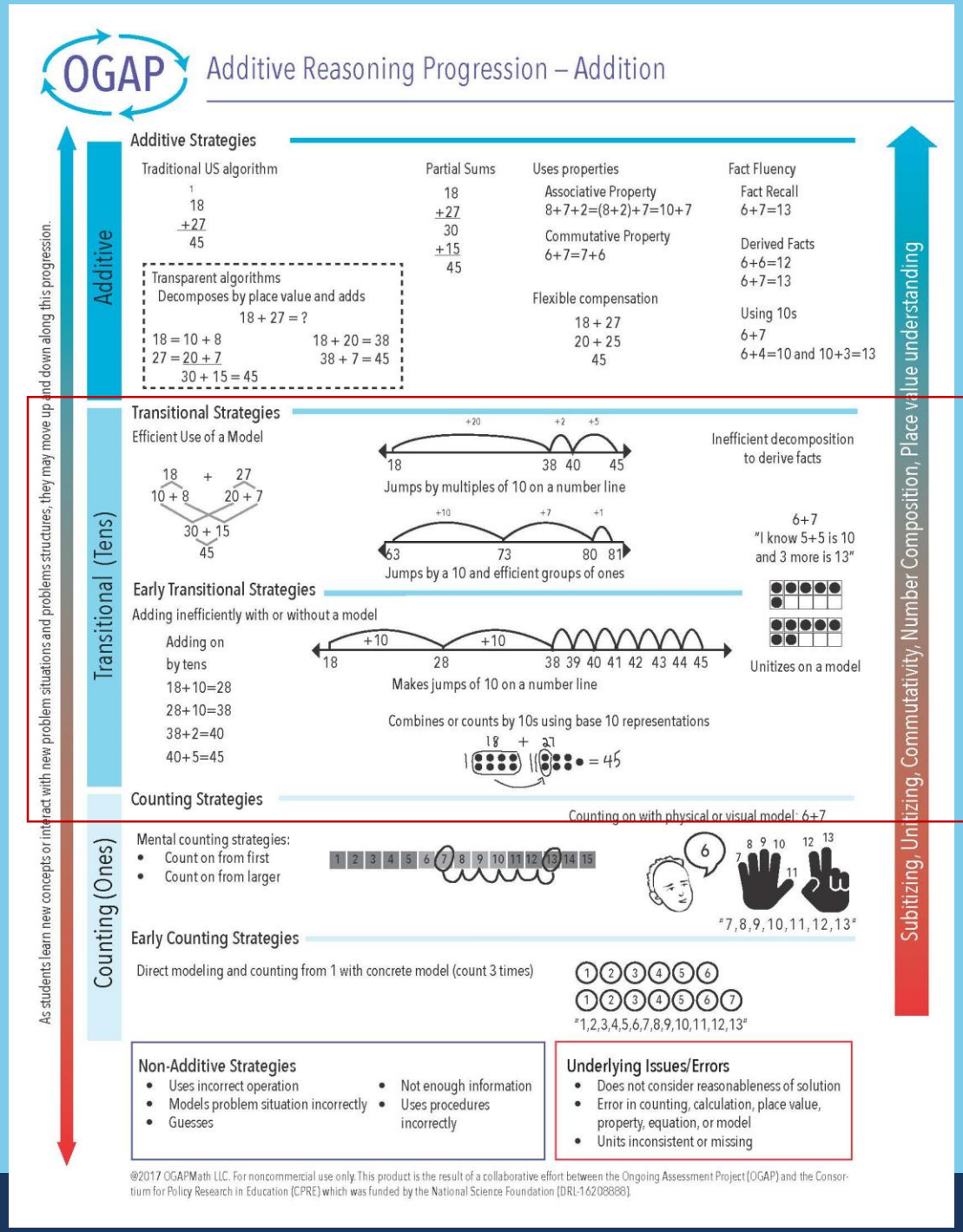


Additive

Transitional

Counting

Non-Additive



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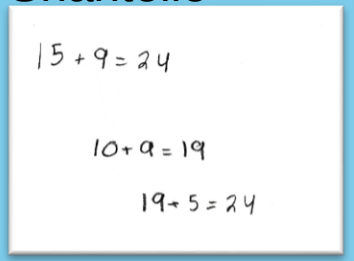
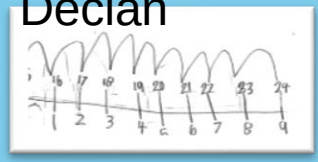


Models that support development of additive reasoning

Declan



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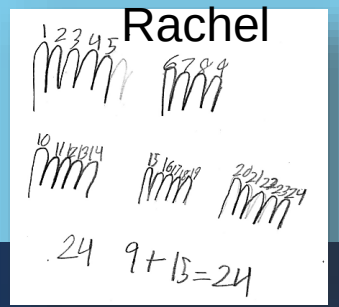

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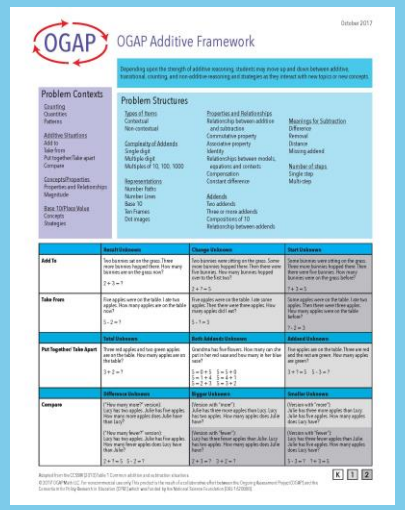
Importance of problem structure and number complexity



OGAP Additive Reasoning Progression – Addition

As students learn new concepts or interact with new problem situations and problems structures, they may move up and down along this progression.

Subitizing, Unitizing, Commutativity, Number Composition, Place value understanding

Additive

Additive Strategies

Traditional US algorithm

18
+27
45

Transparent algorithms
Decomposes by place value and adds
18 + 27 = ?
18 = 10 + 8
27 = 20 + 7
30 + 15 = 45
18 + 20 = 38
38 + 7 = 45

Partial Sums

18
+27
30
+15
45

Uses properties
Associative Property
8+7+2=(8+2)+7=10+7
Commutative Property
6+7=7+6
Flexible compensation
18 + 27
20 + 25
45

Fact Fluency
Fact Recall
6+7=13
Derived Facts
6+6=12
6+7=13
Using 10s
6+7
6+4=10 and 10+3=13

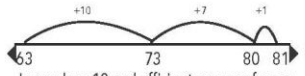
Transitional (Tens)

Transitional Strategies

Efficient Use of a Model

18 + 27
10 + 8 | 20 + 7
30 + 15
45

Jumps by multiples of 10 on a number line



Jumps by a 10 and efficient groups of ones

Inefficient decomposition to derive facts

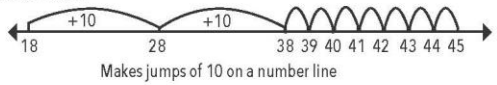
6+7
"I know 5+5 is 10 and 3 more is 13"


Unitizes on a model

Early Transitional Strategies

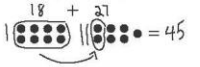
Adding inefficiently with or without a model

Adding on by tens
18+10=28
28+10=38
38+2=40
40+5=45



Makes jumps of 10 on a number line

Combines or counts by 10s using base 10 representations



Counting (Ones)

Counting Strategies

Mental counting strategies:
- Count on from first
- Count on from larger

Counting on with physical or visual model: 6+7

"7,8,9,10,11,12,13"

Early Counting Strategies

Direct modeling and counting from 1 with concrete model (count 3 times)

"1,2,3,4,5,6,7,8,9,10,11,12,13"

Non-Additive Strategies
- Uses incorrect operation
- Models problem situation incorrectly
- Guesses
- Not enough information
- Uses procedures incorrectly

Underlying Issues/Errors
- Does not consider reasonableness of solution
- Error in counting, calculation, place value, property, equation, or model
- Units inconsistent or missing

The progression is not evaluative

Movement along the progression is not linear

Instructional Guidance

Place value understanding
Number composition
Properties of addition
Unitizing
Subitizing

Errors and Misconceptions

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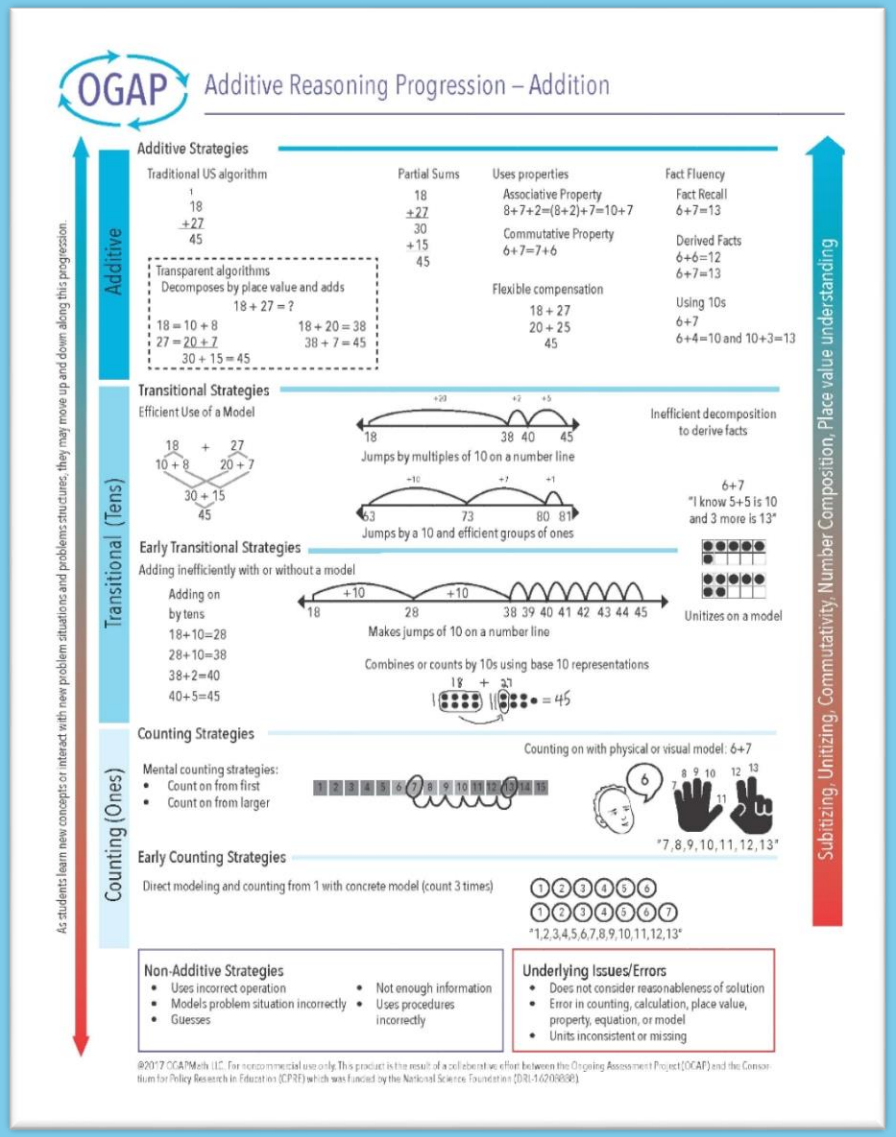
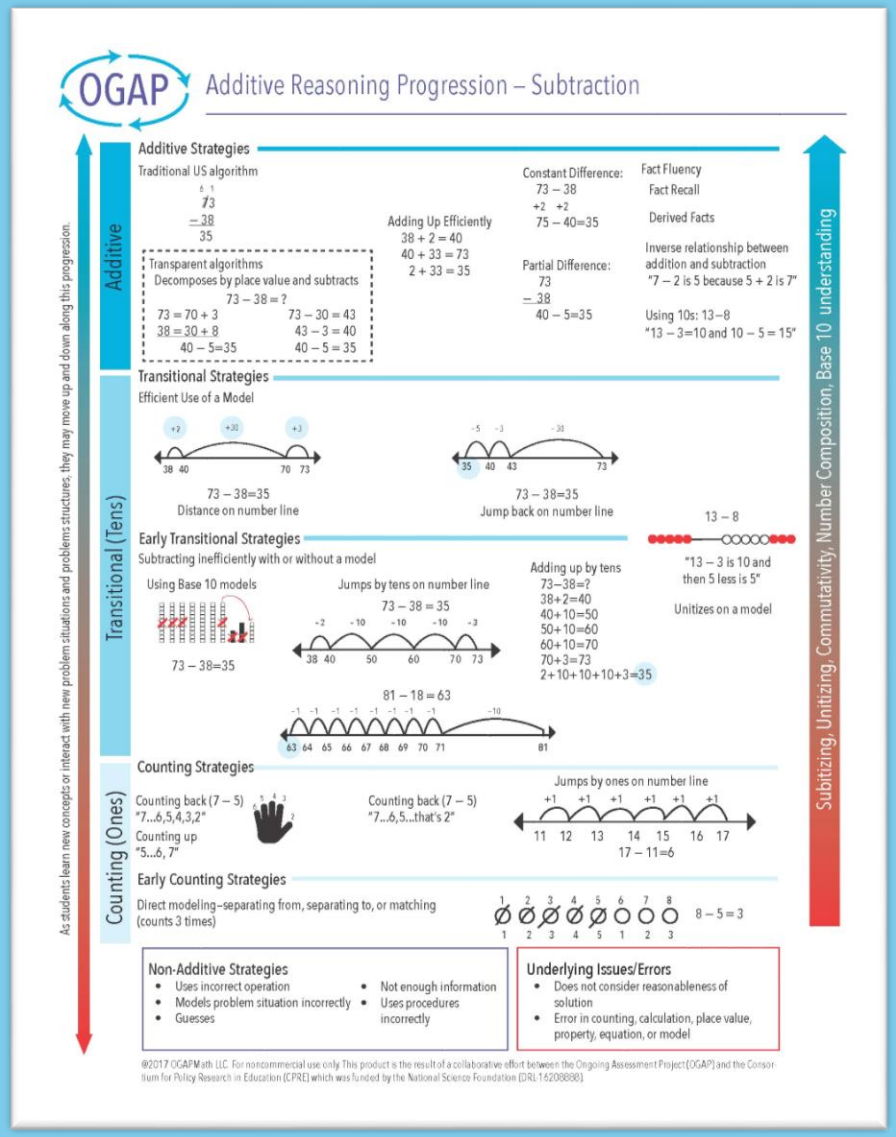
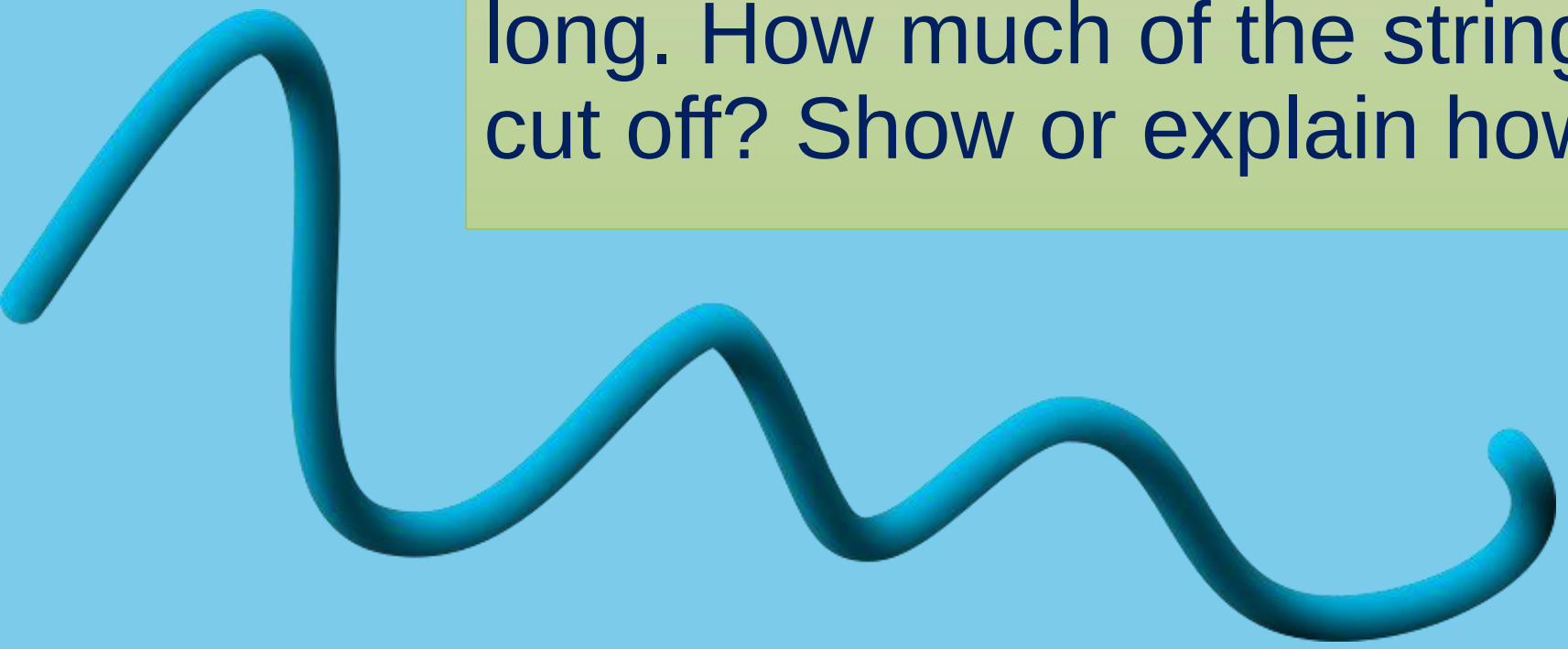
Addition and Subtraction Progressions

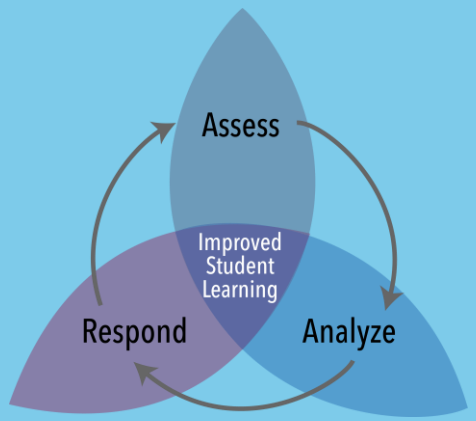




Solve two different ways.

A string was 56 inches long. Dylan cut some off. Now the string is 27 inches long. How much of the string did Dylan cut off? Show or explain how you know.





Sort

OGAP Additive Reasoning Progression – Addition

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As students learn new concepts or interact with new problem situations and problems structures, they may move up and down along this progression.

Subitizing, Unitizing, Commutativity, Number Composition, Place value understanding

OGAP Additive Reasoning Progression – Subtraction

Additive

Additive Strategies

Traditional US algorithm

73
−38
35

Transparent algorithms
Decomposes by place value and subtracts
73 − 38 = ?
73 = 70 + 3
38 = 30 + 8
40 − 5 = 35
73 − 30 = 43
43 − 3 = 40
40 − 5 = 35

Adding Up Efficiently
38 + 2 = 40
40 + 33 = 73
2 + 33 = 35

Constant Difference:
73 − 38
+2 +2
75 − 40 = 35

Partial Difference:
73
− 38
40 − 5 = 35

Fact Fluency
Fact Recall
Derived Facts
Inverse relationship between addition and subtraction
"7 − 2 is 5 because 5 + 2 is 7"
Using 10s: 13−8
"13 − 3 = 10 and 10 − 5 = 15"

Transitional (Tens)

Transitional Strategies

Efficient Use of a Model

73 − 38 = 35
Distance on number line

73 − 38 = 35
Jump back on number line

13 − 8
"13 − 3 is 10 and then 5 less is 5"
Unitizes on a model

Early Transitional Strategies

Subtracting inefficiently with or without a model

Using Base 10 models

73 − 38 = 35

Jumps by tens on number line

73 − 38 = 35

Adding up by tens
73−38=?
38+2=40
40+10=50
50+10=60
60+10=70
70+3=73
2+10+10+10+3=35

81 − 18 = 63

Counting (Ones)

Counting Strategies

Counting back (7 − 5)
"7...6,5,4,3,2"
Counting up
"5...6, 7"

Counting back (7 − 5)
"7...6,5...that's 2"

Jumps by ones on number line

17 − 11 = 6

Early Counting Strategies

Direct modeling–separating from, separating to, or matching (counts 3 times)

8 − 5 = 3

Non-Additive Strategies
- Uses incorrect operation
- Models problem situation incorrectly
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- Not enough information
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Underlying Issues/Errors
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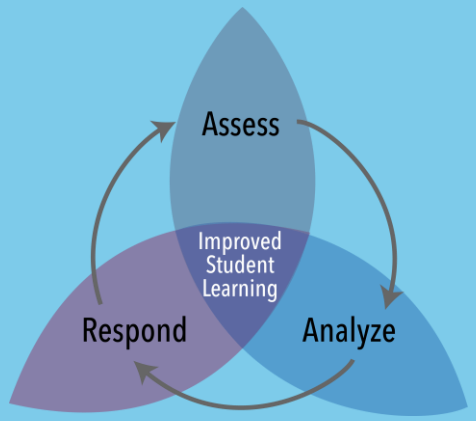


OGAP The Ongoing Assessment Project

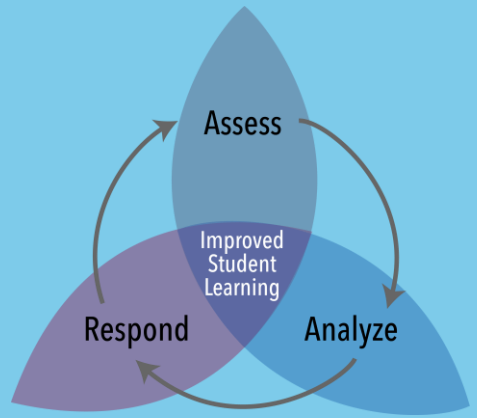
Questions to ask when analyzing student work:

- What is the evidence of developing understanding that can be built upon?
- What are issues or concerns that are evidenced in student work?
- What are the next instructional steps based on that evidence?

OGAP

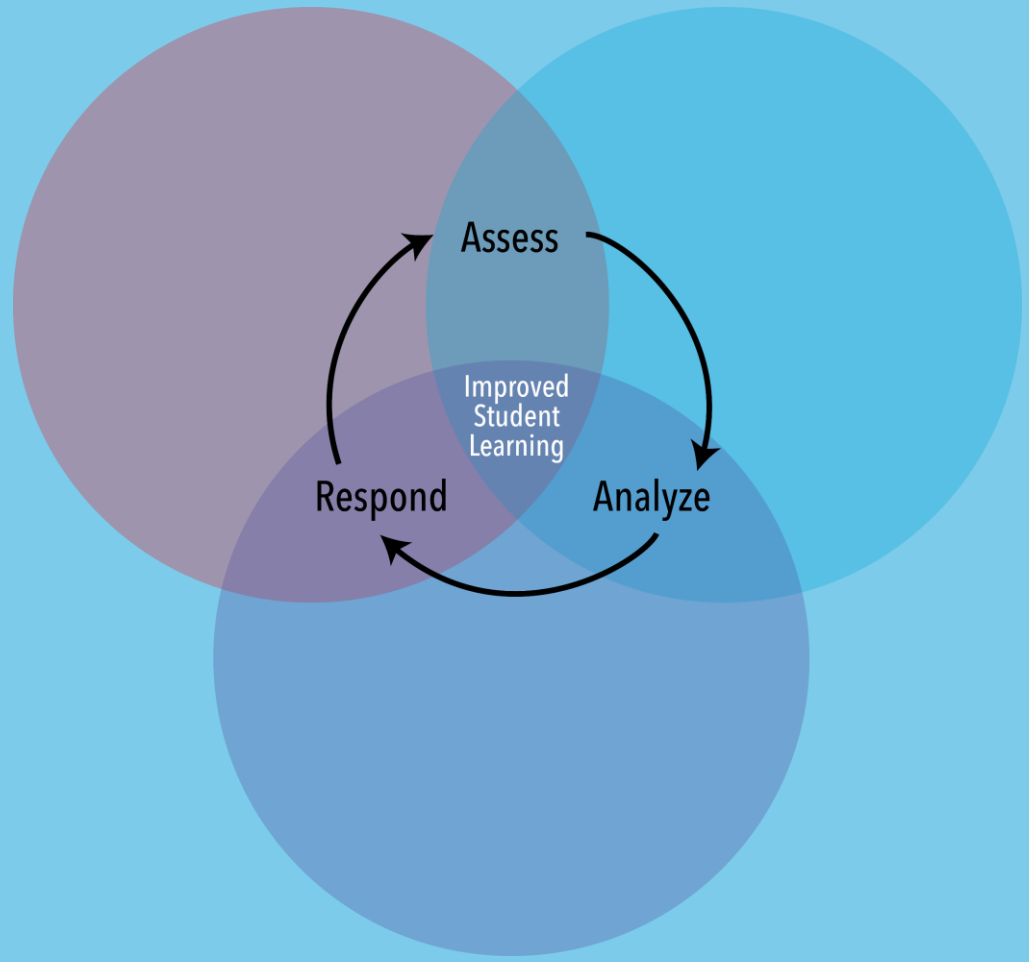
Select and Sequence

- Choose 2 pieces of student work that you could project side-by-side to start the next lesson.
- Think about how to help all students move forward on the progression
- What questions would you ask students to structure the discussion?



Formative Assessment Cycle

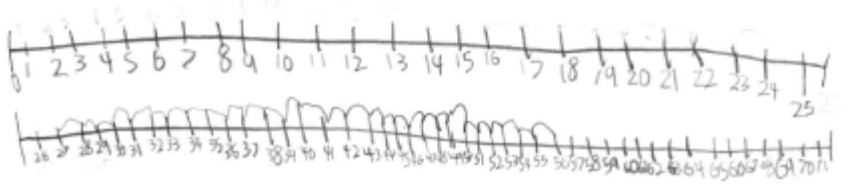
The essence of formative assessment is the relentless attention to evidence of student understanding and intentional and systematic use of the evidence for planning and instruction. (Popham, 2012)



What do these two students understand about number lines? About number?

A string was 56 inches long. Dylan cut some off. Now the string is 27 inches long. How much of the string did Dylan cut off? Show how you know.

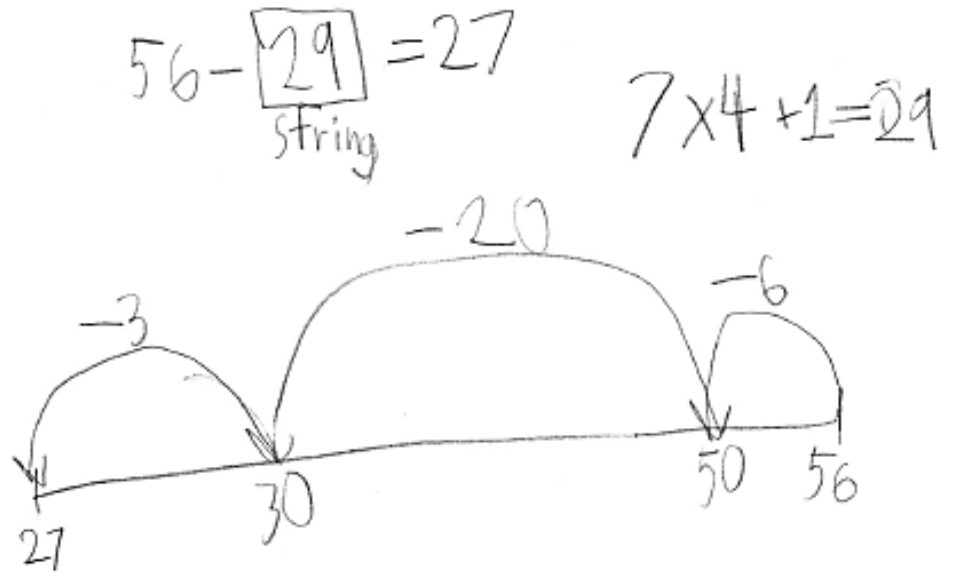
B



29, because I use a number line and hop backwards till 27 and then count the hops

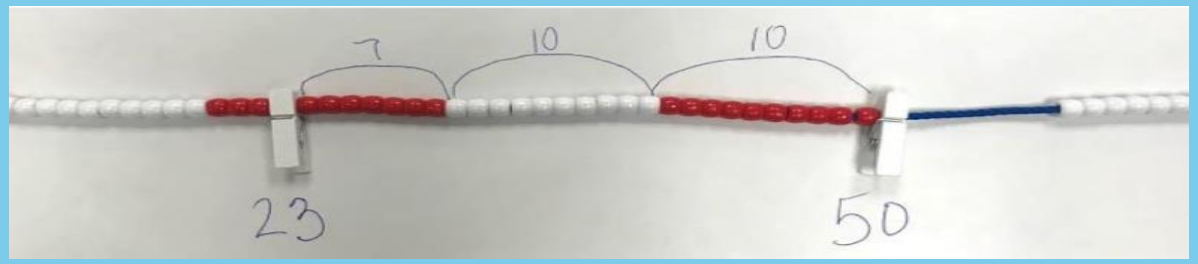
A string was 56 inches long. Dylan cut some off. Now the string is 27 inches long. How much of the string did Dylan cut off? Show how you know.

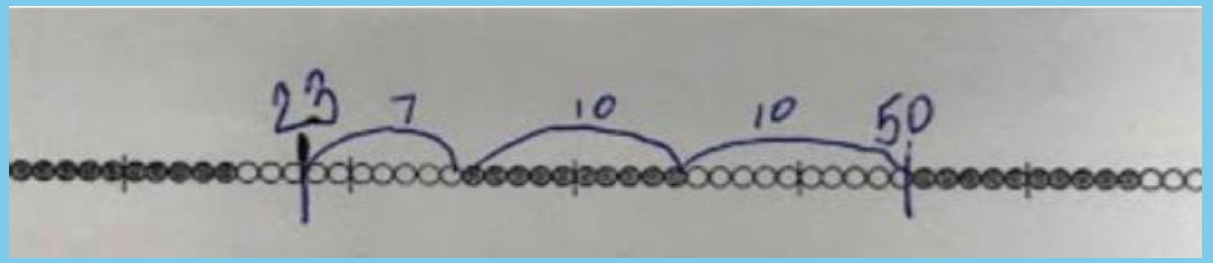
E

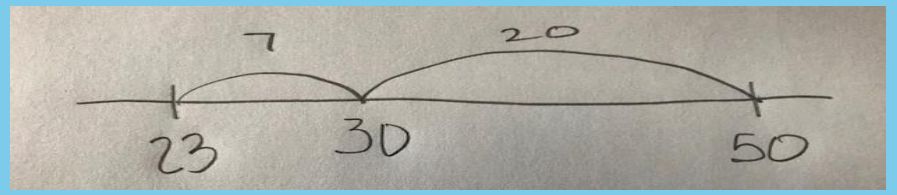


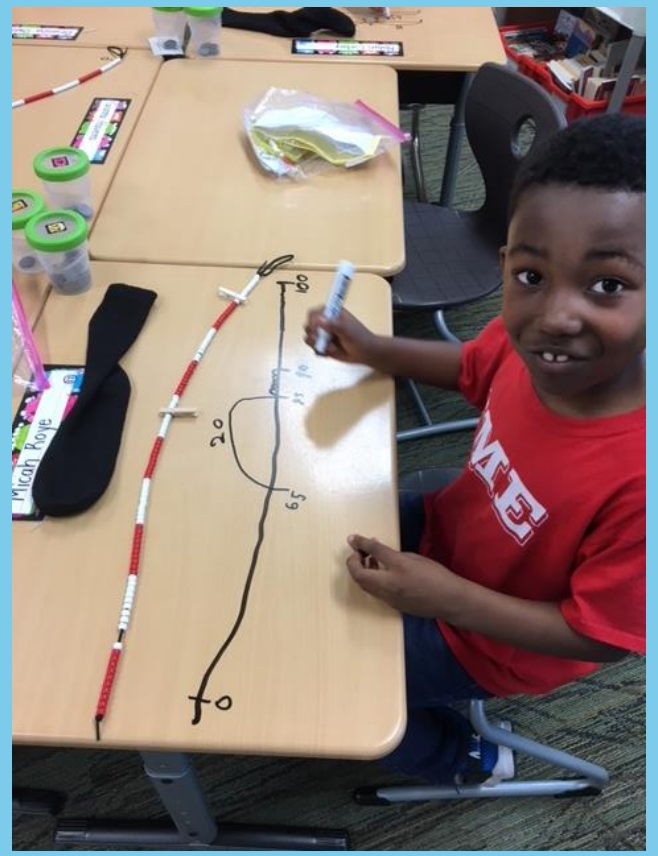
Using Number Lines to Add and Subtract

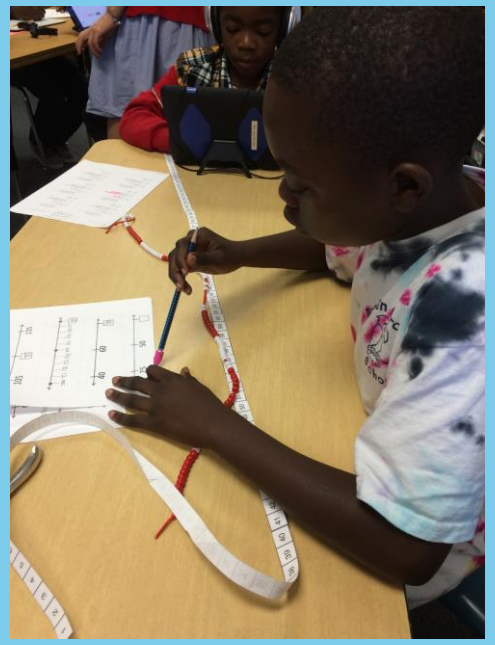
50 - 23 = ____

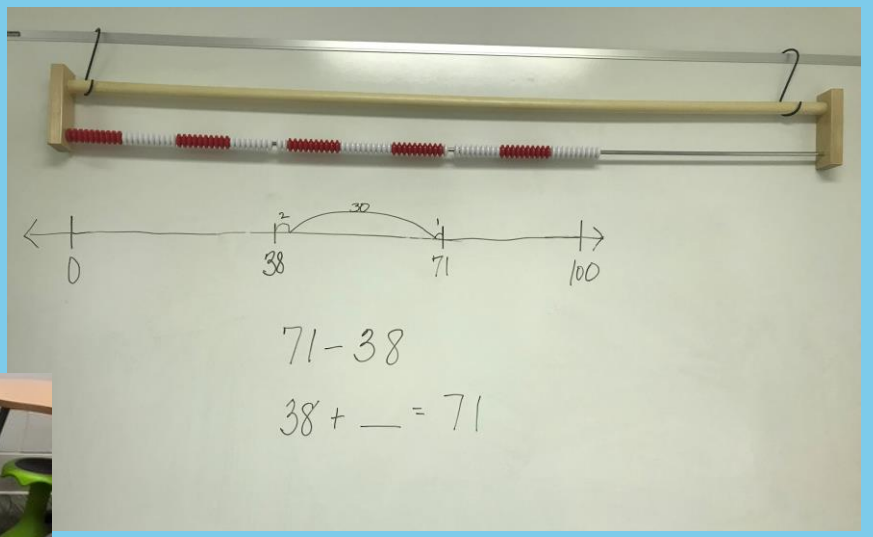


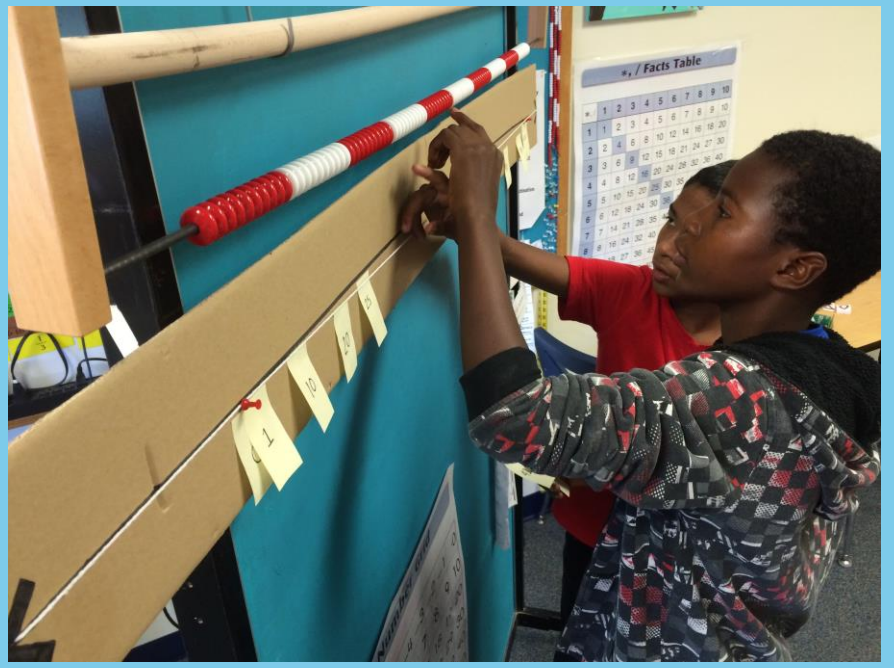












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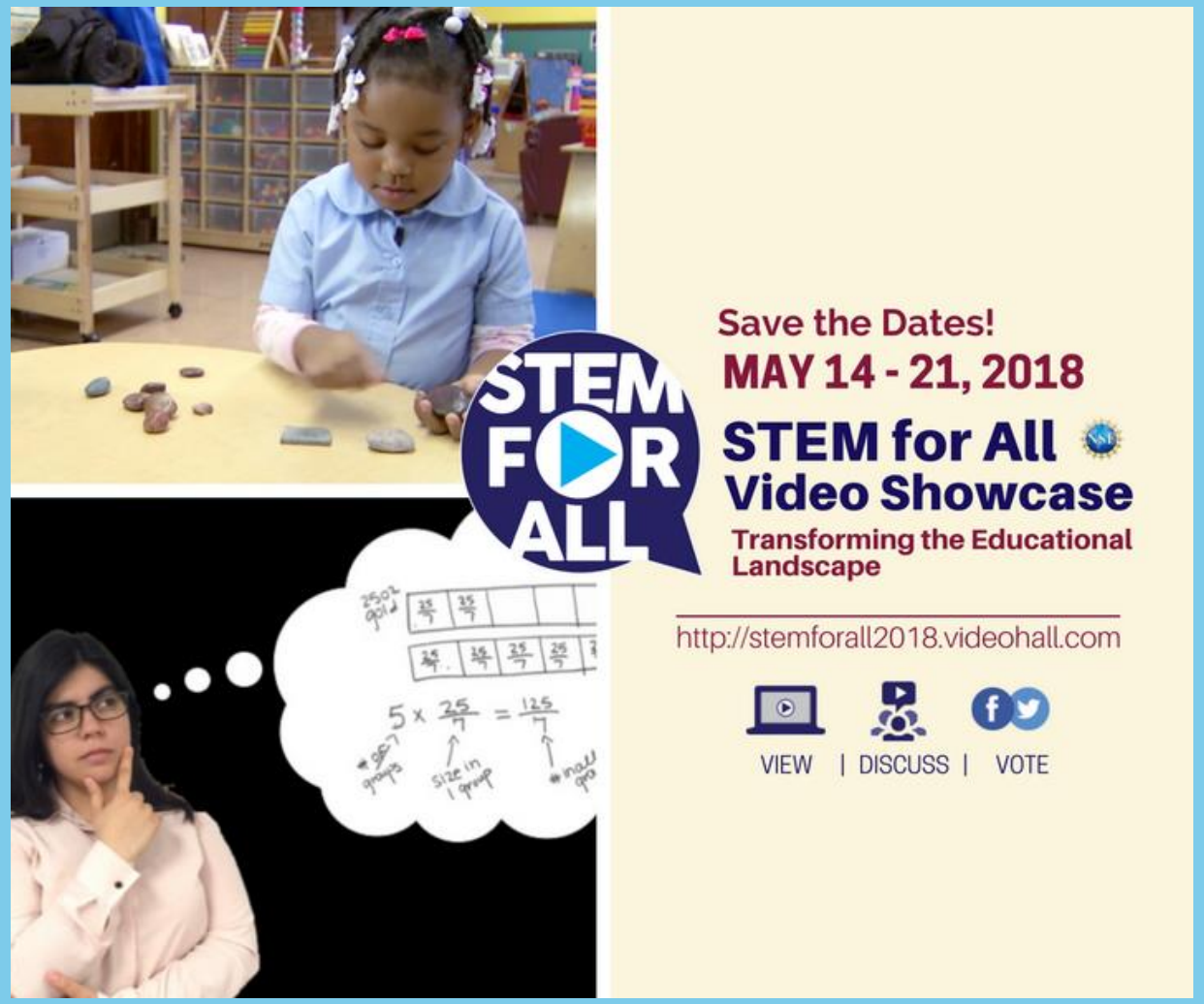

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NCTM Research Conference

Tuesday, 3:45 – 4:15 PM

- *Revealing Base Ten Understanding Through Written Formative Assessment Tasks*

Wednesday, 3:00 – 4:00 PM

- *Improving Mathematics Through Formative Assessmen*t.

Research findings from a 2-year large scale OGAP impact and implementation study and ongoing OGAP implementation and support research.