

Introduction to High School Math

Teacher Resource
Volume 1



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SAMPLE

Introduction

The *Introduction to High School Math* course was developed and piloted with educators. It is designed to prepare high school students for success in Algebra 1 and future math courses.

The program provides teacher materials supportive of a classroom culture and professional practices that encourage accurate and engaging mathematical discussions to make the system of mathematics accessible to students. It offers students hands-on experiences and opportunities for discourse that develop and illuminate the central constructs to organize their mathematical thinking.

Introduction to High School Math is not a remedial course in the traditional sense. It works to develop students' sense of numbers and of how the number system works by engaging them with Algebra 1 in a concrete and meaningful fashion, as well as by developing their sense of the structures found within mathematics. Lessons and activities engage students with important concepts in a purposeful three-step sequence, from concrete experiences, to reasoning/sense-making discussions and activities, to communication and representation of the mathematics. The program includes various hands-on activities that generate data and create models that allow students to explore, develop, and reinforce fundamental math concepts.

Introduction to High School Math cross-references activities from strong, research-based mathematics programs such as the Marilyn Burns Fraction Kit and Henry Borenson's Hands-On Equations. All activities are based on research into mathematical learning, as well as lessons that were developed and piloted in classrooms throughout Wake County, North Carolina, as part of the North Carolina Department of Public Instruction Special Improvement Project II Math Grant.

The program includes:

- 90 days' worth of lessons and activities, based on a block schedule
- Essential Standards and Clarifying Objectives for each unit
- Words to Know
- Day-by-day summaries
- Materials Lists
- Warm-Ups
- Direct and Guided Instruction
- Example Dialogue passages to model classroom discourse
- Worksheets and activity templates
- Formative Assessment journal prompts
- Unit Assessments
- Answer Keys

TEACHER'S GUIDE

Introduction

Structure of the Program

The *Introduction to High School Math* materials are completely reproducible. Tabs allow you to access the units quickly and easily. The program includes nine units representing the system of mathematics:

- Different Forms of Numbers
- Everything Based on Tens
- Simplifying Algebraic Expressions and Solving Equations
- Linear Functions
- Translating Verbal Problems into Equations
- Solving Inequalities
- Working with Data
- Simplifying Polynomials
- Working with Radicals

The lessons in the *Introduction to High School Math* program can be implemented on the day-by-day schedule that is built into the materials, but the design is flexible so that you can mix and match activities as the needs of your students dictate.

Structure of the Units

Each unit begins with Essential Standards and Clarifying Objectives, “Words to Know” vocabulary terms, a summary of the concepts to be addressed, a conceptual diagram of the unit activities as they reflect the concrete → conceptual → communication/representation sequence, a summary of each day’s activities, a materials list, career connections, and suggested unit projects. Each unit ends with a practice assessment, a unit assessment, and a complete answer key for all activities, worksheets, and assessments.

Structure of the Daily Lesson Materials

1. Summary of Day’s Activities and Topics

Overview of the day’s class including warm-up, number theory, and main lesson

2. Teacher Background

References to research and articles where appropriate

TEACHER'S GUIDE

Introduction

3. Learner Objective

Statement about what students will be able to do as a result of learning

4. Materials List

List of any resources, equipment, manipulatives, etc., needed for the day

5. Warm-Up Guide

Specific page references to activities in the Marilyn Burns Fraction Kit and/or implementation suggestions for various games and activities used as warm-ups

6. Formative Assessment Prompt

Journal prompts to elicit student thinking

7. Student Worksheet List

Listing of all student pages including class activities and practice/homework

8. Instruction Outline

Suggestions to the teacher for introducing and exploring instructional concepts or topics; usually includes some combination of activity and discussion

9. Example Dialogue

Snapshots of classroom conversation modeling appropriate math discourse. These are designed to give a sense of the appropriate tone to use with students, as well as to provide a sample of effective questioning and other strategies for supporting student thinking and discussion.

10. Student Practice and Activity Sheets

A list of all the sheets and forms needed for class activities and student assignments. All student pages are reproducible and/or may be found in student workbooks.

Standards Correlations

Instruction

The lessons and activities throughout *Introduction to High School Math* focus on the mathematics standards for the middle grades and provide students the opportunity to learn the prerequisite skills and concepts necessary to be successful in Algebra 1.

Each unit of the program explicitly addresses a series of Essential Standards and Clarifying Objectives. They are listed on the first page of each unit as seen in the example below.

Example Unit Correlation

UNIT 1 • DIFFERENT FORMS OF NUMBERS

Overview

Instruction

Essential Standards and Clarifying Objectives

Understand the relationship between decimals, fractions, and integers.

- Order decimals, fractions, and integers.

Represent percents as decimals, ratios, and fractions of a whole.

- Understand the relationship between percents, decimals, and fractions.
- Illustrate percentages of a whole using area models.

Apply unit rate to solve problems.

- Use ratio tables to represent unit rates.



Goal: To develop the concepts of ratio and rate of change in different forms, including percent and fractional forms of the unit/whole concept

Day-by-Day Overview

Unit	Day	Warm-Up	Number Theory	Main Lesson
Unit 1: Different Forms of Numbers	1	Fraction Kit: Cover Up	Discussion of rate	Trashketball: Courts of different lengths
	2	Fraction Kit: Cover Up	Discussion of measuring cup	Trashketball: Courts of same length, same number of shots
	3	Fraction Kit: Cover Up	Discussion of measuring cup	Trashketball: Courts of same length, different number of shots
	4	Make fraction flash cards, begin collecting and charting data	Associative property	Trashketball: Courts of same length, different number of shots
	5	Fraction flash cards	Distributive property	Bobcat stats
	6	Fraction flash cards	Review for assessment	Bobcat stats
	7	Trashketball Tournament Quiz	Review for assessment	Individual unit assessment with questions on Trashketball, Bobcat stats, measuring cup, predicting from rates
Unit 2: Everything Based on Tens	8	Basketball graphing activity: shoot baskets in gym to collect class data	Money Game I	Powers of 10 and scientific notation
	9	Basketball graphing activity: discussion on finding eyeball line of best fit and making predictions using that line	Money Game II	More scientific notation; finding 10% of something
	10	Basketball graphing activity: line of best fit and making predictions	Bar models/structure of addition	Proportions lesson: Jolly Ranchers®
	11	Fraction Kit: Uncover Version 1	Bar models/structure of subtraction	Scale drawings
	12	Scaling Away	Bar models/mixed addition and subtraction; optional quiz	Percent change (increase)
	13	Fraction Kit: Uncover Version 1	Distributive property (1×2); whole numbers/no variables	Percent change (decrease)

continued

TEACHER'S GUIDE

Day-by-Day Overview

Unit	Day	Warm-Up	Number Theory	Main Lesson
Unit 2: Everything Based on Tens	14	Fraction Kit: Uncover Version 2	Distributive property (2×2); whole numbers/no variables	Mixed practice with percent change: increase and decrease; practice assessment (finish for homework)
	15	Fraction Kit: Uncover Version 2	Review for assessment	Assessment
Unit 3: Simplifying Algebraic Expressions and Solving Equations	16	Journal prompt	Project: Design Your Bedroom	Hands-On Equations®, Lessons 1–3
	17	Fraction Kit: Cover the Whole	Project: Design Your Bedroom	Hands-On Equations®, Lessons 4–6
	18	Fraction Kit: Cover the Whole	Project: Design Your Bedroom	Hands-On Equations®, Lesson 7B
	19	Journal Prompt	Group project: Design a Living Room	Hands-On Equations®, Lessons 8–9
	20	Fraction Kit: Comparing Pairs	Group project: Design a Living Room	Hands-On Equations®, Lessons 10–11
	21	Fraction Kit: Comparing Pairs	Group project: Design a Living Room	Hands-On Equations®, Lesson 12
	22	Ice cream social to evaluate each other's projects using a scoring rubric		Hands-On Equations®, Lessons 13–14
	23	Fraction Kit: What's Missing?	Flex day for projects and assessment review	Hands-On Equations®, Lessons 15 and 16 (Lesson 16 is optional.)
	24	Fraction Kit: What's Missing?	Review for assessment	Hands-On Equations® review
Unit 4: Linear Functions	25	Fraction Kit: What's Missing?	Review for assessment	Assessment
	26	Introduce Conversion games; play Base 10 variation	Operations structures	Exploring the identity function with Dollar Deals—equation, table, graph
	27	Conversion game: Base 10	Operations structures	Dollar Deals with negative slope

continued

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Day-by-Day Overview

Unit	Day	Warm-Up	Number Theory	Main Lesson
Unit 4: Linear Functions	28	Conversion game: Measurement	Operations structures	Reinforcing the y -intercept (Optional projects located at end of unit: Leet's Laser Arcade vs. Ned's Ninja Arcade; Song Store)
	29	Fraction flash cards	Operations structures mixed review	Introduction to the Four Forms of an Equation (verbal, algebraic, tabular, graphic); given verbal form with slope and y -intercept, fill in the other three forms
	30	Fraction Kit: Pick Two	Distributive property (1×2); introduce unknown	More four forms of an equation (group activity)
	31	Fraction Kit: Pick Two	Distributive property (1×2); introduce unknown	Four forms of an equation
	32	Conversion: Measurement	Line of best fit	Four forms of an equation
	33	Conversion: Time	Line of best fit	Slope as the ratio of change in y to change in x
	34	Fraction Kit: Pick Three	Four forms mixed practice	Review for assessment
	35	Fraction Kit: Pick Three	Review for assessment	Assessment
	Unit 5: Translating Verbal Problems into Equations	36	March Madness and exponential growth	Distributive Property (2×2); introduce unknown
37		March Madness and exponential growth	Distributive Property (2×2) with unknown	Hands-On Equations® Verbal Problems 2
38		March Madness and exponential growth	Distributive Property (2×2); introduce symbolic representation	Hands-On Equations® Verbal Problems 3
39		More fun with exponential growth	Operation structures division	Hands-On Equations® Verbal Problems 4
40		Conversion game and exponential growth	Operation structures division	Hands-On Equations® Verbal Problems 5
41		Conversion game and exponential growth	Operation structures mixed practice	Review for assessment
42		Fraction flash cards	Review for assessment	Assessment

continued

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Day-by-Day Overview

Unit	Day	Warm-Up	Number Theory	Main Lesson
Unit 6: Solving Inequalities	43	Mixed practice and review	Operation Structures Mixed Practice	Graphing inequalities
	44	Mixed practice and review	Distributive Property (2×2); unknown/symbolic	Graphing inequalities
	45	Fraction Kit: Roll Five	Distributive Property (2×2); unknown/symbolic	Story problem: inequalities
	46	Fraction Kit: Roll Five	Distributive Property Mixed Practice	Story problem: inequalities
	47	Rational Numbers: Show Me What You Know!	Distributive property mixed practice	Mowing Lawns
	48	Conversion game: time	Integers and money	Mowing Lawns
	49	Fraction Kit: Make a Whole	Integers and football	Review for assessment
	50	Fraction Kit: Make a Whole	Review for assessment	Assessment
Unit 7: Working with Data	51	Fraction Kit: Roll Two	Stem-and-leaf plots	Describing distributions
	52	Fraction Kit: Roll Two	Making sense of rational numbers	Back-to-Back stem plots/comparing two distributions
	53	Flash cards; journal prompt	Histograms/comparing two distributions	Histograms on the calculator
	54	Fraction Kit: student choice	Rational numbers with the operations	Five-number summary/box-and-whisker plots
	55	Fraction flash cards	Rational numbers with the operations	Describing distributions from a box-and-whisker plot
	56	Conversion game: Measurement	Rational numbers with the operations	Box plots on the calculator
	57	Conversion game: Time	Rational numbers with the operations	Side-by-side box plots
	58	Quiz on stats graphs	Rational numbers with the operations	Mean as fair share
	59	Fraction Kit: student choice	Order of operations	Measures of center
	60	Fraction Kit: student choice	Order of operations	Measures of spread

continued

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Day-by-Day Overview

Unit	Day	Warm-Up	Number Theory	Main Lesson
Unit 7: Working with Data	61	Conversion game: Time	Order of operations	Finding outliers
	62	Order of operations puzzle	Order of operations	Effect of outliers on measures of center
	63	Simplifying fractions domino puzzle	Data project	Data project
	64	Conversion game: Measurement	Data project	Data project
	65	Contig	Data project	Data project
	66	Fraction Kit	Data project presentations	Data project presentations
Unit 8: Simplifying Polynomials	67	Fraction flash cards	Structure of real number system	<i>Algebra Tiles Workbook:</i> Introducing Modeling Polynomials, pp. 3–9
	68	Conversion game: Mega Tournament	Misuses of statistics: graphs	<i>Algebra Tiles Workbook:</i> Introducing The Zero Principle, pp. 10–16
	69	Contig	Misuses of statistics: sampling	<i>Algebra Tiles Workbook:</i> Introducing Adding & Subtracting, pp. 17–23
	70	Jeopardy-style review for course assessment	Misuses of statistics: surveys	<i>Algebra Tiles Workbook:</i> Introducing Adding & Subtracting, pp. 17–23
	71	Jeopardy-style review for course assessment	Misuses of statistics: surveys	Naming polynomials
	72	Jeopardy-style review for course assessment	Review of number theory for exam	Multiplying monomials
	73	Mixed practice and review	Review of number theory for exam	Dividing monomials
	74	Who Wants to Be a Millionaire? review game for course assessment	Review for assessment	Review for assessment
	75	Who Wants to Be a Millionaire? review game for course assessment	Review for assessment	Assessment
	76	Who Wants to Be a Millionaire? review game for course assessment	Review of number theory for exam	Perfect squares and square roots

continued

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Day-by-Day Overview

Unit	Day	Warm-Up	Number Theory	Main Lesson
Unit 9: Working with Radicals	77	Practice exam	Review of number theory for exam	Non-perfect squares/ estimating square roots of non-perfect squares
	78	Practice exam	Flex time	Non-perfect squares/ finding exact square root answers
	79	Games: student choice	Review practice exam	More finding exact square root answers
	80	Games: student choice	Review practice exam	Adding and subtracting square roots
	81	Games: student choice	Review practice exam	Pythagorean theorem
	82	Targeted practice worksheets	Flex time	Pythagorean theorem
	83	Targeted practice worksheets	Flex time	Pythagorean theorem: application problems
	84	Targeted practice worksheets	Review for assessment	Review for assessment
	85	Targeted practice worksheets	Review for assessment	Assessment
Review and Final Exam	Days 86–90			

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Ordering information for supplemental materials

Marilyn Burns Fraction Kit: Kits and replacement parts available from www.mathsolutions.com

The Hands-On Equations® Learning System: Kits and replacement parts available from www.borenson.com

Development Partners

This program is the result of a collaboration between Walch Education and the Wake County (N.C.) Public School System. It represents the vision, inspiration, and hard work of Valerie Ness Faulkner and Sonia Dupree, and the input of dozens of teachers who used the pilot materials with their students and provided feedback. *Introduction to High School Math, Version 2.0* reflects the commitment of the Wake County Public School System to providing all their high school students with effective mathematics education and the thought, time, and energy that they have invested in that endeavor.

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SAMPLE

Overview

Instruction

Essential Standards and Clarifying Objectives

Understand the relationship between decimals, fractions, and integers.

- Order decimals, fractions, and integers.

Represent percents as decimals, ratios, and fractions of a whole.

- Understand the relationship between percents, decimals, and fractions.
- Illustrate percentages of a whole using area models.

Apply unit rate to solve problems.

- Use ratio tables to represent unit rates.



Goal: To develop the concepts of ratio and rate of change in different forms, including percent and fractional forms of the unit/whole concept



WORDS TO KNOW

form of number

numbers can be expressed in different ways; the different forms provide different information (e.g., fractions, decimals, percents)

fraction

a number that names a part of a whole or a part of a group

integers

the set of whole numbers and their opposites

per

for each; for every

percent

the ratio of a number to 100; percent means “per hundred”

proportion

an equation that shows that two ratios are equal

rate

a ratio that compares two quantities having different units of measure

ratio

the comparison of two numbers by division

rational numbers

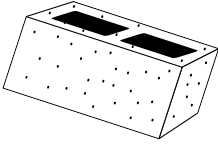

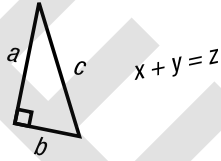
any number that can be written as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$

UNIT 1 • DIFFERENT FORMS OF NUMBERS

Overview

Instruction

Teacher Notes

Concrete Real world 	Sense-making Conceptual understanding 	Communication Representation 
Trashketball Basketball statistics Pricing	predictions, box scores per, per 100 decimal form fractional form percentage form	Free throws made/attempted 1 dollar per item 15 baskets per 100 $35/50 = .7 = 70\%$

Concepts

- A rate is a relationship between two quantifiable things (numbers).
- This relationship allows us to make better predictions.
- This relationship can be written in different ways.
- The same numerical information can be written in different forms.

Students do not tend to think proportionally, unless we develop this thinking for them explicitly, systematically, and with meaningful repetition.

Students need help in thinking through what numbers actually mean, particularly in the form of rates or percentages.

A rate is more than just a number in the sense that it represents a relationship, and not a raw number. For instance, if 80% of your students are on Facebook, you don't know how many of them are on Facebook. We don't know how many unless we have more information to apply the rate to.

Keep in mind that you and your students are used to just looking for an "answer" and not thinking about what the answer represents (i.e., not considering if it is a rate or a raw number).

Make that distinction clear by being consistent and accurate in presenting it.

UNIT 1 • DIFFERENT FORMS OF NUMBERS

Overview

Instruction

An experience that will illustrate rate and provide data for discussion and analysis is basketball, or “Trashketball.” Remind students that some people will make more baskets than others, but this is not a competition, just a way to collect data.

Summary of Activities

Day 1

- Marilyn Burns Fraction Kit—Cover Up Warm-Up
- Discussion of rate
- Introduction to Trashketball and data collection
- Make predictions for rates on different length courts.
- Discussion of how to compare rates on different courts

Day 2

- Marilyn Burns Fraction Kit—Cover Up Warm-Up
- Measuring cup modeling of fractions
- Students play Trashketball on courts of equal length to decide which team is best (equal number of shots).
- Discussion of rate and the accuracy of predictions

Day 3

- Marilyn Burns Fraction Kit—Cover Up Warm-Up
- Measuring cup modeling of fractions
- Trashketball: Students compare team numbers/rates for same number of shots and same-length courts.
- Identify the reasons for wanting to be able to compare “apples to apples”

Day 4

- Make Flashcards for Warm-Up
- Number Theory: Associative Property
- Students (in new teams) shoot different numbers of shots from the same length courts.
- Attempt to compare team numbers/rates
- Introduce decimals as another form of a number that lets you compare two quantities.

UNIT 1 • DIFFERENT FORMS OF NUMBERS

Overview

Instruction

Day 5

- Flash Cards Warm-Up
- Number Theory: Distributive Property
- Bobcats Basketball Statistics

Day 6

- Flash Cards Warm-Up
- More basketball statistics
- Practice Assessment

Day 7

- Trashketball Quiz
- Review
- Unit 1 Assessment

Materials List



- Several trash cans
- Masking tape
- Crumpled-up paper basketballs
- Clear measuring cup or beaker (at least 8 ounces)
- Colored water and extra container
- 25 index cards per student
- Markers (enough for 1 marker per student)
- Scissors (class set)
- Calculators (class set)
- Overhead pens/markers
- Hat or bowl to draw numbers from

Careers Connection

Discuss with students that many professions use rate information—everyone from a meteorologist who predicts wind speed in miles per hour or precipitation in inches per month, to stock brokers who deal with rate of growth or loss, to scientists and statisticians (in sports, finance, education, etc.).

Day 1

Instruction

Summary

- Introduce Marilyn Burns Fraction Kit
- Introduce and discuss concept of rate
- Students make rate predictions based on given data.

Learner Objective

As a result of learning, students should be able to explain the concept of rate.

Materials List

- Marilyn Burns Fraction Kit
- Materials for Trashketball: masking tape, crumpled-up paper basketballs, several trash cans

Warm-Up Activity

- Marilyn Burns Fraction Kit—Cover Up; see pp. 2–3 “Teaching the Games” and “Cover Up.” Use the worksheet on p. 24.

Student Worksheets

- Day 1 Warm-Up: Fraction Kit—Cover Up
- Day 1 Handout: Tracks & Rates
- Day 1 Activity: Trashketball Stats #1
- Day 1 Practice: Predicting Rate I
- Day 1 Practice: Rates I
- Day 1 Practice 2: Rates II

Direct Instruction

Introduction to Different Forms of Numbers

Explain to students that a rate is more than just a number. It **represents** a **relationship**, and not just a raw number. For example, discuss the idea of predicting how long something will take. How many minutes per lap? Miles per hour?

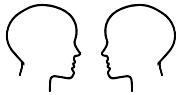
Discuss the term *per*, beginning with their estimates of minutes per lap and identifying other common uses (e.g. miles per hour, hours of work per week, cost per month, etc.). Show some examples on the board or overhead—55 miles per hour, 40 hours per week, \$79 per month for a cell phone, 4 minutes per mile (fast running), 9 inches per month (precipitation). Ask students for additional examples and record. Talk

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about predicting rates; in particular, how long it might take to walk around a track once, and then extending to what that suggests for how long it will take to walk twice, three times, and so on. Point out that knowing the rate does NOT tell you about the raw number. For instance, if I know you went 55 mph, does that mean I know how many miles you actually went? A rate gives us good information, but we need to understand what it is telling us.



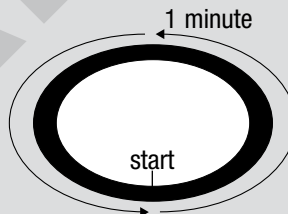
Example Dialogue (*Adapt as necessary.*)

What Is a Rate?

Let's get an idea of how long a minute really is. Everyone look up at the clock and be quiet for a minute. No talking, okay? We want to build our internal sense of what a minute is. What does that mean—our internal sense? It means that we could tell if about a minute went by even without a clock. Okay, ready. Let's wait until it hits the top and we'll all close our eyes and be quiet for a minute. While you're watching the clock inside your head, picture yourself walking around a track and see if you can picture how far along the track you could go in a minute. Ready, go.

Okay, so that's a minute. Turn to your Tracks & Rates page. The first example is someone who thought they would get about this far in a minute. Do you see that in the next minute, if she keeps going at that rate, she'd be here? Then she'd need about another minute to get all the way around. So she would get around the track in 3 min. Three minutes per lap. How far did you think you could get walking around the track? (Answers will vary greatly.) So, let's take X's guess. He thinks he can walk about half the track in a minute. If he walked half the track in a minute, how long would it take for him to walk a full lap? Use your worksheet to show X's guess.

$\frac{1}{2}$ lap per 1 min.



Notice the word “per.” What does that mean? It means that you will get that much of a lap done in 1 minute. Do you see how laps and minutes go together? They are related to each other. This is a rate. A rate can be written in different forms. You can use fractions or division, or you can write it out as a rate: **$\frac{1}{2}$ lap per 1 min.**

Figure out what X's prediction would be for a whole lap. Look at the picture. If it takes 1 minute to go $\frac{1}{2}$ lap, how long will it take to go the whole lap? . . . if we walk at the same rate for the second half, that will be another 1 minute. That's 2 minutes altogether: 2 min. per lap.

Now look at how far you thought you could get in a minute and come up with a prediction for how long it will take to walk around the whole track. Use your Tracks & Rates worksheet to show your thinking.

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Teacher Notes

Make the connection between laps per minute and miles per hour, stressing the use of *per*. Don't do any conversions—just engage students in a discussion about what miles per hour means, once they understand laps per a number of minutes.

Preparation for Trashketball

Mark off sets of three different basketball “courts” with masking tape indicating a shooting line and a wastebasket at each specified distance. You'll need a set of three courts for each group of 9–12 students, so create two sets for 18–24 students, and so on.

- Court A Distance: 3 feet
- Court B Distance: 6 feet
- Court C Distance: 12 feet

Direct Instruction

Develop the idea of rate by having students look at their data from different distances. You will compare these rates through the week to develop the ideas of **rate** and using rate to make a prediction. Focus the discussion on predictions and rates rather than on computation and conversions.

The three different distances will generate results that could be summarized as “always,” “never,” and “half the time.” Discuss what those words mean. Show students that those words represent **rates**. The terms suggest that at 3 feet, students will make every single shot, at 12 feet they'll make no shots, and at 6 feet they'll make every other shot.

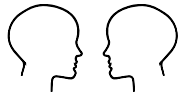
Explain to students that we usually use rates for situations in between **never** and **always**, to help us “quantify” and make sense of situations.

Discuss

- What kind of **rate** information do coaches of various sports use to make decisions about who plays? (batting average, field goal percentage, goals per game, others)
- What kind of **rate** information do teachers use to see who understands the material from class? (test scores)
- What kind of **rate** information do meteorologists use to predict weather? (inches of rain per day or month, wind miles per hour, percent of cloud cover)
- What other uses of **rate** information can you think of?

Review Trashketball Procedures

- Team members rotate shots
- After you shoot, you record your result
- Return to end of line for your next shot
- Team members stay positive with their teammates and with other teams

**Example Dialogue 2****What Is Predictable?**

There are 3 different courts set up. You will use these courts all week and on and off through this math unit. You are going to learn some very important information about **rates, percentages, decimals, and prediction** this quarter. It's important that you work efficiently to get the information you need. I expect that all of you will do well and we can do this in an orderly manner. We'll have fun with it if you all follow the directions. We are using this to learn mathematics, so I need you to be orderly and respond quickly when I call you back to order. Does everyone understand that?

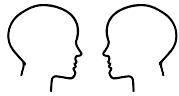
Teacher Notes

Make expectations very clear and be directive. For instance, do not have students crumple up their paper basketball until after they've been assigned groups, and assign one person from the group to make one basketball for the group. Have students move together in their groups, rather than individually. Divide students into sets of three teams with 3–4 members per team. As necessary, have two or three sets of teams and two or three Trashketball court set-ups. Each team gets one trashketball and takes turns shooting.

Give students a few minutes to name their teams and complete Group Name and Group Members on the Trashketball Stats #1 worksheet.

Have students take turns taking shots and recording the results for 30 shots from their masking tape line.

Show students how to complete their worksheets. Every time a shot is taken, the number gets circled—
③. When a shot is made, then put a slash through the circled number: ~~③~~. This way you can tell how many shots have been taken, and how many have been made.

**Example Dialogue 3**

Look at the information you've recorded (Trashketball Stats #1). Wow, the Jellystone Yogis did very well. They made almost all of their baskets and they are 27/30. That's awesome! How did the Fantastics do? 20/30. Not bad. And the Teenage Ninja Squirrels? How did they do? Wow. I'm sorry to hear that Squirrels, 5/30. Not very good. I guess the Squirrels aren't very good basketball players. Is that right? Right, we need to really analyze the situation and think about all the conditions. Good thinking.

Tomorrow you'll go through the next two courts and we'll compare rates to see how hard the different courts are. Now let's talk about predictions. Tomorrow we'll rotate through the courts so that your team will go to the 2 courts you did not go to today.

Today these are our numbers from those three courts:

Court A: 27/30

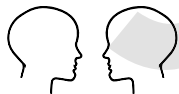
Court B: 20/30

Court C: 5/30

What are your predictions for what your team will do on the courts you have not been to yet? Write them down. Why did you choose those numbers? Squirrels, what do you think the Yogis will get tomorrow from Court C? 4? Out of how many? 30, good. Why did you pick 4 even though they got 20/30 today? How did you know how hard Court C was? You had some numbers, a rate by which to make your prediction.

Teacher Notes

Students will likely say at first "because the court is longer" and that's true. But it's important to help them to see that the information from the team shooting from that distance (5/30) is the best information (even better than the length of the court itself) and gives a rate to use to predict future rates. Continue with reviewing the predictions for tomorrow.

**Example Dialogue 4**

Tomorrow, we'll have even more information and we'll be able to make even better predictions about the courts.

NAME: _____

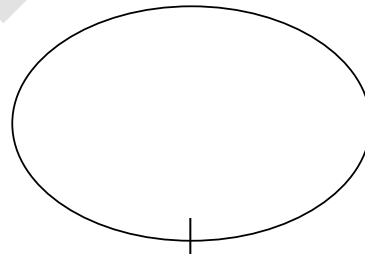
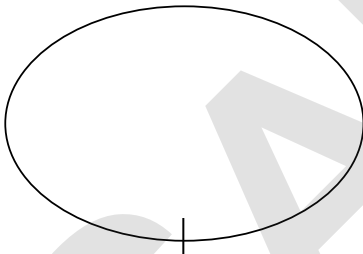
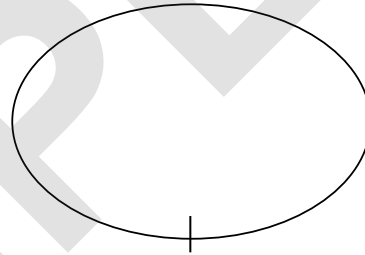
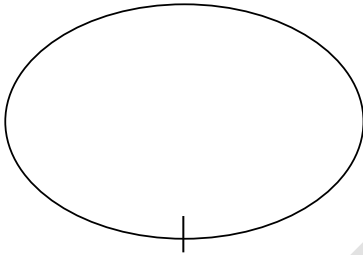
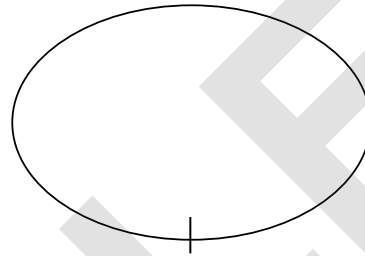
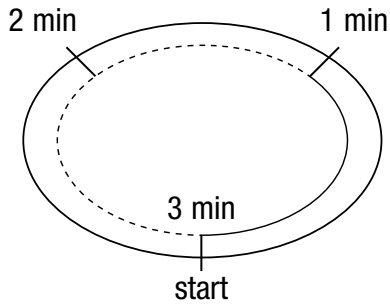
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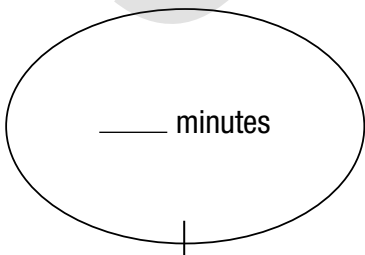
Day 1 Handout

Tracks & Rates

Example

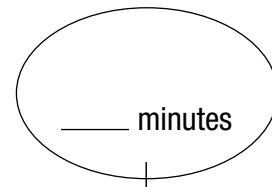
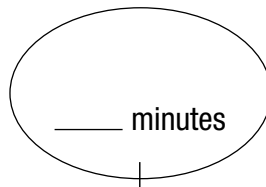
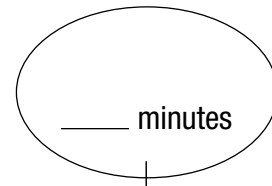
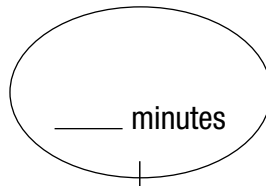


After you've walked the track, record your data:



about _____ minutes per lap

Use the tracks below to predict what will happen tomorrow:



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Trashketball Stats #1

Group name: _____

Group members: _____, _____, _____, _____

Prediction _____

Shots taken—Make or Miss: (On all three courts, indicate “taken” shot with a circle and “made” with a slash.)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Total made/Total taken _____/30

Court A

Prediction _____

Shots taken—Make or Miss:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Total made/Total taken _____/30

Court B

Prediction _____

Shots taken—Make or Miss:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Total made/Total taken _____/30

Court C

NAME: _____

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Day 1 Practice

Predicting Rate I

Read each scenario and make the prediction that follows. Show your work.

1. If it takes you 8 minutes to run a mile, predict how long it would take to run $2\frac{1}{2}$ miles.

2. It takes 22 minutes to build a snowman. Predict how long it would take in minutes to build 3 snowmen one at a time.

3. It takes 1.5 minutes for a printer to print 120 copies of a test. Predict how many copies the printer can print in 4.5 minutes.

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Rates I

Given the rate, make the prediction. Show your work.

1. It takes 3 minutes and 15 seconds to pop one bag of microwave popcorn. Predict how long it would take to pop 4 bags one at a time.

2. A compact fluorescent light bulb (CFL) will save about \$30 over its lifetime. Predict how much money 7 CFL bulbs would save.

3. A three-legged race team takes 2 minutes and 10 seconds to reach the finish line. Predict how long it would take the same team to run the race 3 times.

4. A printer can print 65 pages in 2 minutes. Predict how many pages the printer can print in 6 minutes.

continued

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5. A child can ride her tricycle 1 block in $5 \frac{1}{2}$ minutes. Predict how long it will take her to ride 8 blocks.

6. It takes 6 hours and 30 minutes to install new tile in the kitchen. Predict how long it would take to install the same tile in 3 kitchens of the same size.

7. It takes 2 hours 15 minutes to drive from Asheville to Charlotte. Predict how long it would take to drive back and forth 3 times.

8. It takes a hair stylist 44 minutes to cut and style one customer's hair. Predict how long it would take him to cut and style hair for 6 customers.
