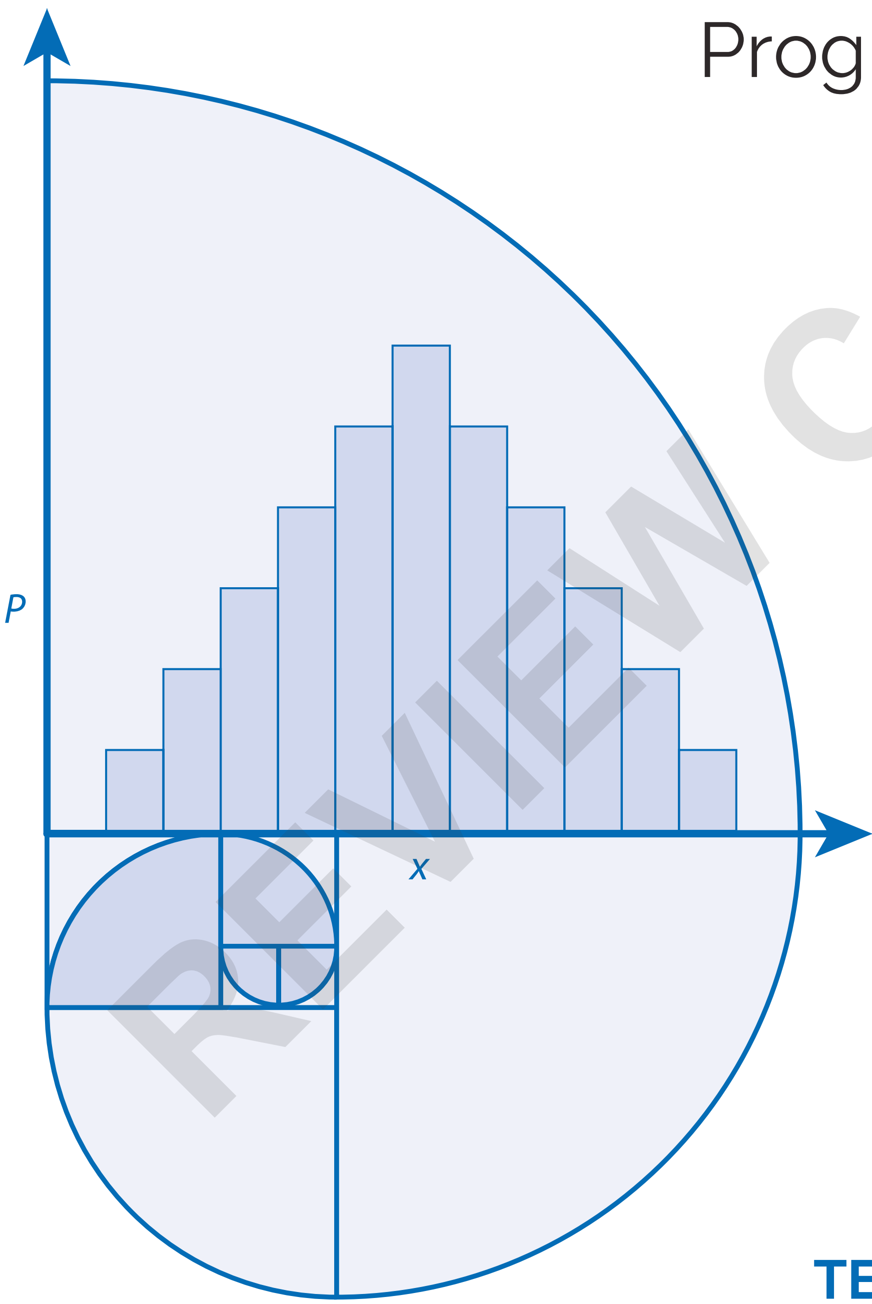
South Carolina CCR Mathematics

Geometry

Program Overview



TEACHER RESOURCE

BW WALCH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

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Introduction to the Program

Introduction

The *South Carolina CCR Mathematics Standards: Geometry* program is a complete set of materials developed around the South Carolina College and Career-Ready (SC CCR) Mathematics Standards. Topics are built around accessible core curricula, ensuring that the South Carolina College and Career-Ready Mathematics Standards Geometry Program is useful for striving students and diverse classrooms.

This program realizes the benefits of exploratory and investigative learning and employs a variety of instructional models to meet the learning needs of students with a range of abilities.

The *South Carolina CCR Mathematics Standards: Geometry* program includes components that support problem-based learning, instruct and coach as needed, provide practice, and assess students' skills. Instructional tools and strategies are embedded throughout.

The program includes:

- More than 150 hours of lessons addressing the SC CCR Mathematics Standards
- Essential Questions for each instructional topic
- Vocabulary
- Instruction and Guided Practice
- Problem-based Tasks and Coaching questions
- Step-by-step graphing calculator instructions for the TI-Nspire and the TI-83/84
- Station activities to promote collaborative learning and problem-solving skills

Purpose of Materials

The *South Carolina CCR Mathematics Standards: Geometry* program has been organized to coordinate with the SC CCR Math Standards Geometry content map and specifications from the SC CCR Mathematics Standards.

Each topic includes activities that offer opportunities for exploration and investigation. These activities incorporate concept and skill development and guided practice, then move on to the application of new skills and concepts in problem-solving situations. Throughout the lessons and activities, problems are contextualized to enhance rigor and relevance.

This program includes all the topics addressed in the South Carolina Geometry content map. These include:

- Transformations, Congruence, and Proof
- Triangle Congruence, Proof, and Constructions
- Similarity, Proof, and Trigonometry
- Extending to Three Dimensions
- Connecting Algebra and Geometry Through Coordinates
- Circles With and Without Coordinates
- Statistics and Probability

The five Mathematical Process Standards are infused throughout:

- **MPS.PS.1:** Make sense of problems and persevere in solving them strategically.
- **MPS.RC.1:** Explain ideas using precise and contextually appropriate mathematical language, tools, and models.
- **MPS.C.1:** Demonstrate a deep and flexible conceptual understanding of mathematical ideas, operations, and relationships while making real-world connections.
- **MPS.AJ.1:** |Use critical thinking skills to reason both abstractly and quantitatively.
- **MPS.SP.1:** Identify and apply regularity in repeated reasoning to make generalizations.

Structure of the Teacher Resource

The *South Carolina Geometry* program is completely reproducible. Online materials can be provided in your Learning Management System (such as Canvas or Schoolology) or in BW Walch's proprietary course management platform, the Curriculum Engine. The nested folder organization in the Curriculum Engine allows you to access the materials quickly and easily. The digital format also facilitates printing and copying student pages and/or making assignments online.

The Program Overview is the first section. This section helps you to navigate the materials, offers a collection of research-based Instructional Strategies along with their literacy connections and implementation suggestions, and shows the correlation between the South Carolina CCR for Mathematics and the South Carolina Geometry course.

The remaining materials focus on content, knowledge, and application of the units in the South Carolina Geometry curriculum: Transformations, Congruence, and Proof; Triangle Congruence, Proof, and Constructions; Similarity, Proof, and Trigonometry; Extending to Three Dimensions; Connecting Algebra and Geometry Through Coordinates; Circles With and Without Coordinates; and Statistics and Probability. These units are designed to be flexible so that you can mix and match activities as the needs of your students and your instructional style dictate.

The Station Activities correspond to the content in the units and provide students with the opportunity to apply concepts and skills, while you have a chance to circulate, observe, speak to individuals and small groups, and informally assess and plan.

Each topic begins with a pre-assessment and ends with a progress assessment. These allow you to assess students' progress as you move from topic to topic, enabling you to gauge how well students have understood the material and to differentiate as appropriate.

Glossary

The Glossary contains vocabulary terms and formulas from throughout the program, organized alphabetically. Each listing provides the term and the definition in both English and Spanish. The listings include the lesson number(s) where the terms can be found in the Words to Know.

Correspondence to NCTM *Principles to Actions* Teaching Practices

How Do Walch's Mathematics Resources Address the NCTM *Principles to Actions* Mathematics Teaching Practices?

Walch's programs for the South Carolina College- and Career-Ready Standards were designed by experienced educators and curriculum developers, informed by best-practice research, and refined through an iterative process of implementation and feedback. Together with professional development, these materials support and sustain good teaching practices.

NCTM Mathematics Teaching Practices	Relevant Attributes of Walch Integrated Math Resources
Establish mathematics goals to focus learning.	Each lesson in Walch's programs addresses specified standards which can be used as goals to focus learning. Essential Questions offer further focus.
Implement tasks that promote reasoning and problem solving.	Each lesson in Walch's programs is built around a Problem-Based Task (PBT), set in a meaningful real-world context and designed to promote reasoning and problem solving. The courses include dozens of PBTs as well as warm-up and practice problems.
Use and connect mathematical representations.	Walch's mathematics programs make frequent use of, and connections among and between, equations, tables, and graphs. PBTs often require students to use and connect two or more of these representations, and the representations are modeled through guided practice.
Facilitate meaningful mathematical discourse.	Several features of the programs support mathematical discourse, including warm-up debriefs with connections to upcoming lessons, optional coaching questions for the PBTs, and discussion guides for Station Activities. Explanations of PBT solutions are another opportunity for discourse. Please note: Mathematical discourse is an important topic for professional development, in conjunction with implementation of these materials.
Pose purposeful questions.	The coaching questions and discussion guides provide samples of purposeful questions. Note that this is another important topic for professional development.
Build procedural fluency from conceptual understanding.	The programs develop conceptual understanding through modeling, guided practice, and application, and then provide additional opportunities to practice and develop fluency.
Support productive struggle in learning mathematics.	The PBTs require "productive struggle;" coaching questions provide an option for additional support as appropriate, allowing students to proceed through the task and ensuring that the struggle remains productive rather than too frustrating.
Elicit and use evidence of student thinking.	Various discussions and PBTs require students to display their thinking. Coaching sample responses offer specific prompts and suggestions for eliciting and responding to student thinking. Professional development supports teachers in using that evidence to respond in instructionally appropriate ways.

PROGRAM OVERVIEW

Unit Structure

All of the instructional units have common features. Each unit begins with a Unit Overview listing all the standards addressed in the topics. The Unit Overview also includes Essential Questions; vocabulary (titled "Words to Know"); and lists of recommended websites and conceptual activities to be used as needed.

Each topic begins with a pre-assessment, lists the specific vocabulary, Essential Questions, and resources for that topic, and ends with a progress assessment to evaluate students' learning.

Each lesson begins with a list of identified prerequisite skills that students need to have mastered in order to be successful with the new material in the upcoming lesson. This is followed by an introduction, key concepts, common errors/misconceptions, scaffolded practice problems, guided practice examples, a problem-based task with coaching questions and sample responses, a closure activity, and practice.

All of the components are described below and on the following pages for your reference.

Pre-Assessment

This can be used to gauge students' prior knowledge and to inform instructional planning.

South Carolina College- and Career-Ready Standards for the Topic

All standards that are addressed in the entire topic are listed.

Essential Questions

These are intended to guide students' thinking as they proceed through the topic. By the end of each topic, students should be able to respond to the questions.

Words to Know

Vocabulary terms and formulas are provided as background information for instruction or to review key concepts that are addressed in the topic.

Recommended Resources

This is a list of websites that can be used as additional resources. Some websites are games; others provide additional examples and/or explanations. (*Note*: Links will be monitored and repaired or replaced as necessary.) Each Recommended Resource is also accessible through Walch's cloud-based Curriculum Engine Learning Object Repository as a separate learning object that can be assigned to students.

Conceptual Activities

Conceptual understanding serves as the foundation on which to build deeper understanding of mathematics. In an effort to build conceptual understanding of mathematical ideas and to provide more than procedural fluency and application, links to interactive open education and Desmos resources are included. (*Note*: These website links will be monitored and repaired or replaced as necessary.) These and many other open educational resources (OERs) are also accessible through the Learning Object Repository as separate objects that can be assigned to students.

Warm-Up

Each warm-up takes approximately 5 minutes and addresses either prerequisite and critical-thinking skills or previously taught math concepts.

South Carolina College- and Career-Ready Standards for the Lesson

When topics are broken down into lessons, the specific standard or standards that are addressed are presented at the beginning of the instructional portion of the lesson.

Warm-Up Debrief

Each debrief provides the answers to the warm-up questions, and offers suggestions for situations in which students might have difficulties. A section titled Connection to the Lesson is also included in the debrief to help answer students' questions about the relevance of the particular warm-up activity to the upcoming instruction. Warm-Ups with debriefs are also provided in PowerPoint presentations.

Identified Prerequisite Skills

This list cites the skills necessary to be successful with the new material.

Introduction

This brief paragraph gives a description of the concepts about to be presented and often contains some Words to Know.

Key Concepts

Provided in bulleted form, this instruction highlights the important ideas and/or processes for meeting the standard.

Graphing Calculator Directions

Step-by-step instructions for using a TI-Nspire and a TI-83/84 are provided whenever graphing calculators are referenced.

Common Errors/Misconceptions

This is a list of the common errors students make when applying Key Concepts. This list suggests what to watch for when students arrive at an incorrect answer or are struggling with solving the problems.

Scaffolded Practice (Printable Practice)

This set of 10 printable practice problems provides introductory level skill practice for the lesson. This practice set can be used during instruction time.

Guided Practice

This section provides step-by-step examples of applying the Key Concepts. The three to five examples are intended to aid during initial instruction, but are also for individuals needing additional instruction and/or for use during review and test preparation.

Enhanced Instructional PowerPoint (Presentation)

Each lesson includes an instructional PowerPoint presentation with the following components: Warm-Up, Key Concepts, and Guided Practice. Selected Guided Practice examples include GeoGebra applets. These instructional PowerPoints are downloadable and editable.

Problem-Based Task

This activity can serve as the centerpiece of a problem-based lesson, or it can be used to walk students through the application of the standard, prior to traditional instruction or at the end of instruction. The task makes use of critical-thinking skills.

Optional Problem-Based Task Coaching Questions with Sample Responses

These questions scaffold the task and guide students to solving the problem(s) presented in the task. They should be used at the discretion of the teacher for students requiring additional support. The Coaching Questions are followed by answers and suggested appropriate responses to the coaching questions. In some cases answers may vary, but a sample answer is given for each question.

Recommended Closure Activity

Students are given the opportunity to synthesize and reflect on the lesson through a journal entry or discussion of one or more of the Essential Questions.

Problem-Based Task Implementation Guide

This instructional overview, found with selected Problem-Based Tasks in each unit, highlights connections between the task and the lesson's key concepts and Mathematical Practices. The Implementation Guide also offers suggestions for facilitating and monitoring, and provides alternative solutions.

Printable Practice (Sets A and B) and Interactive Practice (Set A)

Each lesson includes two sets of practice problems to support students' achievement of the learning objectives. They can be used in any combination of teacher-led instruction, cooperative learning, or independent application of knowledge. Each Practice A is also available as an interactive Learnosity activity with Technology-Enhanced Items.

Progress Assessment

Each lesson ends with 10 multiple-choice questions, as well as one extended-response question that incorporates critical thinking and writing components. This can be used to document the extent to which students grasp the concepts and skills addressed during instruction.

Unit Assessment

Each unit ends with 12 multiple-choice questions and three extended-response questions that incorporate critical thinking and writing components. This can be used to document the extent to which students grasped the concepts and skills of each unit.

Answer Key

Answers for all of the Warm-Ups and practice problems are provided at the end of each unit.

Station Activities

Most units include a collection of station-based activities to provide students with opportunities to practice, reinforce, and apply mathematical skills and concepts. The debriefing discussions after each set of activities provide an important opportunity to help students reflect on their experiences and synthesize their thinking.

Conceptual Tasks

These engaging tasks provide opportunities for students to deepen their understanding and develop their conceptual knowledge of math concepts. These tasks provide multiple entry points and are accessible for ALL learners.

Standards Correlations

Each unit in this *South Carolina CCR Mathematics Standards: Geometry* program was written specifically to address the South Carolina College- and Career-Ready Standards. Each topic lists the standards covered in all the lessons, and each lesson lists the standards addressed in that particular lesson. In this section, you'll find a comprehensive list mapping the lessons to the South Carolina College- and Career-Ready Standards.

Unit 1: Transformations, Congruence, and Proof			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Introducing Transformations**		
	1.1	Defining Terms	GS.MGSR.2.1
	1.2	Transformations As Functions	GS.MGSR.2.1
	1.3	Applying Lines of Symmetry	GS.MGSR.2.1
Topic B	**Rotations, Reflections, and Translations**		
	1.4	Defining Rotations, Reflections, and Translations	GS.MGSR.2.2
	1.5	Applying Rotations, Reflections, and Translations	GS.MGSR.2.2
Topic C	**Exploring Congruence**		
	1.6	Describing Rigid Motions and Predicting the Effects	GS.MGSR.3.1
	1.7	Defining Congruence in Terms of Rigid Motions	GS.MGSR.3.1
Topic D	**Proving Theorems About Lines and Angles**		
	1.8	Proving the Vertical Angles Theorem	GS.MGSR.5.1
	1.9	Proving Theorems About Angles in Parallel Lines Cut by a Transversal	GS.MGSR.5.1

Unit 2: Triangle Congruence, Proof, and Constructions			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Congruent Triangles**		
	2.1	Triangle Congruency	GS.MGSR.3.2
	2.2	Explaining ASA, SAS, SSS, AAS, and HL	GS.MGSR.3.3
Topic B	**Proving Theorems About Triangles**		
	2.3	Proving the Interior Angle Sum Theorem	GS.MGSR.5.1 GS.MGSR.5.2
	2.4	Proving Theorems About Isosceles Triangles	GS.MGSR.5.2
	2.5	Proving the Midsegment of a Triangle	GS.MGSR.5.2
	2.6	Proving Centers of Triangles	GS.MGSR.5.2
Topic C	**Proving Theorems About Parallelograms**		
	2.7	Proving Properties of Parallelograms	GS.MGSR.5.3
	2.8	Proving Properties of Special Quadrilaterals	GS.MGSR.5.3

Unit 3: Similarity, Proof, and Trigonometry			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Investigating Properties of Dilations**		
	3.1	Investigating Properties of Parallelism and the Center	GS.MGSR.4.1
	3.2	Investigating Scale Factors	GS.MGSR.4.1
Topic B	**Defining and Applying Similarity**		
	3.3	Defining Similarity	GS.MGSR.4.3
	3.4	Applying Similarity Using the Angle–Angle (AA) Criterion	GS.MGSR.4.3
Topic C	**Proving Similarity**		
	3.5	Proving Triangle Similarity Using Side-Angle-Side (SAS and Side-Side-Side (SSS) Similarity	GS.MGSR.4.3
	3.6	Working with Ratio Segments	GS.MGSR.4.2
	3.7	Proving the Pythagorean Theorem Using Similarity	GS.MGSR.4.2
	3.8	Solving Problems Using Similarity and Congruence	GS.MGSR.4.3
	3.9	Special Right Triangles	GS.MGSR.4.3

Topic	Lesson number	Title	Standard(s)
Topic D	**Exploring Trigonometric Ratios**		
	3.10	Operating with Radical Expressions	GS.NR.1.1
	3.11	Working with Rational Exponents	GS.NR.1.1
	3.12	Defining Trigonometric Ratios	GS.MGSR.6.2 GS.MGSR.6.1 GS.MGSR.6.3
	3.13	Exploring Sine and Cosine As Complements	GS.MGSR.6.2
Topic E	**Applying Trigonometric Ratios**		
	3.14	Calculating Sine, Cosine, and Tangent	GS.MGSR.6.3 GS.MGSR.6.4
	3.15	Problem Solving with the Pythagorean Theorem and Trigonometry	GS.MGSR.6.5

Unit 4: Extending to Three Dimensions			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Explaining and Applying Area and Volume Formulas**		
	4.1	Circumference and Area of a Circle	GS.MGSR.1.1 GS.MGSR.1.2 GS.PAFR.1.2
	4.2	Volumes of Cylinders, Pyramids, Cones, and Spheres	GS.MGSR.1.1 GS.MGSR.1.2 GS.PAFR.1.2
	4.3	Exploring Volume and Surface Area in Context	GS.PAFR.1.2 GS.PAFR.2.1
Topic B	**Geometric Modeling**		
	4.4	Two-Dimensional Cross Sections of Three-Dimensional Objects	GS.MGSR.1.2 GS.MGSR.1.3
	4.5	Cavalieri's Principle	GS.MGSR.1.2 GS.MGSR.1.3

Unit 5: Connecting Algebra and Geometry Through Coordinates			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Slope and Distance**		
	5.1	Using Coordinates to Prove Geometric Theorems with Slope and Distance	GS.PAFR.2.3 GS.PAFR.3.1
	5.2	Working with Parallel and Perpendicular Lines	GS.MGSR.5.1 GS.PAFR.2.2
Topic B	**Points on Line Segments**		
	5.3	Midpoints and Other Points on Line Segments	GS.MGSR.5.1
Topic C	**Calculating Perimeter and Area**		
	5.4	Calculating Perimeter and Area	GS.PAFR.3.2

Unit 6: Circles With and Without Coordinates			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Introducing Circles**		
	6.1	Similar Circles and Central and Inscribed Angles	GS.MGSR.7.1
	6.2	Chord Central Angles Conjecture	GS.MGSR.7.1
	6.3	Properties of Tangents of a Circle	GS.MGSR.7.2
Topic B	**Inscribed Polygons and Circumscribed Triangles**		
	6.4	Constructing Inscribed Circles	GS.MGSR.7.2
	6.5	Constructing Circumscribed Circles	GS.MGSR.7.2
	6.6	Proving Properties of Inscribed Quadrilaterals	GS.MGSR.7.2
Topic C	**Constructing Tangent Lines**		
	6.7	Constructing Tangent Lines	GS.MGSR.7.2
Topic D	**Finding Arc Lengths and Areas of Sectors**		
	6.8	Defining Radians	GS.PAFR.1.1
	6.9	Deriving the Formula for the Area of a Sector	GS.PAFR.1.1

Unit 7: Statistics and Probability			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Summarizing, Representing, and Interpreting Data**		
	7.1	Data Types and Visualizations	GS.DPSR.1.1
	7.2	Comparing Different Data Sets	GS.DPSR.1.1
	7.3	Interpreting Data and Recognizing Outliers	GS.DPSR.1.1
	7.4	Interpreting Fitted Functions	GS.DPSR.1.2 GS.DPSR.1.3
	7.5	Distinguishing Between Correlation and Causation	GS.DPSR.2.1
Topic B	**Representing Sample Spaces and General Rules**		
	7.6	Drawing and Interpreting Venn and Tree Diagrams	GS.DPSR.3.1 GS.DPSR.1.3 GS.DPSR.3.2 GS.DPSR.3.3
	7.7	Identifying Sample Spaces	GS.DPSR.3.1
	7.8	Identifying Events	GS.DPSR.1.3 GS.DPSR.3.1
	7.9	The Addition Rule	GS.DPSR.3.2
	7.10	The Multiplication Rule	GS.DPSR.3.3

Instructional Strategies

Ensuring Access for All Students

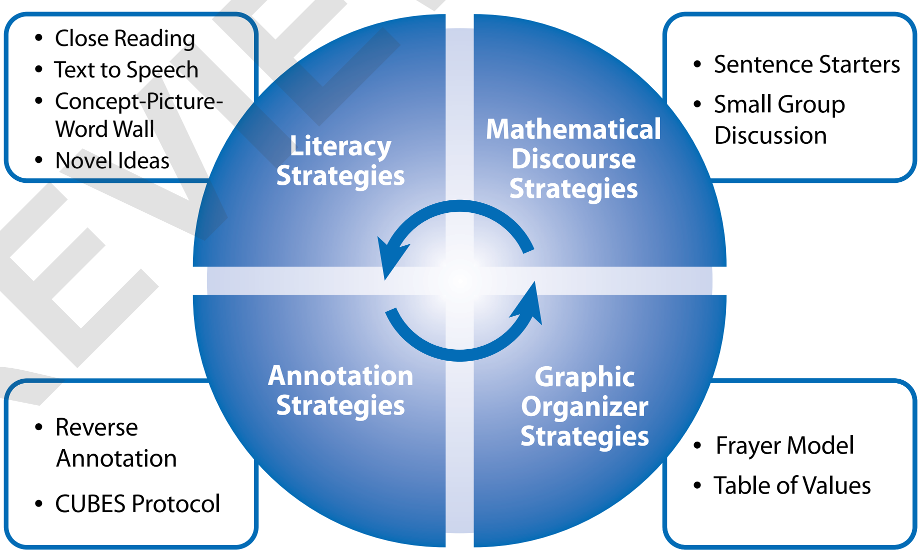
Introduction

The increased focus on literacy in math instruction can help some students navigate mathematical contexts, but for struggling readers, it can further complicate calculations. English language learners struggle to master difficult mathematical concepts while simultaneously processing a new language. Students with learning and behavioral disabilities struggle with the math concepts in their own contexts. This is where teachers and the strategies they select for their classrooms become essential.

The strategies presented here can help all students succeed in math, literacy, school, and, ultimately, in life. These instructional strategies provide teachers with a wide range of instructional support to aid English as a Second Language (ESL) students, students with disabilities (SWD), and struggling readers. These strategies provide support for the Mathematics Standards and the Mathematical Practices (MP), English Language Development (ELD) Standards, English Language Arts Standards, and WIDA English Language Development Standards.

Within each lesson throughout this course, you will find suggested instructional strategies. These instructional strategies are research-based strategies and best practices that work well for all students.

The instructional strategies detailed here fall into four main categories: Literacy, Mathematical Discourse, Annotation, and Graphic Organizers. These strategies provide teachers with research-based strategies to address the needs of all students.



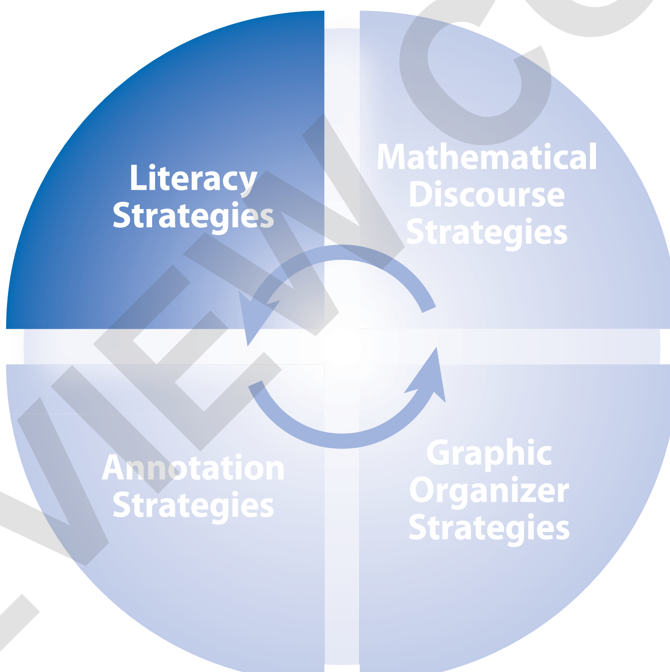
Mathematical Modeling

Source

- WIDA: https://www.walch.com/rr/09052

Understanding the Language of Mathematics: Literacy

Mathematics has its own language consisting of words, notations, formulas, and visuals. In education, the language of mathematics is often regarded solely in the context of word problems and articles. This neglects the vocabulary and other mathematical representations students must be able to interpret. The strategies presented here help students navigate the language of mathematics so that they can understand text and feel confident speaking in and listening to mathematical discussions. For students with disabilities, the stress on repetition and different representations in this approach is essential to their ability to grasp the math concepts. For ESL students, repetition and different representations can strip out some of the English language barriers to understanding the language of mathematics, as well as provide multiple means of accessing the content. Literacy strategies include Close Reading, Text-to-Speech, Concept-Picture-Word Walls, and Novel Ideas.



Literacy Strategies

Close Reading with Guiding Questions

What is Close Reading with Guiding Questions?

Close Reading with Guiding Questions is a process that allows students to preview mathematical reading and problems by answering questions related to the text in advance and reviewing their responses during and/or after reading. Multiple reading protocols can be used in conjunction with guiding questions to enhance their effectiveness.

How do you implement Close Reading with Guiding Questions in the classroom?

When utilizing a textbook, task, or article in a math class, literacy struggles are often a strong barrier to entry into the mathematical ideas. Asking students to answer accessible questions before and/or as they read can lead them to the key information.

Prior to implementation, the teacher should determine the most important information students need to obtain from a text, whether it is a math problem to solve, a task to complete, or an informational lesson or article to read. Then, the teacher should come up with some questions to guide students before they read. These questions can:

- assess and relate prior knowledge
- define key vocabulary words
- discuss non-mathematical concepts in the text

The teacher should also prepare some questions to guide students as they read. These questions can:

- point out key concepts within the text
- relate the text and concepts to future learning
- assist students in identifying key facts in the text
- highlight the importance of text features (graphics, headings, etc.) in the text

To ensure the questions are accessible for students and to encourage reflection and debate after reading, many of these questions should be designed as either "True/False" or "Always True/ Sometimes True/Never True." Students can represent their reasoning for their answer in writing, numbers, or graphic/pictorial representations. Students should complete the guiding questions and reading individually, with discussion to follow.

After students complete the reading, they should be given some time to individually evaluate their initial answers. Then, in partners or in groups, they can discuss their answers and come to final conclusions that will help them find the important information initially identified by the teacher. After deciphering the text through close reading, students will be able to complete the given activity.

When would I use Close Reading with Guiding Questions in the classroom?

Close Reading with Guiding Questions can be used for any activity in which literacy could be a barrier to learning or demonstrating mastery of mathematical concepts. The number of questions and length of the discussions can be altered based on the length, importance, and difficulty of the text and concept. As students become more accustomed to mathematical literacy, the text complexity can be increased, but the adherence to close reading strategies must be maintained to ensure students can access the mathematical concepts. The length of time spent on the literacy aspect can be shortened as students become more skilled, but the questioning and discussions must occur to ensure students are properly interpreting the text in the mathematical context.

How can I use Close Reading with Guiding Questions with students needing additional support?

For struggling readers, including ESLs, Close Reading with Guiding Questions can help make an intimidating lesson, word problem, or task much more accessible. Questions focusing more on Tier 2 and Tier 3 vocabulary, text features, and real-world concepts can help struggling readers relate to the text and learn how to decipher the text in context. Discussions around the questions will help students grasp the math concepts.

Allowing struggling readers to explain their answers using words, numbers, or graphics/pictures ensures that they can express their opinion and rationale despite a potential lack of vocabulary. Through these representations and the ensuing discussion, students will begin to learn the necessary vocabulary to be successful.

What other standards does Close Reading with Guiding Questions address?

Mathematical Practices:

- G.MP.1
- G.MP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7

Sources

- Anne Adams, Jerine Pegg, and Melissa Case. “Anticipation Guides: Reading for Mathematics Understanding.” https://www.walch.com/rr/09053
- Diane Staehr Fenner and Sydney Snyder. “Creating Text Dependent Questions for ELLs: Examples for 6th to 8th Grade.” https://www.walch.com/rr/09054

Literacy Strategies

Text-to-Speech Technology

What is Text-to-Speech Technology?

Text-to-Speech Technology is an adaptive technology that reads text aloud from a text source for students. It is usually accessed through an application or program on a computer, smartphone, or tablet. Some new programs utilize Mathematical Markup Language (MathML) to read mathematical notation in a common, understandable manner for students. Many programs also highlight the words and notation on the screen as the audio plays, which helps students relate the written representation to the words they hear. The use of Text-to-Speech Technology allows students who struggle with literacy to hear the words and notation and access the text in a different way.

How do you implement Text-to-Speech Technology?

A classroom community focused on everyone's learning and a growth mindset is the first step in implementing Text-to-Speech Technology. One of the main barriers to implementation is encouraging students to use the program. Once they do, they will realize how the audio can help them understand the difficult mathematical texts and interpret the math content within them. After students realize the benefits of Text-to-Speech Technology, it can become part of the regular routine for group and independent work.

The use of headphones can be very important for effective use of Text-to-Speech Technology. Students can use the technology to listen to lessons and texts at their own pace. Extra noise from other students working or other students listening at different paces can confuse students attempting to use Text-to-Speech Technology, and headphones can help mitigate these distractions. Many teachers are nervous about the potential disruption headphones can cause in class. However, well-managed use of headphones can help students successfully utilize the technology to learn.

When would I use Text-to-Speech Technology in the classroom?

Text-to-Speech Technology can be used at any time throughout the year, and if the program speaks in MathML, it can be used with any lesson. Without MathML, effective use could be limited to word problems without unusual notation. For example, if x^2 is read as "*x*-two" instead of "*x*-squared" or "*x* to the second power," that could confuse students more.

During a lesson or small group discussion, Text-to-Speech Technology could detract from students' ability to listen, question, and process information. However, during warm-ups, independent work, or assessments, Text-to-Speech Technology can help students process the information and access the activity. It can become a routine for students to automatically listen to the question, problem, or directions first, and then attempt the activity.

How can I use Text-to-Speech Technology with students needing additional support?

Text-to-Speech Technology is an important adaptation and accommodation for struggling readers. Students who have read-aloud accommodations sometimes don't receive them because they are either embarrassed to accept them or because of staffing restrictions. These students can use Text-to-Speech Technology to supplement their math instruction by having text automatically read to them in a manner in which they can process it.

Additionally, for ESL students, hearing the English mathematical language, especially referring to mathematical representations and notation, can help put English words to the ideas they see. Some Text-to-Speech Technology can translate written and mathematical text into other languages, so students can hear the text in their natural language and see the English highlighted on the screen as they hear it. In this way, students are learning English vocabulary as well as learning the mathematical content in a language they can understand.

What other standards does Text-to-Speech Technology address?

Mathematical Practices:

- G.MP.1
- G.MP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7

Source

- Steve Noble. "Using Mathematics eText in the Classroom: What the Research Tells Us." https://www.walch.com/rr/09055

Literacy Strategies

Concept-Picture-Word Wall

more> than
Concept/Word Cards

What is a Concept-Picture-Word Wall?

A Concept-Picture-Word Wall is a classroom display, often a bulletin board or a set of posters, that exposes students to important vocabulary words they will use in math class.

Posting vocabulary words in class helps reinforce the words students will see in textbooks, videos, websites, and test questions on math concepts. These Tier 3 vocabulary words are often not used in everyday language, and the exposure to the words visually through Concept-Picture-Word Walls can help students connect them to the math content.

How do you implement Concept-Picture-Word Walls in the classroom?

Just seeing the vocabulary on a Concept-Picture-Word Wall by itself will help students; more importantly, referring to the words as the teacher uses them in class helps students connect the visual to the application. A simple gesture to the wall makes a very explicit reference to the word as it is used and allows students to connect the unfamiliar word to its meaning in context. Additionally, students can be taught to refer to the wall as they use the words in class, and they can be asked to make sure they say at least 3 words from the wall during each class period in small-group discourse or as answers to whole-class questions. The comfort gained from using these Tier 3 words will help students to use appropriate math vocabulary while solving problems and will help students connect concepts more explicitly.

Postings on the Concept-Picture-Word Wall can be arranged strategically to connect concepts, units of study, or groups of words where appropriate. Having three sections of the Concept-Picture-Word Wall—for example, an "In the Future" section, a "Live in the Present" section, and a "Remember the Past" section—can help students see and remember the vocabulary throughout the entire course. Even without regular use of some words, just seeing the words before a unit can help instill a familiarity with the vocabulary. Leaving the words on the Concept-Picture-Word Wall after a unit is taught can help students connect "old" concepts to the current lesson and ensure that students still have access to the vocabulary.

When would I use Concept-Picture-Word Walls in the classroom?

Concept-Picture-Word Walls can be used for the entire year. The actual words might have to change, or at least be moved to different areas of the Concept-Picture-Word wall. The more exposure students have to the words, the more familiar and comfortable they will become. The constant exposure to the math context is beneficial for students throughout the entire course, especially for words with multiple meanings (bias, tangent, etc.) that could exist as Tier 2 words in everyday conversation but are Tier 3 words in the math classroom.

How can I use Concept-Picture-Word Walls with students needing additional support?

For all students learning mathematics, knowing and using the math vocabulary is often a major barrier. This is a problem especially for ESL students, who are learning the English language along with math content. If teachers try to simplify the words too much for students, it does them a disservice as they seek out information from other teachers, textbooks, and online sources that use the proper vocabulary. Most tests, especially state tests, will expect students to have knowledge of the Tier 3, math-specific vocabulary. The more students see these words, the more familiarity they will have when they apply them.

Concept-Picture-Word Walls can also be written in multiple languages. Especially for students who are on-grade-level in their native language, a multi-lingual Concept-Picture-Word Wall can help students connect the content they already know in another language to the English vocabulary necessary for success on English-language math activities and tests.

This website can help you get started on an English-Spanish Concept-Picture-Word Wall: https://www.walch.com/rr/09056

What other standards do Concept-Picture-Word Walls address?

Mathematical Practices:

- G.MP.1
- G.MP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7

Source

- Janis M. Harmon, Karen D. Wood, Wanda B. Hedrick, Jean Vintinner, and Terri Willeford. "Interactive Word Walls: More Than Just Reading the Writing on the Walls."

  https://www.walch.com/rr/09057

Literacy Strategies

Novel Ideas

What is Novel Ideas?

Novel Ideas is a classroom activity that explores students' understanding of important Tier 2 vocabulary words they will use in math class. Instead of asking students to look up vocabulary words in the dictionary, Novel Ideas allows students to have conversations with their peers about vocabulary words in class. This reinforces the mathematical vocabulary students will see in textbooks, videos, websites, and test questions. These Tier 2 vocabulary words are often used in everyday language, but have specific meaning in mathematics. Exposure to the words through Novel Ideas can help students connect them to the math content.

How do you implement Novel Ideas in the classroom?

While building a rich representation of math content words and connecting the words to other words and concepts has inherent merit, it is more important to consider that pre-teaching the words before they are used in class helps students connect to the application. The understanding gained from discussing these Tier 2 words will help students apply them in a mathematical context to solve problems and connect concepts.

Here is a step-by-step process for implementing Novel Ideas:

1. Students separate into groups of four.
2. Students copy the teacher generated prompt/sentence starters and number their papers 1–8.
3. One student offers an idea, another echoes it, and all write it down.
4. After three minutes, students draw a line under the last item in the list.
5. All students stand, and the teacher calls one student from a group to read the group's list.
6. The student starts by reading the prompt/sentence starters, "We think a _______ called _______ may be about ... ," and then adds whatever ideas the team has agreed on.
7. The rest of the class must pay attention because after the first group has presented all their ideas, the teacher asks them to sit down and calls on a student from another team to add that team's "novel ideas only." Ideas that have already been presented cannot be repeated.
8. As teams complete their turns and sit down, each seated student should record novel ideas from other groups below the line that marks the end of his or her team's ideas.

When would I use Novel Ideas in the classroom?

Novel Ideas can be used for the entire year. The more students are exposed to mathematical vocabulary, the more familiar and comfortable they become, leading to increased usage of these math terms in their conversation and writing. Using math vocabulary in context is beneficial for students throughout the entire course, especially for words with multiple meanings (bias, tangent, etc.) that could exist as Tier 2 words in everyday conversation but are Tier 3 words in the math classroom.

How can I use Novel Ideas with students needing additional support?

Most tests, especially state tests, will expect students to have knowledge of the Tier 3, math-specific vocabulary. The more students use these words in conversation, the more familiarity they will have when they apply them. Understanding Tier 2 words also helps students avoid misconceptions in mathematics. Twice a week before the start of a lesson, allow students to use sentence starters in small groups that include all students. Prepare the sentence starter "When I hear the word _______, I think about _______" to share out with whole class. This will allow students who know the vocabulary words to share their knowledge, and will allow other students to hear the meaning of the vocabulary words. This strategy is particularly helpful for ESL students.

What other standards does Novel Ideas address?

Mathematical Practices:

- G.MP.1
- G.MP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7

Sources

- Colorín Colorado. "Selecting Vocabulary Words to Teach English Language Learners."

  https://www.walch.com/rr/09058

- Elsa Billings and Peggy Mueller, WestEd. "Quality Student Interactions: Why Are They Crucial to Language Learning and How Can We Support Them?"

  https://www.walch.com/rr/09059

Novel Ideas Sentence Starters

Slope

- When I hear the word climb, I think about …
- When I hear the word steep, I think about …

Volume

- When I hear the word filling, I think about …

Equations

- When I hear the word balance, I think about …
- When I hear the word equal, I think about …

Graphing

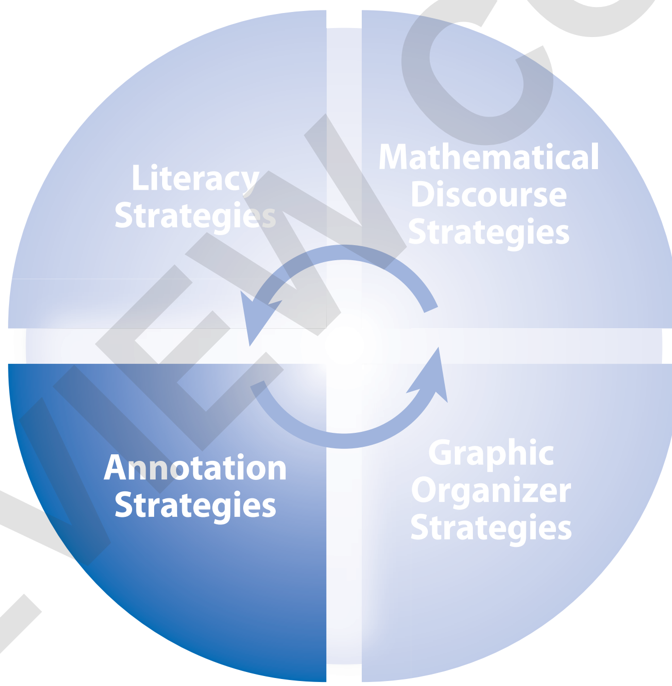
- When I hear the word grid, I think about …
- When I hear the word graph, I think about …

Scatter Plots

- When I hear the word scattered, I think about …

Understanding Mathematical Content: Annotation

Understanding mathematical content is an extremely important skill, both in the math classroom and in life. When students read word problems, articles, charts, graphs, equations, tables, or other forms of mathematical text, they must be able to decode and extract meaning from the text. Annotation can help. The strategies presented here help students identify and focus on key characteristics and facts from various forms of text while ignoring the non-essential information. For students with disabilities, many of whom struggle with the distractions inherent in many high-school level texts, making notes and drawing pictures to explain a problem can help them focus. ESL students will be pointed to certain Tier 3 vocabulary words and determine which Tier 2 vocabulary words they must learn to be proficient in math class and in the English language. Annotation strategies include Reverse Annotation and CUBES protocol.



Annotation Strategies

Reverse Annotation Protocol

What is Reverse Annotation?

Reverse Annotation is a strategy that asks students to identify and write down key information from math problems. This is especially helpful for problems given on a computer or tablet, where students can't annotate directly on the problem. A template is given at the end of this section.

How do you implement Reverse Annotation in the classroom?

Many annotation strategies ask students to write, underline, or mark directly on the text of a problem. While those forms of annotation are also beneficial, they are not always possible with technology. Whether the problem is given on paper or using technology, having students write the answers to these questions will ensure that they are thinking strategically and specifically about the strategies and information needed to solve the problem.

The three questions at the top of the Reverse Annotation template are the key to understanding mathematical problems. For every problem given in class, ask students:

1. What is the problem asking us to solve?
2. What key words tell us the mathematical steps we need to perform?
3. What information in the problem can help us figure it out?

After answering the initial questions, students should make a guess, or estimate, of what they think the answer will be. This helps grow their number sense, and provides an initial, reasonable solution to guide their work. Students can then use the strategies they selected to solve the problem and evaluate their solution using the questions at the bottom of the template.

When students first begin to use Reverse Annotation, the teacher should walk them through the steps individually to ensure they can accurately identify the question, key words, and important information. Teachers can also lead students through the estimation process, making a game out of which student has the closest estimate.

Work through each step individually for several "easy" problems first, so that difficult math doesn't interfere with the process. Increase the problem difficulty incrementally as students begin to master the process. This may seem like a long process at first, but the ultimate result is worth the time investment.

When would I use Reverse Annotation in the classroom?

Reverse Annotation can be used to solve any math problem, and is especially helpful for word problems. When Reverse Annotation is initially implemented, the steps should be discussed in detail. As students become accustomed to Reverse Annotation and begin thinking about problems in this manner automatically, the individual steps become less important and can be scaffolded out to improve efficiency. Students should reach the point where they immediately ask themselves the three initial questions when they first see a problem. However, the teacher should ensure that students are truly evaluating all the key information before routine discussions of the individual steps are removed.

How can I use Reverse Annotation with students needing additional support?

Annotation strategies can help students identify key information, even when certain vocabulary words are not known. As teachers introduce the content-specific Tier 3 vocabulary to their classes, annotation strategies such as reverse annotation can help students use these words to apply appropriate strategies while problem solving. Answering the three initial questions can help students organize the key facts and vocabulary, and the identification of key information can simplify the problem. This strategy is especially beneficial for ESL students.

Using reverse annotation with graphic organizers benefits ESL students by removing a lot of the confusing wording and allowing them to focus on the important pieces of a problem. When using Reverse Annotation, all students, including ESL students, will begin to think about problem solving in a way that encourages them to use the appropriate information to find a solution.

What other standards does the Reverse Annotation Protocol address?

Mathematical Practices:

- G.MP.1
- G.MP.2
- G.MP.5
- G.MP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.SL.9–10.3
- ELA–LITERACY.SL.9–10.2
- ELA–LITERACY.RST.9–10.4

Source

- Alliance for Excellent Education. “Six Key Strategies for Teachers of English Language Learners.” https://www.walch.com/rr/09060

Reverse Annotation Template

Name: ______________________________ **Problem/Assignment:** ______________________________

Analyze the Problem

What is the problem asking us to solve?	
What key words will tell us the mathematical steps we need to perform?	
What information in the problem can help us figure it out?	

Initial estimate of solution:

Work Space

Remember to box in your solution!

Name: ______________________________ Problem/Assignment: ______________________________

Check It Over

How close was your estimate?	
Does your answer make sense? Is it reasonable? How do you know?	
Did you perform the calculations correctly?	
What does your answer mean in context?	

Annotation Strategies

CUBES Protocol

What is the annotation strategy CUBES?

CUBES is an annotation strategy in which students use different written designs to highlight the key aspects of word problems. It can help them choose the correct mathematical strategy to solve the problem accurately.

How do you implement CUBES in the classroom?

The steps for CUBES are:

1. **C**: **C**ircle all the key numbers.
2. **U**: **U**nderline the question.
3. **B**: **B**ox in the key words that will determine the operation(s) necessary and write the mathematical symbol for the operation(s).
4. **E**: **E**valuate the information given to determine the strategy needed. **E**liminate any unnecessary information.
5. **S**: **S**olve the problem, **s**how your work, and check your answer.

As students learn to use CUBES, walk them through the steps individually to ensure they can accurately identify the key numbers, question, key words, unnecessary information, and strategy. Work through each step individually for several "easy" problems first, so that difficult math doesn't interfere with the process. Increase the problem difficulty incrementally as students begin to master the process. This may seem like a long process at first, but the ultimate result is worth the time investment.

A graphic organizer can help students master the process, especially when problems are given on a computer or tablet where students can't always annotate directly on the problem. Students can write down the key numbers and circle them, write down the question and underline it, and so on. This will encourage students to truly think about the different pieces of the problem they are identifying, and how these pieces will guide the strategy and affect the solution.

When would I use CUBES in the classroom?

CUBES can be used to solve any math problem, and is especially helpful for word problems. When CUBES is initially implemented, the steps should be discussed in detail. As students become accustomed to using CUBES and begin thinking about problems in this manner automatically, the individual steps become less important and can be scaffolded out to improve efficiency. However, the teacher should ensure that students are truly evaluating all the key information before routine discussions of the individual steps are removed.

How can I use CUBES with students needing additional support?

Design features can help students identify key words and features, even when certain vocabulary words are not known. As teachers introduce the content-specific Tier 3 vocabulary to their classes, annotation strategies such as CUBES can help students use these words to apply appropriate strategies while problem solving. Using circles, underlines, and boxes can help students organize the key facts and vocabulary, and the elimination of unnecessary information can simplify the problem. This strategy is especially beneficial for ESL students.

Combining CUBES with graphic organizers also benefits ESL students by removing a lot of the confusing wording and allowing them to focus on the important facts of a problem. When using CUBES with a graphic organizer, all students, including ESL students, will begin to think about problem solving in a way that helps encourage them to use the appropriate information to find a solution.

What other standards does the CUBES Protocol address?

Mathematical Practices:

- G.MP.1
- G.MP.2
- G.MP.5
- G.MP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

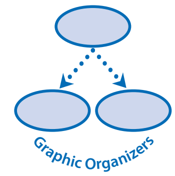
- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.SL.9–10.3
- ELA–LITERACY.SL.9–10.2
- ELA–LITERACY.RST.9–10.4

Source

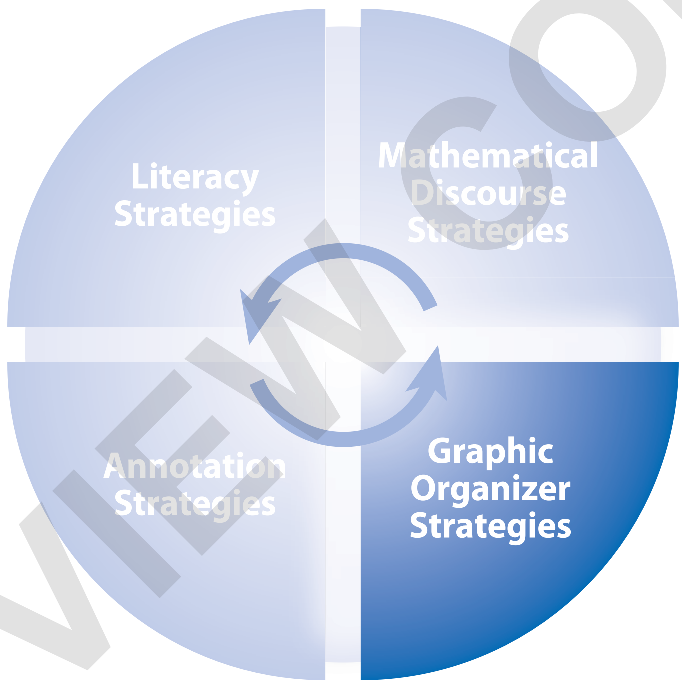
- Margaret Tibbett. “Comparing the effectiveness of two verbal problem solving strategies: Solve It! and CUBES.”

  https://www.walch.com/rr/09061

Organizing Mathematical Content: Graphic Organizers



Organizing mathematical content is a crucial skill for problem solving, exploring other possible methods for finding solutions, and managing math content. All students need strategies for organizing content to build conceptual understanding. For students with disabilities, visual representations and graphic organizers can help them clarify their thoughts and focus on the math. ESL students also benefit from visual representations and graphic organizers. Organizing mathematical knowledge with visuals can help ESL students navigate math content while learning the language. Graphic organizers include Frayer Models and Tables of Values.



Graphic Organizers

Frayer Models

What is a Frayer Model?

A Frayer Model is a graphic organizer that can help students understand new vocabulary words and concepts by exploring their characteristics. A Frayer model lists the definition of a word or concept, describes some key facts, and gives examples and non-examples. Examples and non-examples can come from a mathematical or real-world context.

How do you implement Frayer Models in the classroom?

Students can learn to create Frayer Models the first week of school, and the process can be used throughout the year each time students experience a new word or concept.

While it is important for teachers to give students precise mathematical definitions with appropriate content vocabulary, it is maybe more important for students to understand the application of mathematical words and concepts in their own context. As students learn new information, small group discussions and think-pair-share activities are great ways for students to formulate their own definitions, review the characteristics and facts they have learned, and discuss examples and non-examples.

Discussions of the examples and non-examples can help lead to the mathematical definition. For example, if students use a Frayer Model to define a quadratic function, they would notice that all examples have a highest exponent of 2, and all non-examples would not have a highest exponent of 2. All examples would have parabolic graphs, and all non-examples would have other graphs. Through these comparisons, students will understand the definition of quadratics using different representations, and they will be able to apply it in different contexts.

When would I use Frayer Models in the classroom?

Frayer Models can be used at different points during instruction. They are appropriate as introductions to new concepts, summaries to ensure understanding of new concepts, or as note-organizers throughout the lesson for students to fill in as they learn new concepts. At first, students might need help figuring out how to list and differentiate between the definition, facts and characteristics, examples, and non-examples. As students adapt to the process, they will be able to categorize information on their own or in small groups. As they compare newer Frayer Models to previous models, they will also be able to see how concepts build upon each other.

How can I use Frayer Models with students needing additional support?

Frayer Models can be a point of reference for students as they progress throughout the year. As students determine their own definitions for math-specific words and concepts, and use the examples and non-examples to determine the key facts, they will be able to put them in their own context and apply them to solve complicated problems. As math concepts build upon each other both within a unit and throughout the year, the use of Frayer Models to remind students of their initial definitions of words or concepts can help solidify their understanding. Using Frayer Models as part of a Word Wall or Concept Wall, or having a consistent notebook process to reference past Frayer models, can help consistently reinforce learning.

What other standards do Frayer Models address?

Mathematical Practices:

- G.MP.1
- G.MP.2
- G.MP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.1
- ELA–LITERACY.SL.9–10.1
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4

Source

- Deborah K. Reed. “Building Vocabulary and Conceptual Knowledge Using the Frayer Model.” https://www.walch.com/rr/09062

Frayer Model

<table>
<tr><td>Definition</td><td>Characteristics</td></tr>
<tr><td colspan="2">WORD</td></tr>
<tr><td>Examples from Life</td><td>Non-Examples</td></tr>
</table>

Graphic Organizers

Tables of Values

What is a Table of Values?

A Table of Values is an organized way to list numbers that represent different categories of values. These values can be represented as ordered pairs, graphs, word problems, or lists. Tables can help students see and compare values in a different way.

How do you implement Tables of Values in the classroom?

Tables can be used throughout the year to support various mathematical standards. Some standards mention tables specifically, and in others, tables can be an effective support to help students organize and understand the meaning and application of values.

Tables can be set up with numerical values in rows or columns. The key to understanding the values lies in the headings. The headings must be specific enough to show students the meaning and/or application of the numerical values, but not so wordy that they interfere with the clarity of the numbers in the table. For example:

x (year)	y (population in millions)
1960	219
1970	230
1980	258
1990	312
2000	342

Mean (statistical average)	50	45
Median (middle value)	52	43
Quartile 1 (median of the lower 50%)	40	38
Quartile 3 (median of the upper 50%)	72	80
Range (difference of max and min values)	80	61
Interquartile Range (difference of quartiles)	32	42
Standard Deviation (measure of spread of data)	7.24	10.23

When would I use Tables of Values in the classroom?

Various mathematical topics can be represented by tables. For example:

- An (x, y) table of values to represent coordinates on a graph or independent and dependent variables for a given context
- A table to represent coefficients and/or constants in an equation
- A table to show different statistical measures when comparing sets of data
- A table to compare output values for the same input given different functions

Each time numbers or values are being listed, compared, or graphed, a table can help students differentiate between the values. Tables are easy to create, and students can be encouraged to create them as another representation to clarify and compare numbers for nearly any topic.

How can I use Tables of Values with students needing additional support?

Tables of Values can help students focus on numerical values and their meaning in context without distraction. They clarify what each number represents, what numbers can be compared, and what ordered pairs can be graphed to give a visual representation. Additionally, headings can be used to either highlight the relevant facts from a context or to describe mathematical vocabulary.

In general, graphic organizers benefit students by removing much of the confusing wording and focusing on the important facts and numbers of a problem.

What other standards do Tables of Values address?

Mathematical Practices:

- G.MP.1
- G.MP.2
- G.MP.6

WIDA English Language Development Standards:

- ELD Standard 3

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.1
- ELA–LITERACY.SL.9–10.1
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4

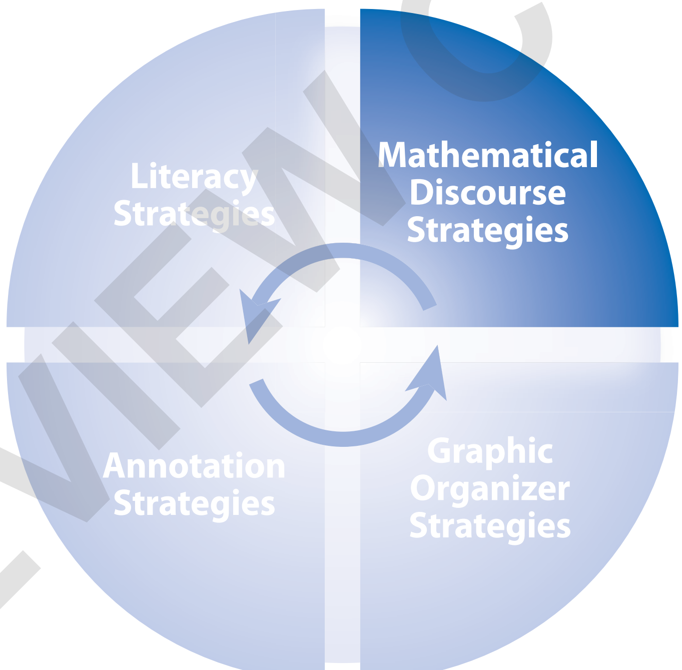
Source

- Alliance for Excellent Education. "Six Key Strategies for Teachers of English Language Learners."

  https://www.walch.com/rr/09063

Communicating Mathematical Content: Mathematical Discourse

Reading, writing, speaking, and listening are all important ways to learn and express information, but the last two ways are often slighted in the math classroom. The mathematical discourse strategies presented here promote speaking and listening in a math-focused literacy context. Working these strategies into the daily routine of a classroom can help students become comfortable speaking and listening in a mathematical context, which will help them become comfortable with the mathematical content. Routines and structures are essential to support students with disabilities, as they often benefit from following a routine. This can lead to developing capability in their mathematical skills. These strategies also remove the barrier to entry for many ESL students, as structure and routine can help them focus on the math content rather than English language deficiencies. Mathematical Discourse strategies include Sentence Starters and Small Group Discussion.



Mathematical Discourse Strategies

Sentence Starters

What is a Sentence Starter?

A Sentence Starter is a common phrase or mathematical sentence frame that can help students begin and sustain academic conversations around mathematical content. It helps guide students through the discussion and bring out pertinent ideas that can lead to greater understanding.

How do you implement Sentence Starters in the classroom?

Many people view math class as a place to calculate solutions to math problems. However, to ensure the conceptual understanding and proper application of a math concept, students need to be able to explain the concepts and reasoning behind a solution to a problem. As many students are not accustomed to having academic conversations about math, sentence starters can help begin and continue these conversations in a productive manner.

There are two main types of sentence starters for mathematical discussions: discourse starters and math starters. For example, a poster with these or other sentence starters can be displayed from the beginning of the year, and the expectation can be set that any answer to a question or comment in a discussion should be framed using one of these starters. As students become accustomed to framing mathematical conversations in this way, they can expand on the given sentence starters and create some of their own. They will begin to realize how these statements ensure that their conversations revolve around math, enhance understanding of the concept, and force them not only to state, but also to explain their thinking. They will gain confidence from the ability to engage, as the first step has already been taken for them.

When would I use Sentence Starters in the classroom?

Sentence Starters can be used throughout the entire school year with any concept. However, they are most important to use at the beginning of the school year to build a mathematical community in the classroom centered on a comfort with mathematical discourse. Especially at the beginning of the year, students should be encouraged to use these sentence starters for every math statement. Appropriate settings include during small group discussion, while responding to whole class questions, and when writing explanations for problem solutions.

Modifications can be introduced so that students must use certain mathematical vocabulary within the sentences, or must use certain sentence starters at different points in conversations or for different conversation types and situations. However the starters are implemented, it is important for students to realize that these are intended to enhance and focus their conversations, not limit them.

How can I use Sentence Starters with students needing additional support?

Often, students are reluctant to talk about math concepts because they either lack confidence in their knowledge, are afraid to be "wrong," or don't know how to start or continue the conversation. Sentence starters can help students overcome this reluctance. The non-threatening, easy-to-interpret sentence starters remove the barrier to entry for students who don't know how to engage, and the respectful, mathematical focus promoted by sentence starters can help build confidence and provide a structure so that students will not fear being wrong.

For ESL students specifically, sentence starters can provide the English language support to help students engage with and discuss the math. The support of sentence structure removes language barriers to entry for students who don't fully understand English sentence structure.

Discourse Starters	Math Starters
I agree/disagree with … because …	My answer was … because …
I understand/don't understand …	The next step is … because …
First/Next/Finally I … because …	I used (insert formula/equation/concept) because …
I noticed that …	
I wonder …	My answer is right/reasonable because …

What other standards do Sentence Starters address?

WIDA English Language Development Standards

- ELD Standard 3

Mathematical Practices:

- G.MP.1
- G.MP.3
- G.MP.6

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.1
- ELA–LITERACY.SL.9–10.1
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4

Source

- AVID. "Sentence Starters."
  https://www.walch.com/rr/09064

Mathematical Discourse Strategies

Small Group Discussion

What is Small Group Discussion?

Small Group Discussion is a structured way for students to verbalize their mathematical thinking in a comfortable setting to solve a problem, build conceptual understanding, or summarize a concept.

How do you implement Small Group Discussion?

Small Group Discussion in math class depends on a trusting relationship between the teacher and the students. From there, students can build trusting relationships among themselves. Once this trust has been built, students will feel free to explore mathematical topics in groups, take risks, and engage in a productive struggle toward understanding or a solution.

Once these relationships have been established, certain structures should be established for Small Group Discussion to be effective. Discussion norms can be set by the class to ensure discussions are respectful and productive, and discussions should have predetermined time limits. The group composition is also important and should be based on instructional measures. For different activities, homogeneous groups, heterogeneous groups, or groups based on specific data by standard could be appropriate. Students should always be aware that the groups were chosen to maximize their learning.

Another structure that can be effective for Small Group Discussion is assigning group roles. These roles can include group leader, note taker, timekeeper, resource manager, culture keeper, or other roles determined to be appropriate for the classroom context. During the discussion, assigning each student a letter within the group (A, B, C, D, etc.) can help structure the discussion. Different roles can specify certain time limits for talk, which sentence starters to use, or other structured aspects of the discussion.

When implementing a Small Group Discussion, the question or task should inspire students to think in different ways about a concept. Through the structured format of the discussion, students will compare their ideas and arrive at an answer or explanation of the concept. Within the trusting framework of the class and group, students can focus on the common goal of the discussion and develop their thinking around the math concept. These rich discussions will enhance their understanding.

When would I use Small Group Discussion in the classroom?

Small Group Discussion can be used for nearly any topic, and it can be used at a variety of times in the classroom. The questions and tasks may need to change depending on when it is used. Opening activities for lessons can be Small Group Discussions where students explore properties of new math concepts or review/build upon their prior learning. Turn and talks throughout the lesson can be structured as Small Group Discussions if a consistent framework is in place. At the end of class, a Small Group Discussion can be used to come to a common understanding about an essential question from the lesson.

Depending on when the Small Group Discussion is used in class, and what the goal of the discussion is, the discussion reporting may vary. For a warm-up, each group might be asked to share their thinking. For a guided practice, recording answers on chart paper and a gallery walk could be appropriate. For a closing activity, individual written responses to a question could be appropriate.

How can I use Small Group Discussion with students needing additional support?

As discussed in other Mathematical Discourse strategies, struggling students are reluctant to talk about math concepts because they lack confidence in their knowledge and don't always have the needed vocabulary in their toolbox. Structured discussions with effective grouping can help students through these barriers. After a trusting and respectful classroom environment has been established, struggling students often feel more comfortable sharing their ideas with just a few classmates rather than the whole class. Additionally, adding structure can help students engage by providing the expectation that they participate in the process.

The intentional grouping of students can also help them succeed using Small Group Discussion. At times, heterogeneous groups could be appropriate so that stronger students can help struggling students, and at other times, homogeneous groups could be appropriate so the teacher can work with an entire group of struggling students. ESL students can be grouped with other students with the same dominant language to help remove the language barrier from the conversation.

What other standards does Small Group Discussion address?

WIDA English Language Development Standards:

- ELD Standard 3

Mathematical Practices:

- G.MP.1
- G.MP.3
- G.MP.6

Language Arts Standards:

- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.1
- ELA–LITERACY.SL.9–10.1
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4

Source

- Jessie C. Store. "Developing Mathematical Practices: Small Group Discussions."
  https://www.walch.com/rr/09065

Modeling Strategies

Mathematical Modeling

What is Mathematical Modeling?

Mathematical modeling is generally understood as the process of applying mathematics to a real-world problem with a view of understanding the connection. According to the CCSSM, mathematical modeling is the ability to apply concepts learned in class to real-world applications and to use the model to analyze a situation, draw conclusions, and make predictions.

How do you implement Mathematical Modeling in the classroom?

Modeling can be implemented by demonstrating how to make or generate mathematical representations or models, how to validate them, and how to use them to solve real-world problems. There are many ways to show understanding in a math classroom, such as using words, drawings or sketches, physical models, computer programs, or math formulas.

The following is a list of questions and answers suggested in order to create a mathematical modeling classroom environment:

- **Why?** What are we looking for? Identify the need for the model.
- **Find?** What do we want to know? List the data we are seeking.
- **Given?** What do we know? Identify the available relevant data.
- **Assume?** What can we assume? Identify the circumstances that apply.
- **How?** How should we look at this model? Identify the parameters.
- **Predict?** What will our model predict? Identify the equations that will be used, the calculations that will be made, and the answers that will result.
- **Valid?** Are the predictions valid? Identify tests that can be made to validate the model; i.e., is it consistent with its principles and assumptions?
- **Verified?** Are the predictions good? Identify tests that can be made to verify the model; i.e., is it useful in terms of the initial reason it was done? *(inspired by Carson and Cobelli, 2001)*

Teachers should expect these questions to recur often during the modeling process, and should regard this list as a fairly general approach to ways of thinking about mathematical modeling.

In a classroom where mathematical modeling is the expectation, teachers will need to establish that students are responsible for coming up with methods for solving the problems presented and that the teacher will only assist and facilitate.

When would I use Mathematical Modeling in the classroom?

It should come as no surprise that many students find mathematics boring. The most common question posed to any mathematics teacher is "When will I ever need to use this?" Often teachers fail to find problems in which students are interested or to even take student interest into account when planning a lesson. Problems that spark students' interest and curiosity will increase their attention and desire to learn. These types of real-world problems provide students an opportunity to think and respond as a mathematician. Students should be exposed to rigorous learning tasks that allow opportunities for mathematical modeling in the classroom.

How can I use Mathematical Modeling with struggling students?

When struggling readers, which includes ELLs and students with learning disabilities, are exposed to rigorous math learning tasks, there must be a level of scaffolding that includes coaching and guided questions that help to make a word problem or learning task much more accessible. Teachers should come up with questions to guide the students before and during the engagement of the task. Teachers should also:

- assess prior knowledge;
- define Tier 2 and 3 vocabulary words;
- discuss non-mathematical concepts in the task; and
- assist students in identifying key concepts and facts within the tasks.

Allowing struggling readers to explain their answers using words, numbers, or graphics/pictures ensures that they can express their opinion and rationale despite a potential lack of vocabulary. Through these representations and the ensuing discussion, students will begin to learn the necessary math concepts to be successful.

What other standards does Mathematical Modeling address?

WIDA English Language Development Standards:

- ELD Standard 3

Mathematical Practices:

- G.MP.1
- G.MP.2
- G.MP.4
- G.MP.5

English Language Development for Mathematics:

- ELD–A.9–12: Explain (Interpretive)
- ELD–MA.9–12: Explain (Expressive)

English Language Arts standards:

- ELA–LITERACY.SL.9–10.2
- ELA–LITERACY.SL.9–10.4
- ELA–LITERACY.RST.9–10.3
- ELA–LITERACY.RST.9–10.4
- ELA–LITERACY.RST.9–10.7
- ELA–LITERACY.WHST.9–10.4
- ELA–LITERACY.WHST.9–10.9

Sources

- Carson, Ewart, and Cobelli, Claudio. *Modeling Methodology for Physiology and Medicine*. Academic Press, San Diego, CA, 2001.
- Dym, Clive L. *Principles of Mathematical Modeling*. Amsterdam: Elsevier Academic Press, 2008.
- MathisFun.com. "Mathematical Models." https://www.walch.com/rr/09066
- Oswalt, Selena. "Mathematical Modeling in the High School Classroom." LSU Digital Commons. Accessed February 13, 2024. https://www.walch.com/rr/09067

Mathematical Modeling Implementation Guide

Introduction

Walch resources support the framework of the Mathematical Teaching Practices (MPs) and the NCTM Principles of Teaching Practices. Implementing strategies and support from both practices lead to true conceptual understanding of the math standards. One of which includes mathematical modeling, the process of designing and revising representations to solve a problem.

Mathematical modeling is essential to building a deep conceptual understanding of math concepts for students. Teaching students to model boosts engagement, builds student confidence in math concepts, helps them to make sense of problems, and allows them to make connections to the world around them for better understanding. Students then make decisions about the information, create models, interpret the results, and form conclusions.

A Mathematical Modeling Framework

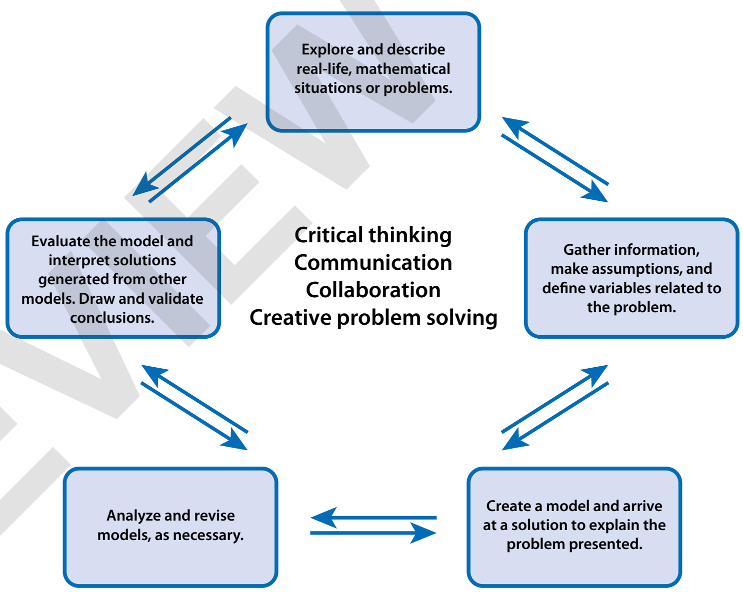


Image adapted from: Suh, Matson, Seshaiyer, 2017

The following is a brief description of how this framework can be applied in the classroom.

Critical Thinking

Students will explore and describe real-life mathematical situations or problems. We want students to discover new ways of thinking and ideas in mathematics. Students do this by developing questions to ask, gathering information, and coming up with solutions. Fostering critical thinking in the classroom not only makes students better at math, but also prepares them for the real world. Below are some ideas and probing questions teachers may use to implement critical thinking.

- Allow for pair-share and small group discussions.
- Encourage students to think and form their own conclusions.
- Encourage the revision of their own thinking and the thinking of others.
- Ask students to think out loud as they work.
- Create a classroom environment that embraces and values student ideas.

Ask students:

- What is the problem asking you to solve?
- Can you think of other strategies you could use to solve this problem?
- What conclusions can you make from this particular problem?
- Will this strategy work in all problems like this? Why or why not? How can we test that?
- Explain how you got to your answer.
- Explain your reasoning.
- How would you respond to a different answer to the same problem?

Communication

When students gather information, make assumptions, and define variables related to the problem, communication allows for them to show their understanding of the math content. Encourage discourse by allowing students to explain their thinking and challenge each other. This encourages students to justify their reasoning. If students communicate their thinking in various ways (including written and oral responses) while doing math, it will improve their understanding of math concepts.

Teachers can do the following to foster communication in the classroom:

- Ask open-ended questions.
- Encourage oral and visual (written and pictorial) communication through journal writing.
- Provide students with detailed feedback.

Ask students:

- Can you explain your thinking?
- How did you get your answer?
- What strategies did you use?
- What information was necessary for you to solve this problem?

Collaboration

Collaboration is an essential component of student success. It allows students to rely on each other during their problem solving. During collaboration, students work in groups, share ideas, ask questions, and discuss math concepts and additional solution strategies while supporting and defending their thinking. Collaboration is most beneficial to students with the use of effective grouping strategies such as assigning students to heterogeneous groups or random grouping.

The following procedures and probing questions can help you implement collaboration in your classroom.

- Establish a classroom culture where all ideas are valued.
- Establish expectations and routines of collaborative learning.
- Discuss "math talk" passages with students.
- Allow students to teach each other.
- Incorporate an accountability piece for students.
- Arrange student seating to support collaboration (group seating).
- Create heterogeneous student groups with varying skill levels.
- Randomize student groups.
- Keep group sizes between 3 and 6 students.
- Assign group roles.

Ask students:

- Come up with as many strategies to solve the problem as you can.
- Explain how you made your calculations.
- Why did you choose that strategy? Why did that strategy work?
- Describe in your own words how your peer-solved the problem.
- Can you make any connections between your strategies?
- Were there any methods that were better than others when solving this problem? Why or why not?
- What did you learn from your group?
- Defend your reasoning behind that solution.

Creative Problem Solving

Creative problem solving is the ability for students to perform math tasks that allow for challenges that increase their conceptual understanding. While performing these tasks, we want students to use mathematical modeling. We want students to evaluate their models and to interpret solutions from other models.

In creative problem solving, students solve problems using different approaches and models, draw on prior knowledge, and justify their thinking. This results in students becoming better problem solvers and increases their understanding of math concepts. Problem solving should be integrated into their math learning and should not be separated.

Here are some tips for implementing creative problem solving.

- Encourage students to challenge different approaches and strategies from their peers as well as the teacher.
- Encourage discourse.
- Allow appropriate wait time for student responses.
- Refrain from telling students how to solve the problem. Instead, allow students to engage and come to their own solutions.
- Allow students to struggle productively.

Ask students:

- How is the information in the problem important to determining the solution?
- How did you go about solving this?
- Can you explain why you chose that model and strategy?
- Are there other ways to model this particular problem? Can you model the problem another way?
- Why did you make that calculation?
- Justify your solution.
- What generalizations can you make about the math concepts based on this particular problem?

Recommended Resource

- Georgia Department of Education. “Scaffolding Instruction for English Learners: A Georgia Mathematics Instructional Resource Guide.”

  https://www.walch.com/rr/09047

  The purpose of this document is to provide mathematics teachers and leaders with evidence-based, pragmatic scaffolds and supports for English Learners (ELs). This guide is a useful tool to help teachers provide high-quality instruction aligned to Georgia’s K-12 Mathematics Standards.

Source

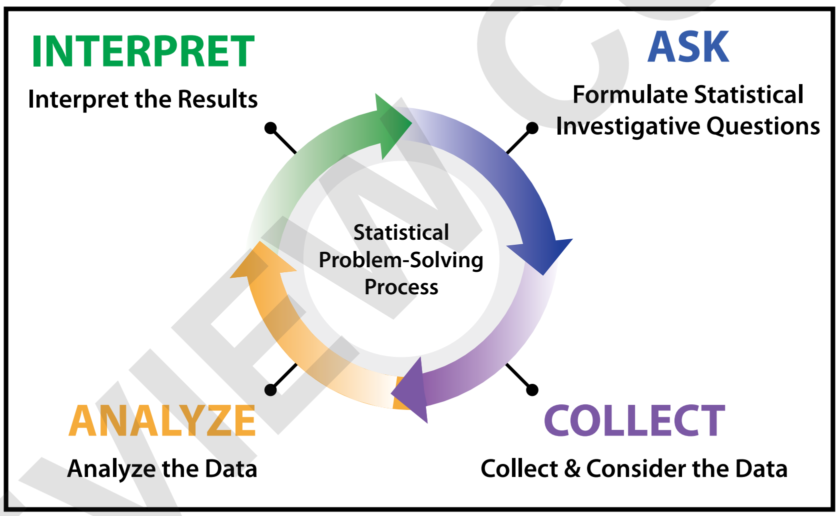
National Council of Teachers of Mathematics. “Problem Solving.” Accessed January 11, 2023. https://www.walch.com/rr/09048.

Statistical Reasoning Implementation Guide

Introduction

Statistical reasoning allows students to make sense of ideas, information, and the changing world through questioning and exploration. It provides the foundation necessary for students to fully understand the concept. Statistical reasoning is a continuous cycle consisting of students asking questions, collecting, analyzing, and interpreting data. In order to guide students in this sense-making process, Walch resources support this four-step statistical problem-solving strategy to help students develop their understanding in statistical reasoning.

Georgia Framework for Statistical Reasoning, 2021



Source: Georgia Department of Education

Here is a brief description of how this framework can be applied in the classroom.

Formulate Statistical Investigative Questions

Students will form and ask investigative questions that allow for various answers. These questions will clarify the problem and lead to questions that can be answered with the data. Best practices and teacher prompts that can foster this framework include:

- Using a student-centered approach.
- Having students prepare ahead of time with an assigned reading to familiarize themselves with words and techniques.

Ask students:

- What do you think?
- What do you notice? What do you wonder?
- What criteria need to be met in order for the question to be statistical?
- How did you determine your question?
- What changes would you make to the question?

Collect and Consider the Data

Students will collect data by creating a plan in order to collect real and relevant data. Making sure the data is relevant to students will increase engagement and lead to more math talk and discussion. Strategies include:

- Refraining from presenting students with procedures.
- Allowing students to use real data sets and to generate their own data.
- Encouraging students to discuss the questions and possible ideas.

Ask students:

- What do you notice about the data?
- In what other ways can the data be collected?
- What are some other methods you can use to collect the data? How do these different methods affect your data collection?
- How can you represent your data? Can you represent it with a visual?
- Are there representations better fit for particular findings? Justify your answer.

Analyze the Data

Students will analyze the data by selecting methods that are appropriate. Exploration of various methods will allow for students to make connections and draw conclusions based on the data. This will deepen their understanding of statistical reasoning. Strategies include:

- Allowing students to use technology tools to explore and analyze their findings.
- Refraining from giving students all the information. Allow students to form their own analysis of the data.
- Creating a classroom environment in which student ideas are valued.

Ask students:

- What conclusions can you draw from the data?
- Do you notice any trends in the data? How can you tell?
- What is the relationship between the data points?
- What evidence may help you distinguish between results?
- Do you agree or disagree? Justify your thinking.
- How can we test that conclusion?
- What do you do about outliers in your data? What do they tell you?
- If extreme values are removed, what happens to the data representation?
- Compare your data with a classmate's. What do you notice?

Interpret the Results

Students will interpret and discuss the results by relating all findings to the original question. Students will discuss these findings and justify their reasoning. Best practices and teacher prompts include:

- Encouraging discourse. Encourage students to present their ideas, answer classmates' questions, and support their responses.
- Focusing on key ideas instead of procedures and calculated answers.
- Making sure students have answered their "I wonder" questions.

Ask students:

- What do the results tell you about the original question?
- Have your "I wonder" questions been answered?
- What conclusions can you make from the results?
- Compare your interpretations to those of your classmates. What connections can you make?
- What do your interpretations represent in a real-world context?

Source

Garfield, Joan and Ben-Zvi, Dani. "Helping Students Develop Statistical Reasoning: Implementing a Statistical Reasoning Learning Environment." Accessed Jan. 11, 2023. https://www.walch.com/rr/09049

Conceptual Activities

Use these interactive open education and/or Desmos resources to build conceptual understanding of mathematical ideas. (*Note*: Activity links will be monitored and repaired or replaced as necessary.)

Unit 1

- Desmos. "Lines, Transversals, and Angles."

  https://www.walch.com/ca/01033

  In this activity, students explore the relationship of angles formed by a transversal and a system of two lines. In particular, students consider what happens when the two lines are parallel versus when they are not.

- Desmos. "Polygraph: Angle Relationships."

  https://www.walch.com/ca/01034

  This activity is designed to spark vocabulary-rich conversations about angle relationships. Key vocabulary terms that may appear in student questions include *parallel*, *transversal*, *adjacent*, *opposite*, *alternate interior*, *corresponding*, *alternate exterior*, *vertical*, and *right*.

- Desmos. "Polygraph: Figure It Out."

  https://www.walch.com/ca/01035

  This activity is intended as an introduction to geometric notation and vocabulary. Depending on prior knowledge, students could use the following to distinguish figures: points, lines, rays, segments, parallel, perpendicular, angles, congruence, midpoints, bisectors, betweenness, collinearity, and more.

- Desmos. "Polygraph: Transformations."

  https://www.walch.com/ca/01025

  This activity is designed to spark vocabulary-rich conversations about transformations. Key vocabulary terms that may appear in student questions include translation, rotation, reflection, dilation, scale factor, preimage, and image.

- Desmos. "Transformation Golf: Rigid Motion."

  https://www.walch.com/ca/10016

  In this activity, students will use their knowledge of rigid motions to maneuver shapes around obstacles in a coordinate plane.

- Illuminations. "Finding Lines of Symmetry."

  https://www.walch.com/ca/10017

  Students will develop their understanding of symmetry using folded paper cutouts.

Unit 2

- Desmos. "Parallel Lines and Triangles."

  https://www.walch.com/ca/10014

  In this activity, students will investigate the interior angle sum theorem for triangles.

- Desmos. "Parallelograms in the Coordinate Plane."

  https://www.walch.com/ca/10015

  In this activity, students will connect their understanding of slope with geometric understanding of parallel lines. They will determine whether a set of four points in the coordinate plane forms a parallelogram using rate of change arguments.

- Desmos. "Polygraph: Angle Relationships."

  https://www.walch.com/ca/01034

  This activity is designed to spark vocabulary-rich conversations about angle relationships. Key vocabulary terms that may appear in student questions include *parallel*, *transversal*, *adjacent*, *opposite*, *alternate interior*, *corresponding*, *alternate exterior*, *vertical*, and *right*.

- Desmos. "Polygraph: Figure It Out."

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Unit 3

- Desmos. "Special Right Triangles."

  https://www.walch.com/ca/10018

  In this activity, students work with the side length ratios of 45°–45°–90° and 30°–60°–90° right triangles.

- Desmos. "Working with Dilations."

  https://www.walch.com/ca/10019

  This activity is a basic introduction to dilations.

- Illustrative Mathematics. "Mt. Whitney to Death Valley."

  https://www.walch.com/ca/10020

  In this task, students will apply trigonometric ratios to solve a real-life problem.

- Illuminations. "Trigonometry Square."

  https://www.walch.com/ca/10021

  This activity allows students to practice evaluating trigonometric ratios for specific values.

Unit 4

- Inside Mathematics. "Problem of the Month: Cutting Cubes."

  https://www.walch.com/ca/10002

  In this activity, students explore the attributes of polygons, faces, edges, vertices, spatial visualization, counting strategies, classification, and geometric solids.

- Inside Mathematics. "Problem of the Month: Piece it Together."

  https://www.walch.com/ca/10001

  In this activity, students use two- and three-dimensional geometry to solve problems involving polygons and polyhedra.

Unit 5

- Desmos. "Slopes of Parallel and Perpendicular Lines."

  https://www.walch.com/ca/10023

  In this activity, students will investigate the slopes of parallel and perpendicular lines in the coordinate plane.

- Gizmos. "Distance Formula."

  https://www.walch.com/ca/10024

  This activity relates the distance formula to the Pythagorean Theorem. Students will learn to see any two points in the coordinate plane as the endpoints of the hypotenuse of a right triangle.

- Illustrative Mathematics. "A Midpoint Miracle."

  https://www.walch.com/ca/10003

  Students will prove a theorem about quadrilaterals that is somewhat difficult to prove with a straightedge and ruler but relatively easy to prove using coordinates. This task requires that students be comfortable using the formula for the midpoint of a line segment and the parallel line criterion.

Unit 6

- Desmos. "Circle Patterns."

  https://www.walch.com/ca/01039

  In this activity, students notice similarities and differences in a set of circles. They then use this information to practice writing equations of circles that extend a given pattern or match a given set of conditions.

- Desmos. "Equations of Circles."

  https://www.walch.com/ca/01040

  In this activity, students write equations of circles with different given information. The activity involves writing equations in both standard and general form.

- Desmos. "Sector Area."

  https://www.walch.com/ca/01041

  In this proportional reasoning activity, students explore the relationship between circle area, sector area, and sector angle.

Unit 7

- Desmos. "Binomial Distribution."

  https://www.walch.com/ca/20007

  This resource is an interactive plot of the binomial distribution. There are sliders for the probability of the favorable outcome and the number of trials. Moving the slider changes the plot.

- Desmos. "Chance Experiments."

  https://www.walch.com/ca/20006

  This activity uses spinners as a chance event to predict empirical probabilities from theoretical probabilities. There are no calculations involved; probabilities are visually interpreted from the segments of the spinners.

- Desmos. "Geometric Distribution."

  https://www.walch.com/ca/20008

  This resource is an interactive plot of the geometric distribution. There is a slider for the probability of the favorable outcome. Moving the slider changes the plot.

- Desmos. "Polygraph: Histograms."

  https://www.walch.com/ca/01024

  This activity is designed to spark vocabulary-rich conversations about histograms. Key vocabulary terms that may appear in student questions include *shape, center, spread, roughly symmetric*, *skew right*, *skew left*, *mean*, *median*, *range*, *peak*, *unimodal*, and *bimodal*.

Station Activities Guide

Introduction

Each unit includes a collection of station-based activities to provide students with opportunities to practice and apply the mathematical skills and concepts they are learning. You may use these activities in addition to the instructional lessons, or, especially if the pre-test or other formative assessment results suggest it, instead of direct instruction in areas where students have the basic concepts but need practice. The debriefing discussions after each set of activities provide an important opportunity to help students reflect on their experiences and synthesize their thinking. Debriefing also provides an additional opportunity for ongoing, informal assessment to guide instructional planning.

Implementation Guide

The following guidelines will help you prepare for and use the activity sets in this section.

Setting Up the Stations

Each activity set consists of four or five stations. Set up each station at a desk, or at several desks pushed together, with enough chairs for a small group of students. Place a card with the number of the station on the desk. Each station should also contain the materials specified in the teacher's notes, and a stack of student activity sheets (one copy per student). Place the required materials (as listed) at each station.

When a group of students arrives at a station, each student should take one of the activity sheets to record the group's work. Although students should work together to develop one set of answers for the entire group, each student should record the answers on his or her own activity sheet. This helps keep students engaged in the activity and gives each student a record of the activity for future reference.

Forming Groups of Students

All activity sets consist of four or five stations. You might divide the class into four or five groups by having students count off from 1 to 4 or 5. If you have a large class and want to have students working in small groups, you might set up two identical sets of stations, labeled A and B. In this way, the class can be divided into eight groups, with each group of students rotating through the "A" stations or "B" stations.

Assigning Roles to Students

Students often work most productively in groups when each student has an assigned role. You may want to assign roles to students when they are assigned to groups and change the roles occasionally. Some possible roles are as follows:

- Reader—reads the steps of the activity aloud
- Facilitator—makes sure that each student in the group has a chance to speak and pose questions; also makes sure that each student agrees on each answer before it is written down
- Materials Manager—handles the materials at the station and makes sure the materials are put back in place at the end of the activity
- Timekeeper—tracks the group's progress to ensure that the activity is completed in the allotted time
- Spokesperson—speaks for the group during the debriefing session after the activities

Timing the Activities

The activities in this section are designed to take approximately 10 minutes per station. Therefore, you might plan on having groups change stations every 10 minutes, with a two-minute interval for moving from one station to the next. It is helpful to give students a "5-minute warning" before it is time to change stations.

Since each activity set consists of four or five stations, the above time frame means that it will take about 50 to 60 minutes for groups to work through all stations.

Guidelines for Students

Before starting the first activity set, you may want to review the following "ground rules" with students. You might also post the rules in the classroom.

- All students in a group should agree on each answer before it is written down. If there is a disagreement within the group, discuss it with one another.
- You can ask your teacher a question only if everyone in the group has the same question.
- If you finish early, work together to write problems of your own that are similar to the ones on the activity sheet.
- Leave the station exactly as you found it. All materials should be in the same place and in the same condition as when you arrived.

Debriefing the Activities

After each group has rotated through every station, bring students together for a brief class discussion. At this time, you might have the groups' spokespersons pose any questions they had about the activities. Before responding, ask if students in other groups encountered the same difficulty or if they have a response to the question. The class discussion is also a good time to reinforce the essential ideas of the activities. The questions that are provided in the teacher's notes for each activity set can serve as a guide to initiating this type of discussion.

You may want to collect the student activity sheets before beginning the class discussion. However, it can be beneficial to collect the sheets afterward so that students can refer to them during the discussion. This also gives students a chance to revisit and refine their work based on the debriefing session. If you run out of time to hold class discussions, you might want to have students journal about their experiences and follow up with a class discussion the next day.

PROGRAM OVERVIEW

Digital Enhancements Guide

Introduction

With this program, you have access to the following digital components, described here with guidelines and suggestions for implementation.

Digital Instruction PowerPoints (Presentations)

These optional versions of the Warm-Ups, Warm-Up Debriefs, Introductions, Key Concepts, and Guided Practices for each lesson run on PowerPoint. (*Please note*: Computers may render PowerPoint images differently. For best viewing and display, use a PowerPoint Viewer and adjust your settings to optimize images and text.)

Each PowerPoint begins with the lesson's Warm-Up and is followed by the Warm-Up Debrief, which reveals the answers to the Warm-Up questions.

In the notes section of the last Warm-Up slide, you will find the "Connections to the Lesson," which describes concepts students will glean or skills they will need in the upcoming lesson. The "Connections" help transition from the Warm-Up to instruction.

GeoGebra Applets (Interactive Practice Problems)

One or two interactive GeoGebra applets are provided for most lessons. The applets model the mathematics in the Guided Practice examples for these lessons. Links to these applets are also embedded within the Instructional PowerPoints. With an Internet connection, simply click on the "Play" button slide that follows selected examples.

Once you've accessed the GeoGebra applet, please adjust your view to maximize the image. Each applet illustrates the specific problem addressed in the Guided Practice example. The applets allow you to walk through the solution by visually demonstrating the steps, such as defining points and drawing lines. Variable components of the applets (usually fill-in boxes or sliders) allow you to substitute different values in order to explore the mathematics. For example, "What happens to the line when we increase the amount of time?" or "What if we cut the number of students in half?" This experimentation and discussion supports development of conceptual understanding.

GeoGebra for PC/MAC

GeoGebra is not required for using the applets, but can be downloaded for free for further exploration at the following link:

https://www.geogebra.org/download

Curriculum Engine Learning Object Repository

Walch's Curriculum Engine comes loaded with thousands of curated learning objects that can be used to build formative and summative assessments, as well as practice worksheets, instructional components, and an item bank. District leaders and teachers can search for items by standard and create assessments or worksheets in minutes using the three-step assessment builder.

For more information about the Curriculum Engine, or for additional support, please contact us at (207) 828-8800 or success@bwwalch.com.

PROGRAM OVERVIEW

Graphic Organizers

Overview

Graphic organizers can be a versatile tool in your classroom. Organizers offer an easy, straightforward way to visually present a wide range of material. Research suggests that graphic organizers support learning in the classroom for all levels of learners. Gifted students, students on grade level, and students with learning difficulties all benefit from the use of graphic organizers. They reduce the cognitive demand on students by helping them access information quickly and easily. Using graphic organizers, learners can understand content more clearly and can take concise notes. Ultimately, learners find it easier to retain and apply what they've learned.

Graphic organizers help foster higher-level thinking skills. They help students identify main ideas and details in their reading. They make it easier for students to see patterns such as cause and effect, comparing and contrasting, and chronological order. Organizers also help students master critical-thinking skills by asking them to recall, evaluate, synthesize, analyze, and apply what they've learned. Research suggests that graphic organizers contribute to better test scores because they help students understand relationships between key ideas, and enable them to be more focused as they study.

Types of Graphic Organizers

There are four main purposes for using graphic organizers in mathematics and a variety of tools within each category:

Purpose 1: Organizing, Categorizing, and Classifying	**Purpose 2: Problem Solving**	**Purpose 3: Understanding Mathematical Information**	**Purpose 4: Communicating Mathematical Information**
Tables Flowcharts Webs Venn Diagrams	Number Lines Geometric Drawings Factor Trees Attribute Tables Cause and Effect Maps Coordinate Plane Probability Trees	Frayer Model Semantic Map/ Concept Map Compare-and-Contrast Diagram	Line Graphs Bar Charts

Tables

A table is simply a grid with rows and columns. Tables are useful because information stored in a table is easy to find—much easier than the same information embedded in text.

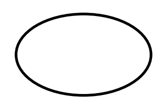
Usually, a table has a row (horizontal) for each item being listed. The columns (vertical) provide places for details about the listed items—the things they have in common. The places where the rows and columns meet are called cells. In each cell, we write information that fits both the topic of the row (the thing being listed) and the topic of the column (the aspect being examined). To create a table, we make rows and columns to fit the number of items and attributes.

Flowcharts

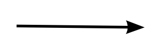
Flowcharts are graphic organizers that show the steps in a process. Flowcharts can be very simple—just a series of boxes with one step in each box. However, there is also a more formal type of flowchart. These flowcharts use special symbols to show different things, such as starting and stopping points, or points where decisions must be made. These symbols make flowcharts especially useful for showing complicated processes.

Each step in a flowchart is written in a box. The boxes are connected by arrows to show the sequence of steps. The boxes aren't all rectangular; different shapes are used to indicate different actions. The shapes and symbols are a kind of visual shorthand. Whenever a certain symbol is used, it always has the same meaning.

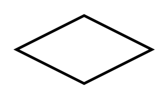
- Circles and ovals show starting and stopping points. They often contain the words start or stop. The "start" circle or oval has no arrows in and one arrow out. The "stop" circle or oval has one arrow in and no arrows out.



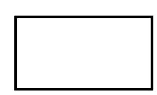
- Arrows show the direction in which the process is moving.



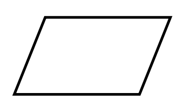
- Diamonds show points where a decision must be made or a question must be answered. The question can usually be answered either "yes" or "no."



- Rectangles and squares show steps where a process or an operation takes place.



- Parallelograms show input or output, such as writing or printing a result or solution.



Webs

Webs are graphic organizers that help take notes, identify important ideas, and show relationships between and among pieces of information. In a web, the main idea is written in the center circle. Details are recorded in other circles with lines to connect related topics. Circles or lines can be added or deleted as necessary.

Number Lines

In its simplest form, a number line is any line that uses equally spaced marks to show numbers. Number lines are used to visualize equalities and inequalities, positive and negative numbers, and measurements of all kinds. They can "map" math problems, especially ones that involve negative numbers or distances.

Geometric Drawings

A geometric drawing is a representation on paper (or some other surface) of a geometric figure. The geometric drawings we make can never be as perfect as the geometric figures they represent, but as long as they are reasonably accurate, they can help us visualize the figures. In fact, it's often impossible to solve a geometry problem without making a drawing.

Factor Trees

There are several ways to find factors. One that helps to visually keep track of all the factors is called a factor tree. This is a diagram with a tree-like shape. It uses "branches" to show the factors of a number.

All whole numbers other than 1 can be written as the product of factors. A prime number is a number that has only two factors, itself and 1. An example of a prime number is 13. Its only factors are 13 and 1. A composite number is a number that has more than two factors. An example of a composite number is 6. Its factors include 6, 3, 2, and 1. Prime factors are factors that are also prime numbers. The greatest common factor (GCF) of two numbers is the largest number that is a factor of both numbers.

Coordinate Plane

This is the plane determined by a horizontal number line, called the *x*-axis, and a vertical number line, called the *y*-axis, intersecting at a point called the origin. A coordinate plane can be used to illustrate locations and relationships using ordered pairs of numbers.

Venn Diagrams

A set is a list of objects in no particular order. Items in a set can be numbers, but they can also be letters or words. Venn diagrams are a visual way of showing how sets of things can include one another, overlap, or be distinct from one another.

Venn diagrams are often used to compare and contrast things. But they are also a useful tool to sort and classify information. You can use Venn diagrams to take notes on material that shows relationships between things or ideas. You can also use them to solve certain types of word problems. When a word problem names two or three different categories and asks you how many items fall into each category, a Venn diagram can be a useful problem-solving tool.

A Venn diagram begins with a rectangle representing the universal set. Then each set in the problem is represented by a circle. Circles can be separate, overlapping, or one within another. When two circles overlap, it means that the two sets intersect. Some members of one set are also members of the other set.

Venn Diagrams AND **Compare-and-Contrast Diagrams**

The Venn diagram is an organizing device for planning comparisons and contrasts. A completed Venn diagram helps students categorize and organize similarities and differences, and provides a blueprint for a comparison-and-contrast exercise. The compare-and-contrast diagram provides a structure to identify or list similarities and differences between two objects.

Attribute Tables

To solve logic problems, you need a way to keep track of the subjects and which attributes they have or don't have. An attribute table can help. This is a table with a row for each subject in the problem, and a column for each attribute. The rows and columns meet to form cells. Because the attributes in logic problems are usually exclusive, you can use Xs or check marks (✔) to show which attribute belongs to which subject.

Cause and Effect Maps

Cause and effect maps help you work through information to make sense of it. Write each cause in the oval. Write all its effects in the boxes. Add or delete ovals and boxes as needed.

Frayer Model

The Frayer Model is a word categorization activity that helps learners to develop their understanding of concepts. Using this model, students provide a definition, list characteristics, and provide examples and non-examples of the concept.

Semantic Map

A semantic word map allows students to conceptually explore their knowledge of a new term or concept by mapping it with other related words, concepts, or phrases that are similar in meaning. Semantic maps portray the schematic relations that compose a concept. It assumes that there are multiple relations between a concept and the knowledge that is associated with the concept.

Line Graphs

Line graphs are often used to show how things change over time. They clearly show trends in data and can let you make predictions about future trends, too. Line graphs use two number lines, one horizontal and one vertical. The horizontal number line is called the x-axis. The vertical line is called the y-axis. The x-axis often shows the passage of time. The y-axis often shows a quantity of some kind, such as height, speed, cost, and so forth.

Bar Charts

Bar charts are useful when you want to compare things or to show how one thing changes over time. They are a good way to show overall trends. Bar charts use horizontal or vertical bars to represent data. Longer bars represent higher values. Different colors can be used to show different variables. When you look at a bar chart, it's easy to see which element has the greatest value—the one with the longest bar.

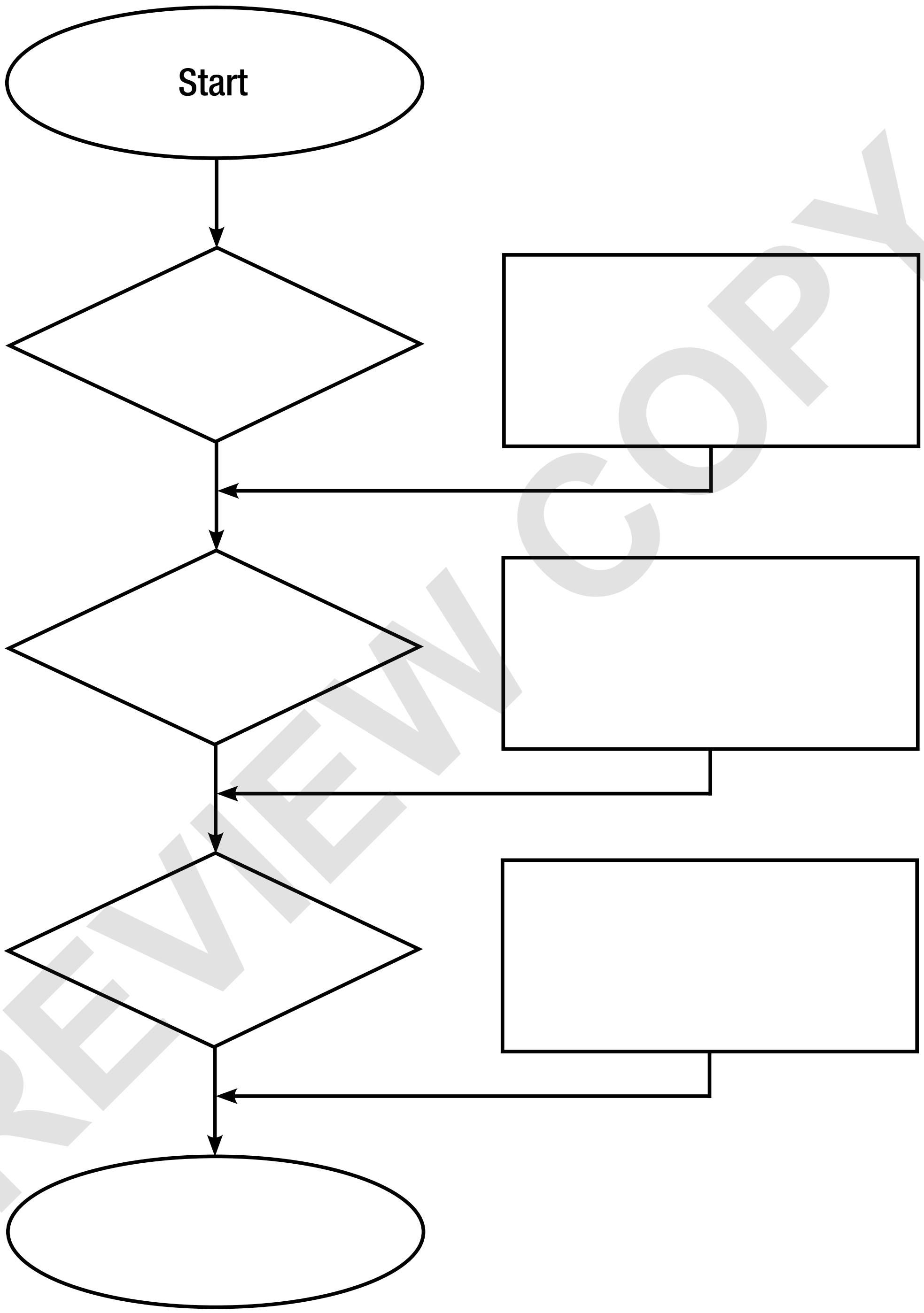
Bar charts have an x-axis (horizontal) and a y-axis (vertical). If the graph is being used to show how something changes over time, the x-axis has numbers for the time period. If the graph is being used to compare things, the x-axis shows which things are being compared. The y-axis has numbers that show how much of each thing there is.

Probability Trees

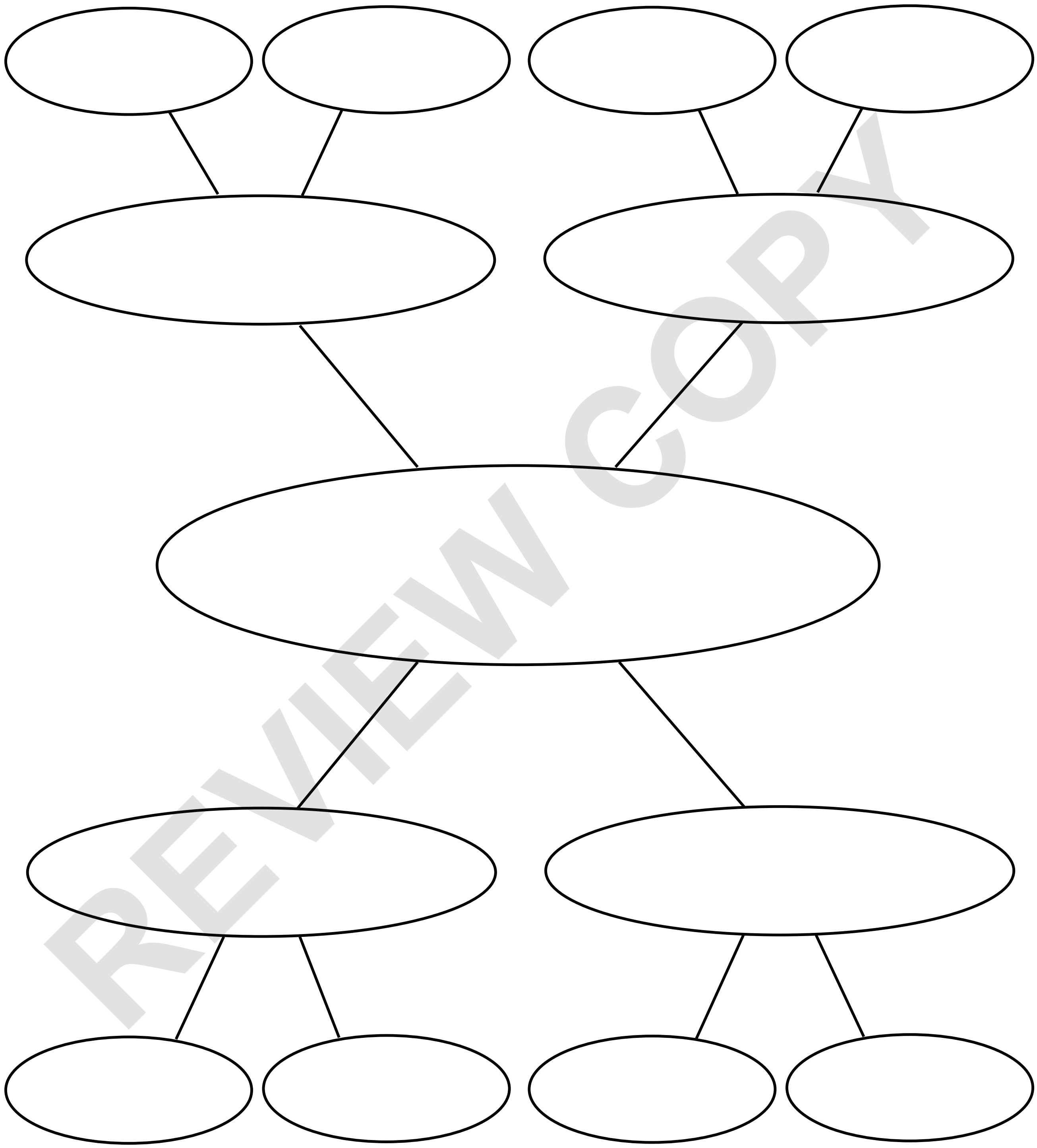
When we have probability problems with many possible outcomes, or events that depend on one another, probability trees can help. Probability trees show all the possible outcomes of an event. Whenever a problem calls for figuring out how many possible outcomes there are, and the probability that any one of them will happen, a probability tree can be useful.

Table

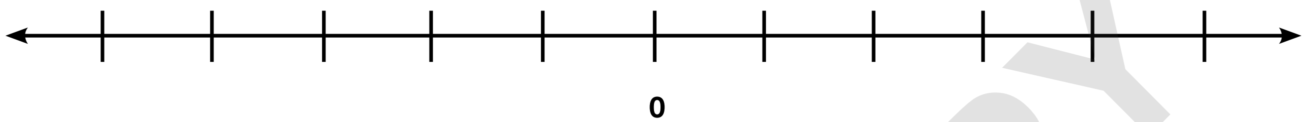
Flowchart

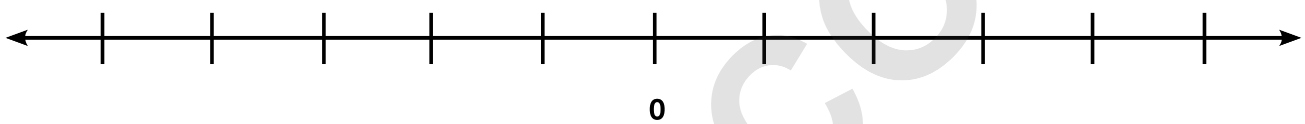


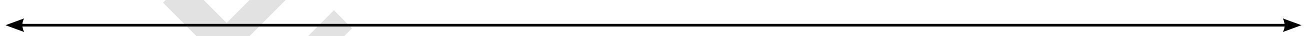
Web



Number Line

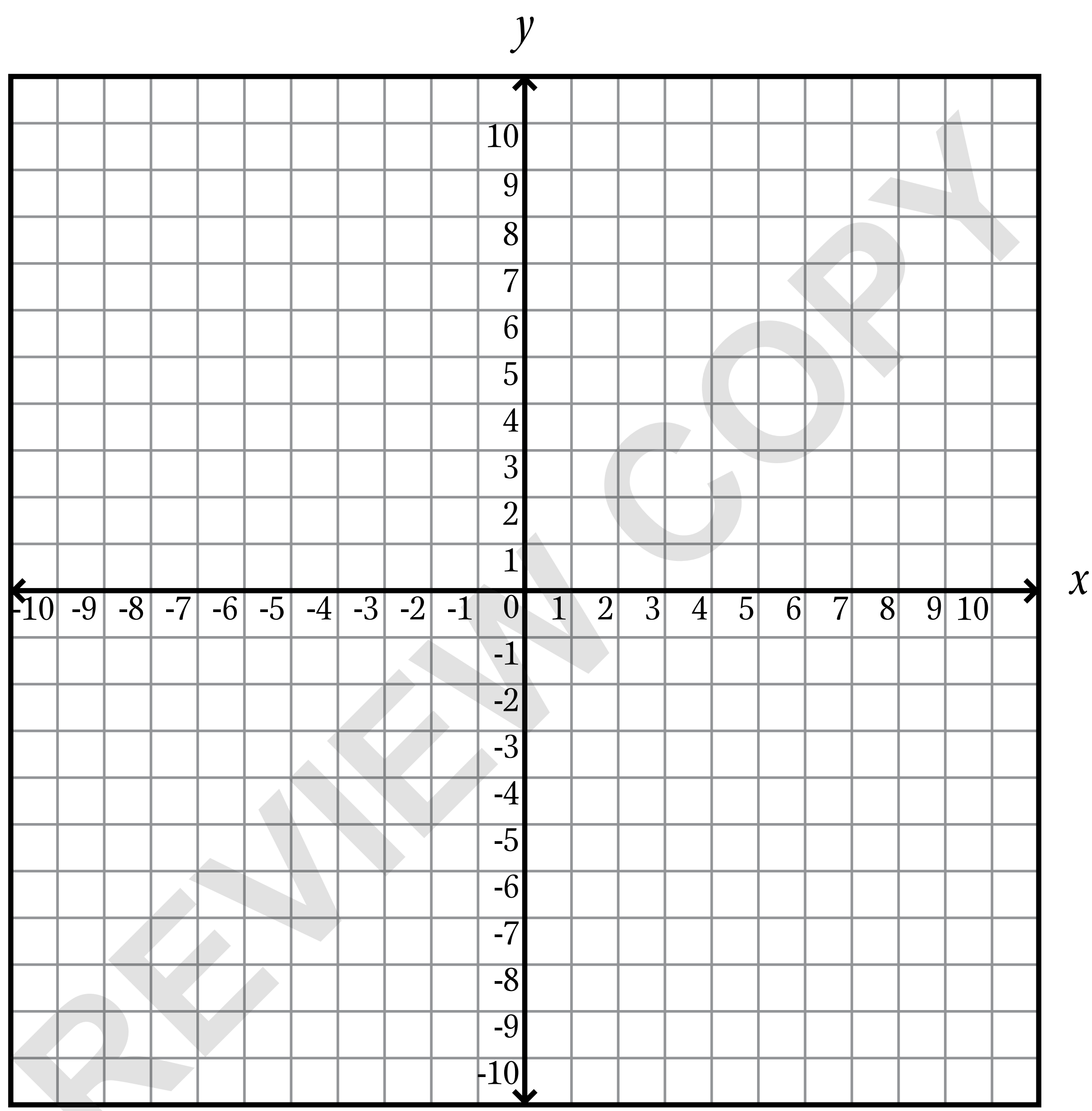




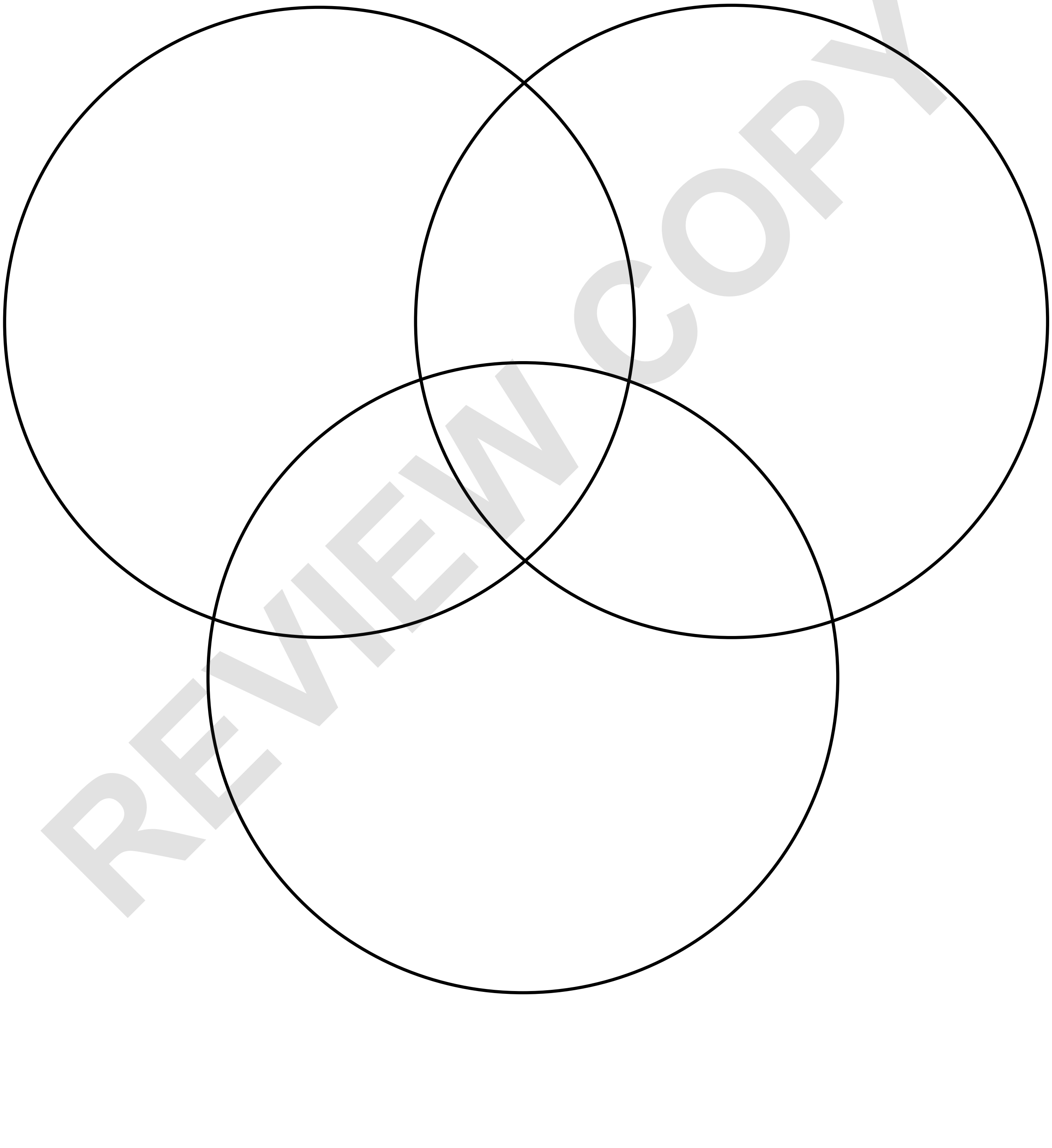


Geometric Drawing

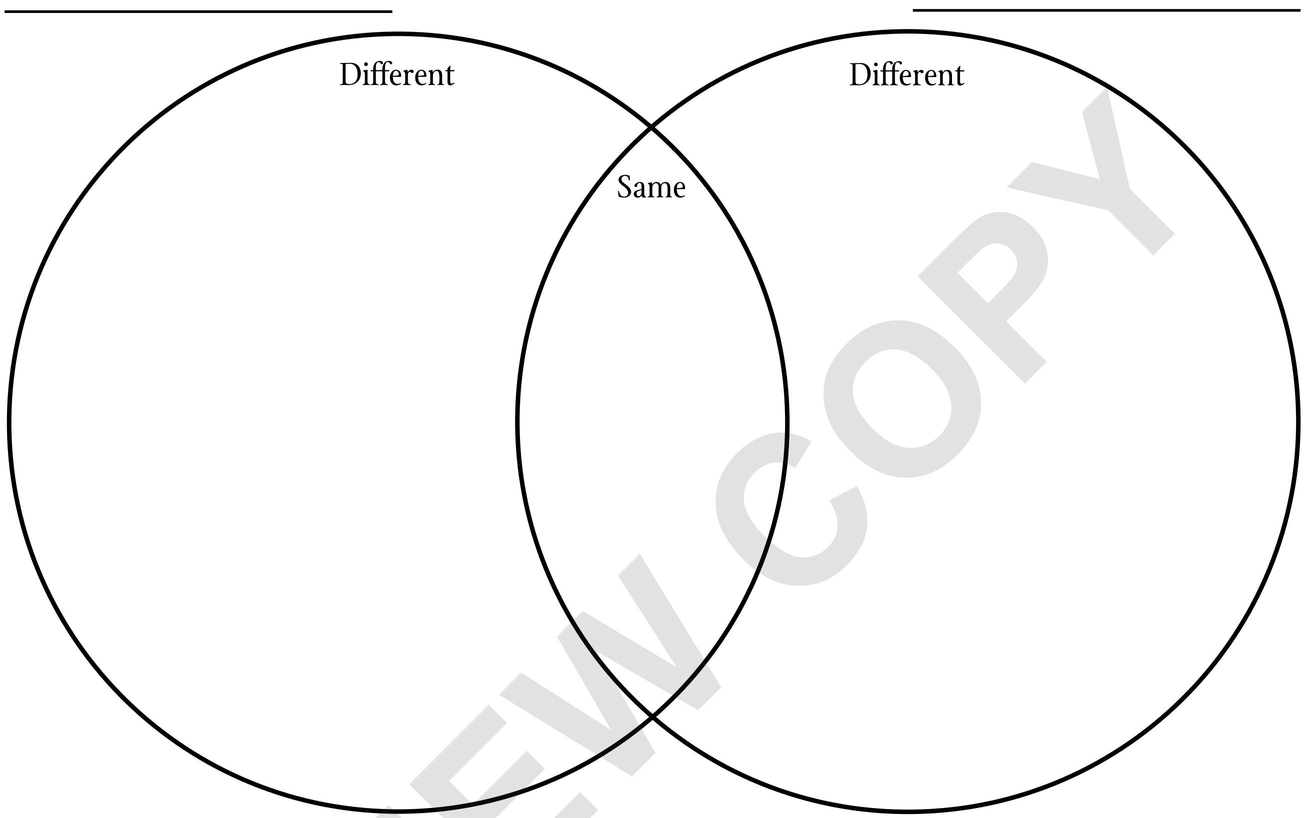
Coordinate Plane



Venn Diagram



Venn Diagram



Compare-and-Contrast Diagram

Item 1 ______________________________ Item 2 ______________________________

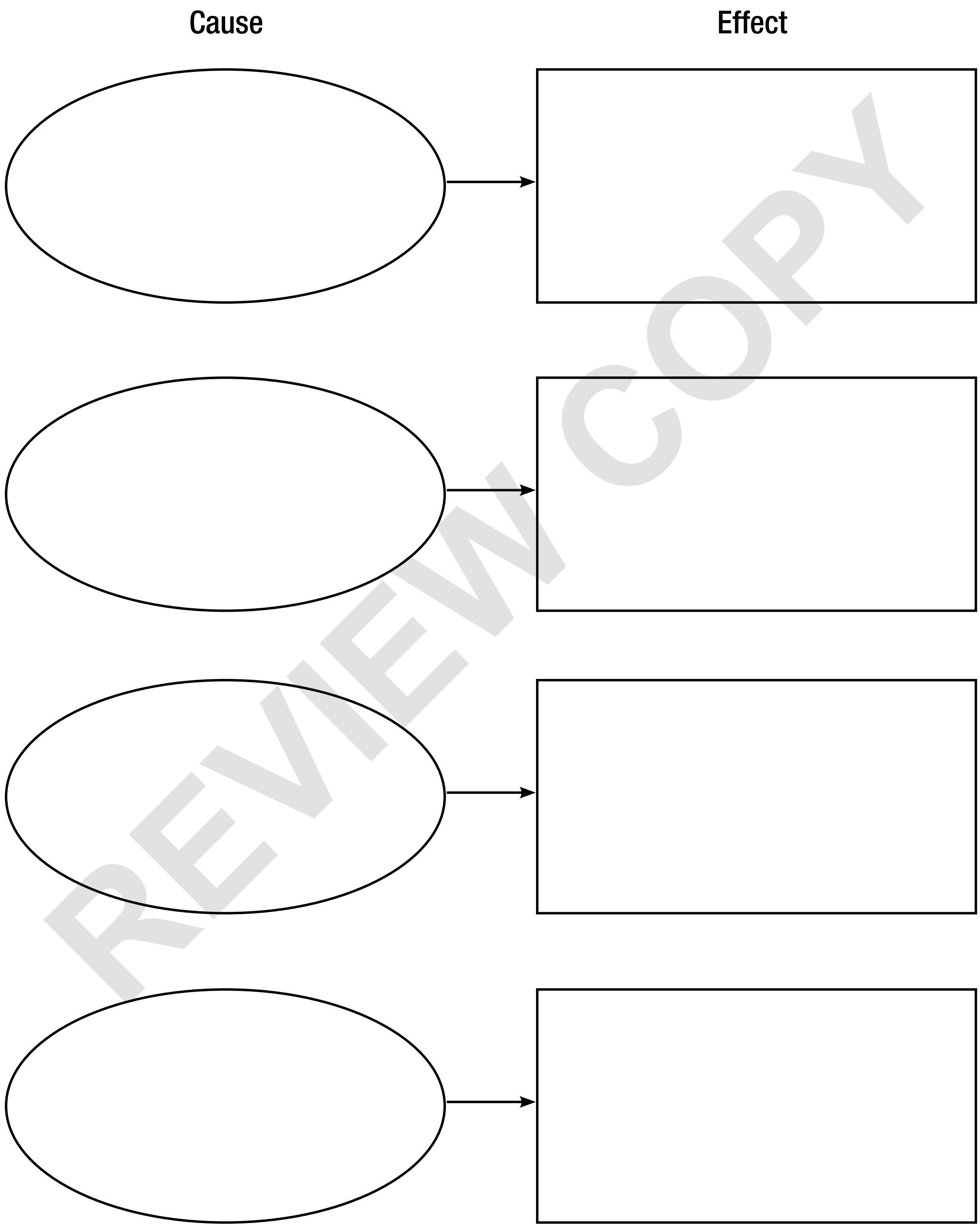
How Alike?

How Different?

With Regard To

Attribute Table

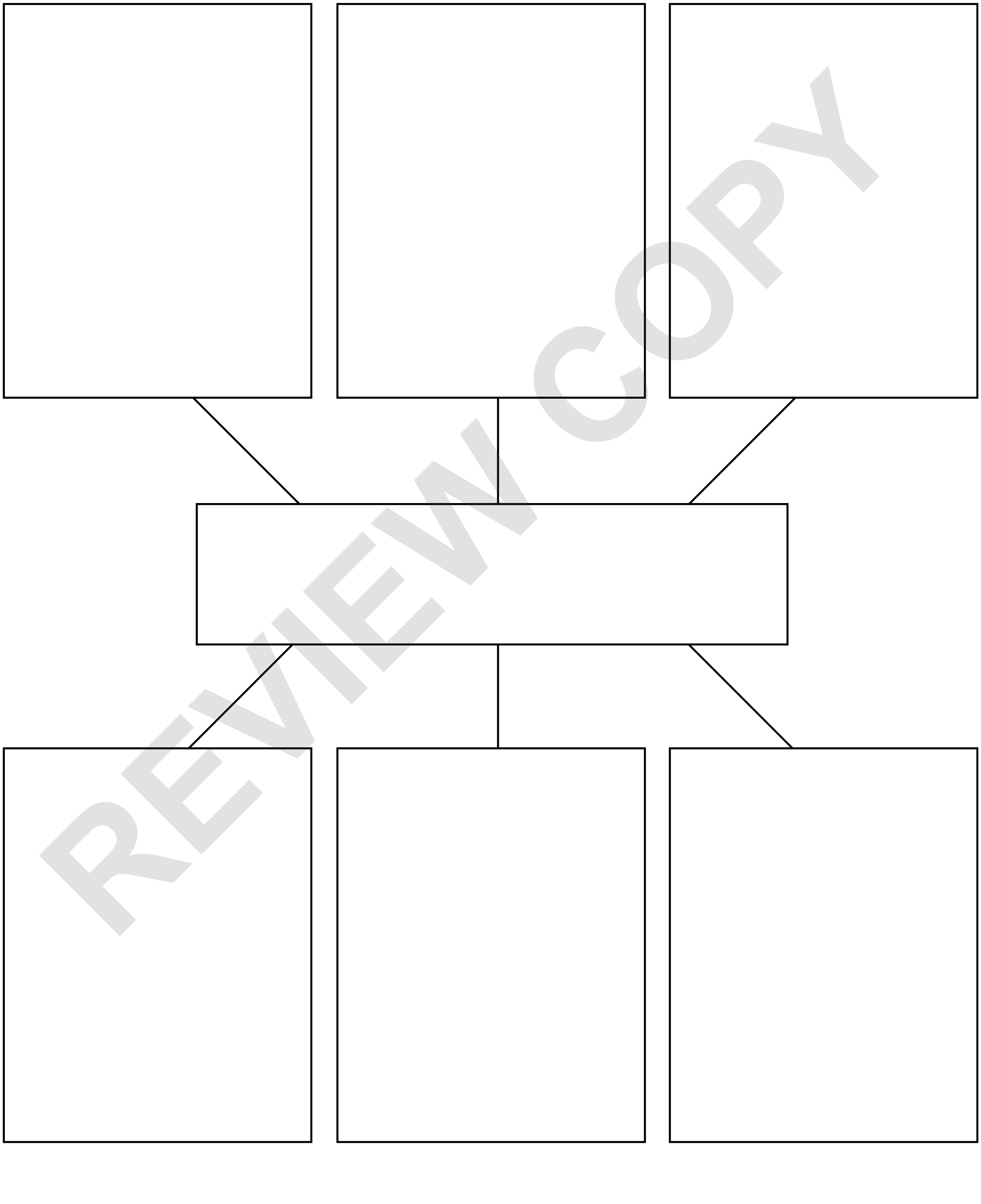
Cause and Effect Map



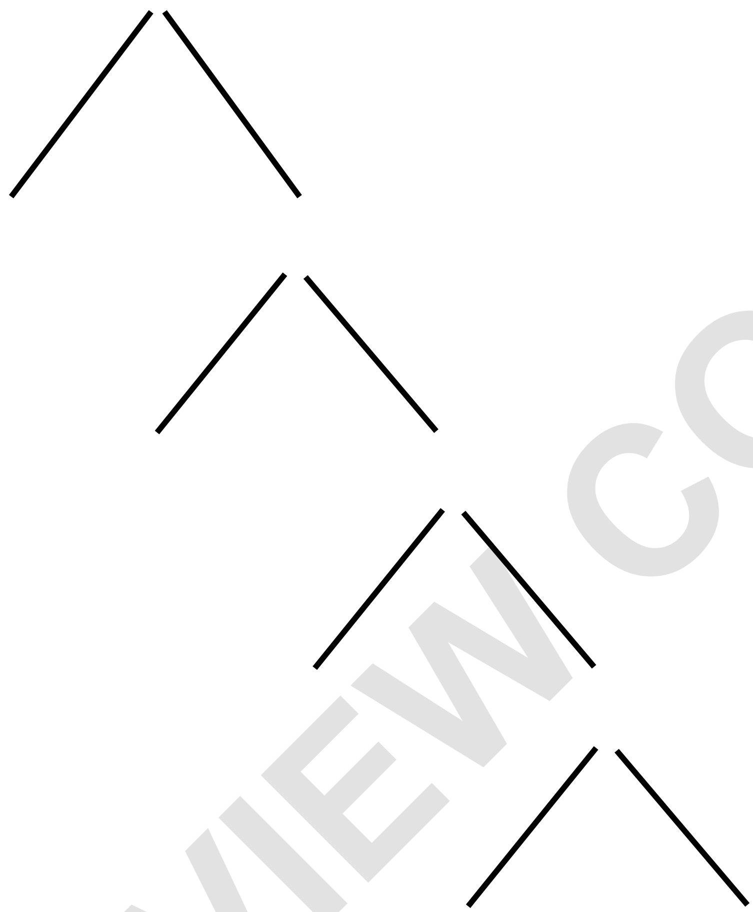
Frayer Model

Definition	Characteristics
WORD	
Examples from Life	Non-Examples

Semantic Map/Concept Map



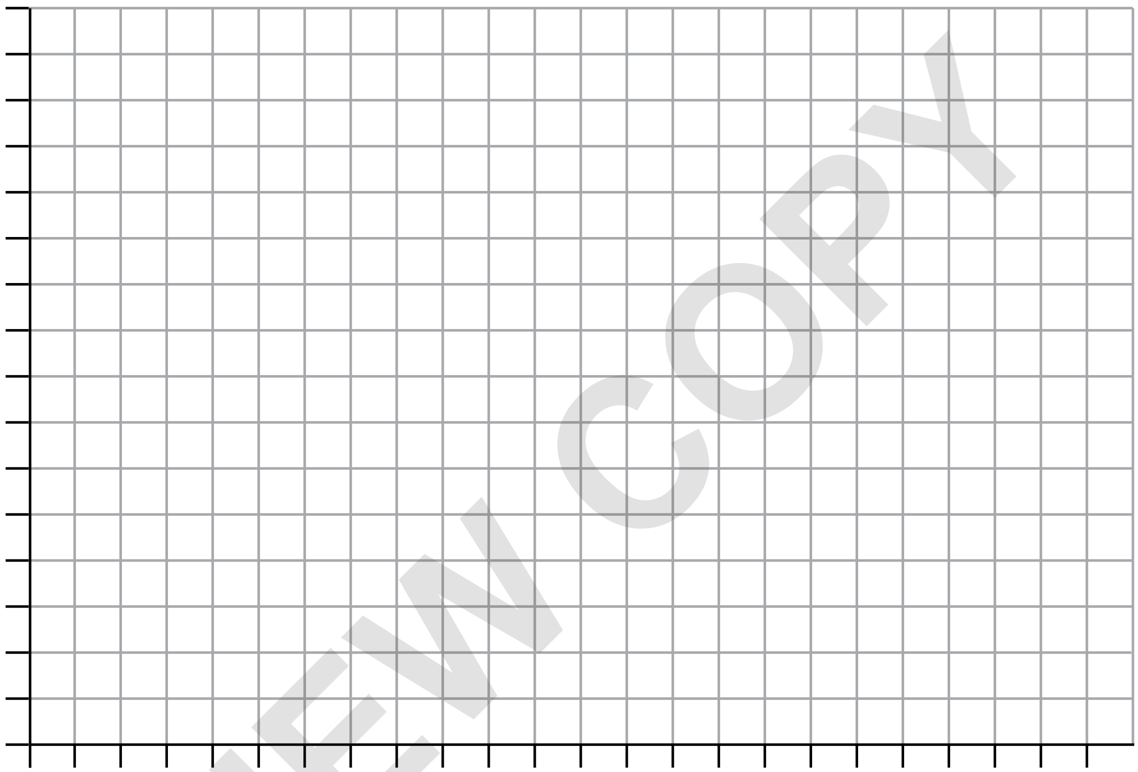
Factor Tree



Line Graph

Graph title ____________________

Axis title ____________________

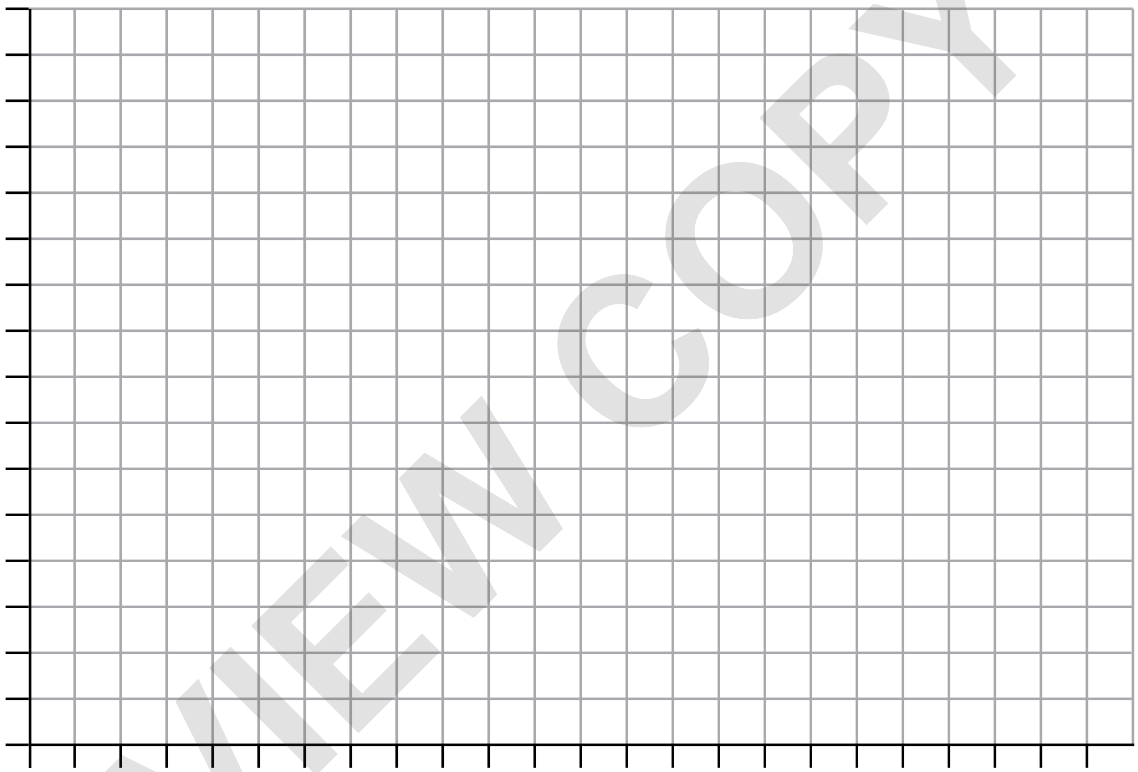


Axis title ____________________

Bar Chart/Histogram

Graph title ______________________

Axis title ______________________



KEY

- ☐
- ☐
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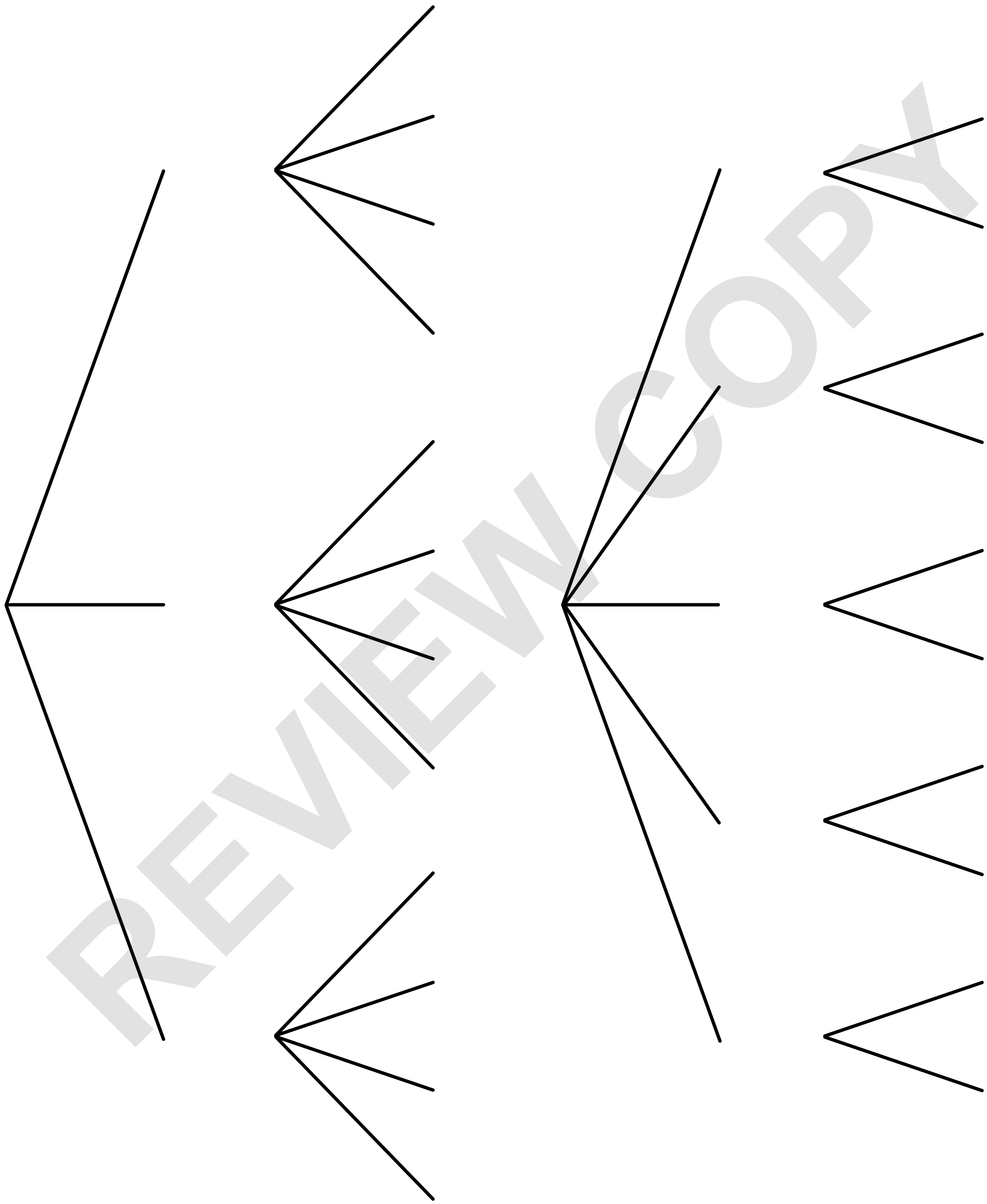
- ☐

Axis title ______________________

Probability Trees



ALGEBRA

Functions	
$f(x)$	Function notation, "f of x"
$f^{-1}(x)$	Inverse function notation
$f(x) = mx + b$	Linear function
$f(x) = b^x + k$	Exponential function
$(f + g)(x) = f(x) + g(x)$	Addition
$(f - g)(x) = f(x) - g(x)$	Subtraction
$(f \bullet g)(x) = f(x) \bullet g(x)$	Multiplication
$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$	Division
$\dfrac{f(b) - f(a)}{b - a}$	Average rate of change
$f(-x) = -f(x)$	Odd function
$f(-x) = f(x)$	Even function
$f(x) = \lfloor x \rfloor$	Floor/greatest integer function
$f(x) = \lceil x \rceil$	Ceiling/least integer function
$f(x) = a\sqrt[3]{(x-h)} + k$	Cube root function
$f(x) = a\sqrt[n]{(x-h)} + k$	Radical function
$f(x) = a\lvert x - h\rvert + k$	Absolute value function
$f(x) = \dfrac{p(x)}{q(x)};\quad q(x) \neq 0$	Rational function

Symbols	
$\approx$	Approximately equal to
$\neq$	Is not equal to
$\lvert a \rvert$	Absolute value of a
$\sqrt{a}$	Square root of a
∞	Infinity
[	Inclusive on the lower bound
]	Inclusive on the upper bound
(	Non-inclusive on the lower bound
)	Non-inclusive on the upper bound

Linear Equations	
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	Slope
$y = mx + b$	Slope-intercept form
$ax + by = c$	General form
$y - y_1 = m(x - x_1)$	Point-slope form

Exponential Equations	
$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$	Compounded interest formula
Compounded...	n (number of times per year)
Yearly/annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

Quadratic Functions and Equations	
$x=\frac{-b}{2a}$	Axis of symmetry
$x=\frac{p+q}{2}$	Axis of symmetry using the midpoint of the x-intercepts
$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$	Vertex
$f(x) = ax^2 + bx + c$	General form
$f(x) = a(x - h)^2 + k$	Vertex form
$f(x) = a(x - p)(x - q)$	Factored/intercept form
$b^2 - 4ac$	Discriminant
$x^2+bx+\left(\frac{b}{2}\right)^2$	Perfect square trinomial
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	Quadratic formula
$(ax)^2-b^2=(ax+b)(ax-b)$	Difference of squares

Exponential Functions	
$1+r$	Growth factor
$1-r$	Decay factor
$f(t)=a(1+r)^t$	Exponential growth function
$f(t)=a(1-r)^t$	Exponential decay function
$f(x)=ab^x$	Exponential function in general form

General	
(x, y)	Ordered pair
$(x, 0)$	x-intercept
$(0, y)$	y-intercept

Equations of Circles	
$(x-h)^2+(y-k)^2=r^2$	Standard form
$x^2+y^2=r^2$	Center at (0, 0)
$Ax^2+By^2+Cx+Dy+E=0$	General form

Properties of Radicals
$\sqrt{ab}=\sqrt{a}\bullet\sqrt{b}$
$\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$

Imaginary Numbers
$i=\sqrt{-1}$
$i^2=-1$
$i^3=-i$
$i^4=1$

Radicals to Rational Exponents
$\sqrt[n]{a}=a^{\frac{1}{n}}$
$\sqrt[n]{x^m}=x^{\frac{m}{n}}$

Multiplication of Complex Conjugates
$(a+bi)(a-bi)=a^2+b^2$

Properties of Exponents	
Property	General rule
Zero Exponent	$a^0=1$
Negative Exponent	$b^{-\frac{m}{n}}=\frac{1}{b^{\frac{m}{n}}}$
Product of Powers	$a^m\bullet a^n=a^{m+n}$
Quotient of Powers	$\frac{a^m}{a^n}=a^{m-n}$
Power of a Power	$\left(b^m\right)^n=b^{mn}$
Power of a Product	$\left(bc\right)^n=b^nc^n$
Power of a Quotient	$\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}$

DATA ANALYSIS

Rules and Equations	
$P(E)=\dfrac{\text{\# of outcomes in } E}{\text{\# of outcomes in sample space}}$	Probability of event *E*
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$	Addition rule
$P(\overline{A})=1-P(A)$	Complement rule
$P(B\vert A)=\dfrac{P(A\cap B)}{P(A)}$	Conditional probability
$P(A\cap B)=P(A)\bullet P(B\vert A)$	Multiplication rule
$P(A\cap B)=P(A)\bullet P(B)$	Multiplication rule if *A* and *B* are independent
${}_nC_r=\dfrac{n!}{(n-r)!r!}$	Combination
${}_nP_r=\dfrac{n!}{(n-r)!}$	Permutation
$n!=n\bullet(n-1)\bullet(n-2)\bullet\cdots\bullet 1$	Factorial
$E(X)=X_1\bullet P(X_1)+X_2\bullet P(X_2)+X_3\bullet P(X_3)+\cdots+X_n\bullet P(X_n)$	Expected Value

Symbols	
$\varnothing$	Empty/null set
$\cap$	Intersection, "and"
$\cup$	Union, "or"
$\subset$	Subset
$\overline{A}$	Complement of Set A
!	Factorial
${}_nC_r$	Combination
${}_nP_r$	Permutation

GEOMETRY

Symbols	
$\overset{\frown}{ABC}$	Major arc length
$\overset{\frown}{AB}$	Minor arc length
$\angle$	Angle
$\odot$	Circle
$\cong$	Congruent
$\overleftrightarrow{PQ}$	Line
$\overline{PQ}$	Line segment
$\overrightarrow{PQ}$	Ray
$\parallel$	Parallel
$\perp$	Perpendicular
$\bullet$	Point
$\triangle$	Triangle
$\square$	Parallelogram
A'	Prime
$\circ$	Degrees
θ	Theta
ϕ	Phi
π	Pi

Area	
$A = lw$	Rectangle
$A=\frac{1}{2}bh$	Triangle
$A=\pi r^2$	Circle
$A=\frac{1}{2}(b_1+b_2)h$	Trapezoid

Trigonometric Ratios		
$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}}$	$\cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan\theta=\frac{\text{opposite}}{\text{adjacent}}$
$\csc\theta=\frac{\text{hypotenuse}}{\text{opposite}}$	$\sec\theta=\frac{\text{hypotenuse}}{\text{adjacent}}$	$\cot\theta=\frac{\text{adjacent}}{\text{opposite}}$

Trigonometric Identities
$\sin\theta=\cos(90°-\theta)$
$\cos\theta=\sin(90°-\theta)$
$\tan\theta=\frac{\sin\theta}{\cos\theta}$
$\csc\theta=\frac{1}{\sin\theta}$
$\sec\theta=\frac{1}{\cos\theta}$
$\cot\theta=\frac{1}{\tan\theta}$
$\cot\theta=\frac{\cos\theta}{\sin\theta}$
$\sin^2\theta+\cos^2\theta=1$

Pythagorean Theorem
$a^2 + b^2 = c^2$

Volume	
$V = lwh$	Rectangular prism
$V = Bh$	Prism
$V=\frac{1}{3}\pi r^2 h$	Cone
$V=\frac{1}{3}Bh$	Pyramid
$V=\pi r^2 h$	Cylinder
$V=\frac{4}{3}\pi r^3$	Sphere

Distance Formula
$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Dilation
$D_k(x,y)=(kx,ky)$

Pi Defined
$\pi=\frac{\text{circumference}}{\text{diameter}}=\frac{\text{circumference}}{2\bullet\text{radius}}$

Circumference of a Circle	
$C=2\pi r$	Circumference given the radius
$C=\pi d$	Circumference given the diameter

Inverse Trigonometric Functions
Arcsin $\theta = \sin^{-1}\theta$
Arccos $\theta = \cos^{-1}\theta$
Arctan $\theta = \tan^{-1}\theta$

Converting Between Degrees and Radians
$\frac{\text{radian measure}}{\pi}=\frac{\text{degree measure}}{180}$

Arc Length	
$s=\theta r$	Arc length (θ in radians)

Midpoint Formula
$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

MEASUREMENTS

Length
Metric
1 kilometer (km) = 1000 meters (m)
1 meter (m) = 100 centimeters (cm)
1 centimeter (cm) = 10 millimeters (mm)
Customary
1 mile (mi) = 1760 yards (yd)
1 mile (mi) = 5280 feet (ft)
1 yard (yd) = 3 feet (ft)
1 foot (ft) = 12 inches (in)

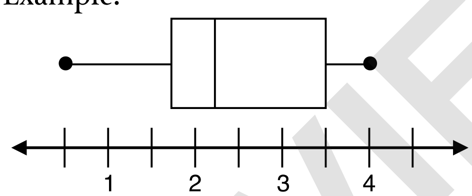
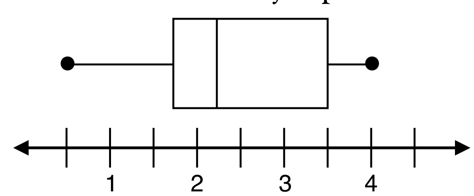
Volume and Capacity
Metric
1 liter (L) = 1000 milliliters (mL)
Customary
1 gallon (gal) = 4 quarts (qt)
1 quart (qt) = 2 pints (pt)
1 pint (pt) = 2 cups (c)
1 cup (c) = 8 fluid ounces (fl oz)

Weight and Mass
Metric
1 kilogram (kg) = 1000 grams (g)
1 gram (g) = 1000 milligrams (mg)
1 metric ton (MT) = 1000 kilograms
Customary
1 ton (T) = 2000 pounds (lb)
1 pound (lb) = 16 ounces (oz)

English	Unit/Topic	Español
	A	
acute angle an angle measuring less than 90° but greater than 0°	1/A	**ángulo agudo** ángulo que mide menos de 90˚ pero más de 0˚
acute triangle a triangle in which all of the angles are acute (less than 90˚)	2/B	**triángulo agudo** triángulo en el que todos los ángulos son agudos (menos de 90°)
Addition Rule If *A* and *B* are any two events, then the probability of *A* or *B*, denoted *P*(*A* or *B*), is given by: *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*). Using set notation, the rule is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.	7/B	**Regla de la suma** Si *A* y *B* son dos eventos cualquiera, entonces la probabilidad de *A* o *B*, que se indica con *P* (*A* o *B*), está dada por: *P*(*A* o *B*) **=** *P*(*A*) + *P*(*B*) – *P*(*A* y *B*). Con el uso de notación de conjuntos, la regla es $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
adjacent angles angles that lie in the same plane and share a vertex and a common side. They have no common interior points.	1/D	**ángulos adyacentes** ángulos en el mismo plano que comparten un vértice y un lado común. No tienen puntos interiores comunes.
adjacent side the leg next to an acute angle in a right triangle that is not the hypotenuse	3/D	**lado adyacente** el cateto junto a un ángulo agudo en un triángulo rectángulo que no es la hipotenusa
alternate exterior angles angles that are on opposite sides of the transversal and lie on the exterior of the two lines that the transversal intersects	1/D	**ángulos exteriores alternos** ángulos en lados opuestos de la transversal que se sitúan en el exterior de las dos líneas que corta la transversal
alternate interior angles angles that are on opposite sides of the transversal and lie within the interior of the two lines that the transversal intersects	1/D	**ángulos interiores alternos** ángulos que están en los lados opuestos de la transversal y se ubican en el interior de las dos líneas que corta la transversal
altitude the perpendicular line segment from a vertex of a figure to its opposite side; height	3/C 3/E	**altitud** línea perpendicular desde el vértice de una figura hasta su lado opuesto; altura
angle two rays or line segments sharing a common endpoint. Angles can be measured in degrees or radians; written as $\angle A$.	1/A	**ángulo** dos semirrectas o segmentos de línea que comparten un extremo común; Los ángulos se pueden medir en grados o radianes; se expresa como $\angle$

English	Unit/Topic	Español
Angle Addition Postulate If D is in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$. If $m\angle ABD + m\angle DBC = m\angle ABC$, then D is in the interior of $\angle ABC$.	1/D	**Postulado de la suma de ángulos** Si D está en el interior de $\angle ABC$, entonces $m\angle ABD + m\angle DBC = m\angle ABC$. Si $m\angle ABD + m\angle DBC = m\angle ABC$, entonces D está en el interior de $\angle ABC$.
Angle-Angle (AA) Similarity Statement If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.	3/B	**Criterio de semejanza ángulo-ángulo (AA)** Si dos ángulos de un triángulo son congruentes con dos ángulos de otro triángulo, entonces los triángulos son similares.
angle-angle-side (AAS) if two angles and a non-included side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the triangles are congruent	2/A	**ángulo-ángulo-lado (AAS)** si dos ángulos y un lado no incluido de un triángulo son congruentes con los correspondientes ángulos y dos lados de un segundo triángulo, entonces los triángulos son congruentes
angle-side-angle (ASA) if two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent	2/A	**ángulo-lado-ángulo (ASA)** si dos ángulos y el lado incluido de un triángulo son congruentes con los dos ángulos y el lado incluido de otro triángulo, entonces los dos triángulos son congruentes
angle bisector a ray that divides an angle into two congruent angles	3/C 6/B	**bisectriz del ángulo** semirrecta que divide un ángulo en dos ángulos congruentes
angle of depression the angle created by a horizontal line and a downward line of sight to an object that is below the observer	3/E	**ángulo de depresión** ángulo creado por una línea horizontal y una línea de mira descendente en relación a un objeto que se encuentra por debajo del observador
angle of elevation the angle created by a horizontal line and an upward line of sight to an object that is above the observer	3/E	**ángulo de elevación** ángulo creado por una línea horizontal y una línea de mira ascendente en relación a un objeto que se encuentra por encima del observador
angle-side-angle (ASA) if two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent	2/A	**ángulo-lado-ángulo (ASA)** si dos ángulos y el lado incluido de un triángulo son congruentes con los dos ángulos y el lado incluido de otro triángulo, entonces los dos triángulos son congruentes

English	Unit/Topic	Español
angle of rotation the angle through which a figure is rotated about a center point; the measure of the angle created by connecting the center of rotation to a point on the preimage and to the corresponding point on the image	1/C	**ángulo de rotación** el ángulo a través del cual se hace girar una figura alrededor de un punto central; la medida del ángulo creado por la conexión del centro de rotación a un punto de la imagen inversa y para el punto correspondiente en la imagen
annulus the region between two circles that have different radii	4/B	**anillo (o corona circular)** es la región entre dos círculos de diferentes radios
arc part of a circle's circumference	6/A	**arco** parte de la circunferencia de un círculo
arc length the distance between the endpoints of an arc; written as $m\overset{\frown}{AB}$	6/D	**longitud de arco** distancia entre los extremos de un arco; se expresa como $m\overset{\frown}{AB}$
arccosine the inverse of the cosine function, written $\cos^{-1}\theta$ or $\arccos\theta$	3/E	**arcocoseno** inversa de la función coseno; se expresa $\cos^{-1}\theta$ o $\arccos\theta$
Archimedes a Greek mathematician, physician, engineer, and inventor who lived from 287–212 B.C.; considered to be one of the greatest mathematicians of all time	4/A	**Arquímedes** fue un matemático, físico, ingeniero e inventor griego que vivió entre 287 y 212 A.C.; se lo considera uno de los matemáticos más importantes de todos los tiempos
arc length the distance between the endpoints of an arc; written as $d\left(\overset{\frown}{ABC}\right)$	1/A	**longitud de arco** distancia entre los extremos de un arco; se expresa como $d\left(\overset{\frown}{ABC}\right)$
arcsine the inverse of the sine function, written $\sin^{-1}\theta$ or $\arcsin\theta$	3/E	**arcoseno** inversa de la función seno; se expresa $\text{sen}^{-1}\theta$ o $\text{arcsen}\,\theta$
arctangent the inverse of the tangent function, written $\tan^{-1}\theta$ or $\arctan\theta$	3/E	**arcotangente** inversa de la función tangente; se expresa $\tan^{-1}\theta$ o $\arctan\theta$
area the amount of space inside the boundary of a two-dimensional figure	5/C	**área** cantidad de espacio dentro del límite de una figura bidimensional
axis of rotation a line about which a plane figure can be rotated in three-dimensional space to create a solid figure, such as a diameter or a symmetry line	4/B	**eje de rotación** línea alrededor de la cual puede girar la figura de un plano en un espacio de tres dimensiones para crear una figura sólida, como un diámetro o una línea de simetría

English	Unit/Topic	Español
	B	
base the side that is opposite the vertex angle of an isosceles triangle	2/B	**base** el lado que es opuesto al ángulo vértice de un triángulo isósceles.
base the quantity that is being raised to an exponent in an exponential expression; in a^x, a is the base; or, the quantity that is raised to an exponent which is the value of the logarithm, such as 2 in the equation $\log_2 g(x) = 3 - x$	7/A	**base** cantidad que es elevada a un exponente en una expresión exponencial; en a^x, a es la base; o, la cantidad que se eleva a un exponente que es el valor del logaritmo, tal que 2 en la ecuación $\log_2 g(x) = 3 - x$
base angle an angle formed by the base and one of the legs of an isosceles triangle	2/B	**ángulo base** ángulo formado por la base y un lado congruente de un triángulo isósceles
box plot a plot showing the minimum, maximum, first quartile, median, and third quartile of a data set; the middle 50% of the data is indicated by a box. Example: 	7/A	**diagrama de caja** diagrama que muestra el mínimo, máximo, primer cuartil, mediana y tercer cuartil de un conjunto de datos; se indica con una caja el 50% medio de los datos. Ejemplo: 
	C	
causation a relationship between two events where a change in one event is responsible for a change in the second event	7/B	**causalidad** relación entre dos eventos en la que un cambio en un evento es responsable por un cambio en el segundo evento
Cavalieri's Principle The volumes of two objects of equal height are equal if the areas of their corresponding cross sections are in all cases equal.	4/A	**Principio de Cavalieri** Los volúmenes de dos objetos de igual altura son iguales si las superficies de sus correspondientes secciones transversales son en todos los casos iguales.
center of dilation a point through which a dilation takes place; all the points of a dilated figure are stretched or compressed through this point	3/A	**centro de dilatación** punto a través del cual se produce una dilatación; todos los puntos de una figura dilatada se alargan o comprimen a través de este punto

English	Unit/Topic	Español
central angle an angle with its vertex at the center of a circle	6/A 6/D	**ángulo central** ángulo con su vértice en el centro de un círculo
centroid the intersection of the medians of a triangle	2/B	**centroide** intersección de las medianas de un triángulo
chord a segment whose endpoints lie on the circumference of the circle	6/A	**cuerda** segmento cuyos extremos se ubican en la circunferencia del círculo
circle the set of points on a plane at a fixed distance, or radius, from a given point, the center	1/A 6/A	**círculo** conjunto de puntos en un plano a determinada distancia, o radio, de un único punto, el centro.
circular arc on a circle, the set of points between the endpoints of two radii	1/A	**arco circular** en un círculo, conjunto de puntos no compartidos entre los extremos de dos radios
circumcenter the intersection of the perpendicular bisectors of a triangle	2/B 6/B	**circuncentro** intersección de las bisectrices perpendiculares de un triángulo
circumference the distance around a circle; $C = 2\pi r$ or $C = \pi d$, for which C represents circumference, r represents the circle's radius, and d represents the circle's diameter	6/A 6/D	**circunferencia** distancia alrededor de un círculo; $C = 2\pi r$ o $C = \pi d$, en donde C representa la circunferencia, r representa el radio del círculo y d, su diámetro
circumscribed angle the angle formed by two lines that are tangent to a circle and whose vertex is outside of the circle	6/A	**ángulo circunscrito** ángulo formado por dos líneas líneas que son tangentes a un círculo y cuyo vértice está fuera del círculo
circumscribed circle a circle that passes through all vertices of a polygon	2/B 6/B	**círculo circunscrito** círculo que pasa a través de todos los vértices de un polígono
circumscribed triangle triangle whose sides are tangent to an interior circle	6/B	**triángulo circunscrito** triángulo cuyos lados son tangentes a un círculo interior
clockwise rotating a figure in the direction that the hands on a clock move	1/B 1/C	**sentido horario** rotación de una figura en la dirección en que se mueven las agujas de un reloj
cofunction a trigonometric function whose ratios have the same values when applied to the two acute angles in a right triangle. The sine of one acute angle in a right triangle is equal to the cosine of the other acute angle, so sine and cosine are cofunctions.	3/D	**cofunción** función trigonométrica cuyas proporciones tienen los mismos valores cuando se aplican a los dos ángulos de un triángulo rectángulo. El seno de un ángulo agudo en un triángulo rectángulo es igual al coseno del otro ángulo agudo, por lo seno y coseno son cofunciones.

English	Unit/Topic	Español
collinear points points that lie on the same line	3/A	**puntos colineales** puntos que se ubican en la misma línea
combination a subset of a group of objects taken from a larger group of objects; the order of the objects does not matter, and objects may be repeated. A combination of size *r* from a group of *n* objects can be represented using the notation ${}_nC_r$, where ${}_nC_r = \frac{n!}{(n-r)!r!}$.	7/B	**combinación** subconjunto de un grupo de objetos tomado de un grupo de objetos más grande; el orden de los objetos no importa y los objetos pueden repetirse. Una combinación de tamaño *r* de un grupo de *n* objetos puede representarse con la notación ${}_nC_r$, donde ${}_nC_r = \frac{n!}{(n-r)!r!}$.
common external tangent a tangent that is common to two circles and does not intersect the segment joining the radii of the circles	6/C	**tangente común externa** tangente común a dos círculos que no corta el segmento que une los radios de los círculos
common internal tangent a tangent that is common to two circles and intersects the segment joining the radii of the circles	6/C	**tangente común interna** tangente común a dos círculos que corta el segmento que une los radios de los círculos
common tangent a line tangent to two circles	6/C	**tangente común** recta tangente a dos círculos
complement a set whose elements are not in another set, but are in some universal set being considered. The complement of set *A*, denoted by $\overline{A}$, is the set of elements that are in the universal set, but not in *A*. The event does not occur. The probability of an event not occurring is 1 minus the probability of the event occurring, $P\left(\overline{A}\right)=1-P(A)$.	7/B	**complemento** conjunto cuyos elementos no se encuentran en otro conjunto, pero están en algún conjunto universal que se considera. El complemento del conjunto *A*, que se indica con $\overline{A}$, es el conjunto de elementos que se encuentran en el conjunto universal, pero no en *A*. El evento no se produce. La probabilidad de que un evento no se produzca es 1 menos la probabilidad de que se produzca, $P\left(\overline{A}\right)=1-P(A)$.
complementary angles two angles whose sum is 90°	1/D 3/D	**ángulos complementarios** dos ángulos cuya suma es 90°
compound event the combination of two or more simple events	7/B	**evento compuesto** combinación de dos o más eventos simples

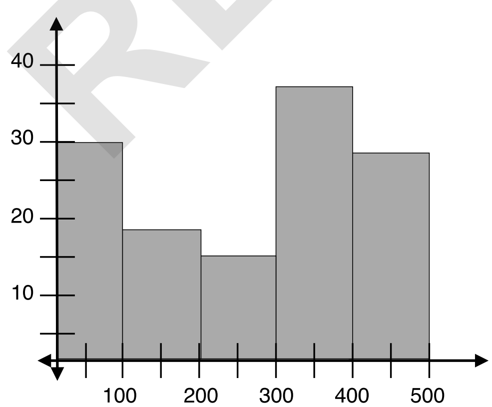
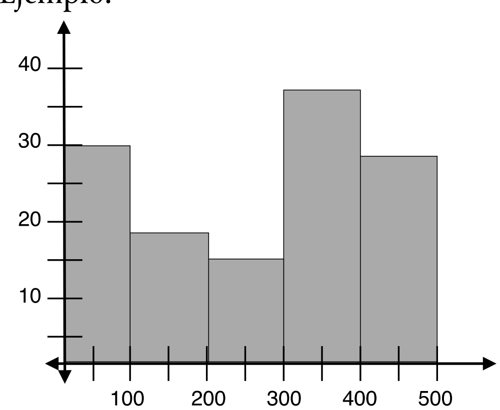
English	Unit/Topic	Español
compound probability the probability of compound events	7/B	**probabilidad compuesta** probabilidad de eventos compuestos
compression a transformation in which a figure becomes smaller; compressions may be horizontal (affecting only horizontal lengths), vertical (affecting only vertical lengths), or both	1/C 3/A	**compresión** transformación en la que una figura se hace más pequeña; las compresiones pueden ser horizontales (cuando afectan sólo la longitud horizontal), verticales (cuando afectan sólo la longitud vertical), o en ambos sentidos
concave polygon a polygon with at least one interior angle greater than 180° and at least one diagonal that does not lie entirely inside the polygon	2/C	**polígono cóncavo** polígono con al menos un ángulo interior de más de 180° y con al menos una diagonal que no se ubica por completo dentro de él
concentric circles coplanar circles that have the same center	6/A	**círculos concéntricos** círculos coplanares que tienen el mismo centro
concurrent lines three or more lines that intersect at a single point	2/B	**rectas concurrentes** tres o más rectas con intersección en un solo punto
cone a solid or hollow object that tapers from a circular or oval base to a point	4/A	**cono** objeto sólido o hueco que se estrecha desde una base circular u ovalada hasta un punto
congruency transformation a transformation in which a geometric figure moves but keeps the same size and shape; a dilation where the scale factor is equal to 1	1/C 3/A	**transformación de congruencia** transformación en la cual una figura geométrica se mueve pero mantiene el mismo tamaño y la misma forma; dilatación en la que el factor de escala es igual a 1
congruent having the same shape, size, lines, and angles; the symbol for congruent is $\cong$.	1/A 1/C 4/B 5/A	**congruente** tiene la misma forma, tamaño, líneas y anglos; el símbolo paracongruente es $\cong$.
congruent angles two angles that have the same measure	2/A	**ángulos congruentes** dos ángulos con la misma medida
congruent arcs two arcs that have the same measure and are either of the same circle or of congruent circles	6/A	**arcos congruentes** dos arcos que tienen la misma medida y son parte del mismo círculo o de círculos congruentes
congruent sides two sides that have the same length	2/A	**lados congruentes** dos lados con la misma longitud

English	Unit/Topic	Español
congruent triangles triangles having the same angle measures and side lengths	2/A	**triángulos congruentes** triángulos con las mismas medidas de ángulos y longitudes de lados
consecutive angles angles that lie on the same side of a figure	2/C	**ángulos consecutivos** ángulos ubicados en el mismo lado de una figura
converse of the Pythagorean Theorem If the sum of the squares of the measures of the two shorter sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.	3/C	**conversa del teorema de Pitágoras** Si la suma de los cuadrados de las medidas de dos lados más cortos de un triángulo equivale al cuadrado de la medida del lado más largo, entonces el triángulo es rectángulo.
convex polygon a polygon with no interior angle greater than 180°; all diagonals lie inside the polygon	2/C	**polígono convexo** polígono sin ángulo interior de más de 180°; todas las diagonales están dentro del polígono
coordinate proof a proof that involves calculations and makes reference to the coordinate plane	2/B	**prueba de coordenadas** prueba que involucra cálculos y hace referencia al plano de coordenadas
correlation coefficient a quantity that assesses the strength of a linear relationship between two variables, ranging from –1 to 1; a correlation coefficient of –1 indicates a strong negative correlation, a correlation coefficient of 1 indicates a strong positive correlation, and a correlation coefficient of 0 indicates a very weak or no linear correlation	7/B	**coeficiente de correlación** cantidad que evalúa la fuerza de una relación lineal entre dos variables, que varía de –1 a 1; un coeficiente de correlación de –1 indica una fuerte correlación negativa, un coeficiente de correlación de 1 indica una fuerte correlación positiva, y un coeficiente de correlación de 0 indica una correlación muy débil o no lineal
corresponding angles 1. angles of two figures that lie in the same position relative to the figure. In transformations, the corresponding angles are the preimage and image angles, so $\angle A$ and $\angle A'$ are corresponding angles and so on. 2. angles in the same relative position with respect to the transversal and the intersecting lines	1/C 1/D 2/A	**ángulos correspondientes** 1. ángulos de dos figuras que se ubican en la misma posición relativa a la figura. En las transformaciones, los angulos correspondientes son los angulos de preimagen e imagen, de manera que $\angle A$ y $\angle A'$ son los angulos correspondientes, etc. 2. ángulos en la misma posición relativa con respecto a las líneas transversal y de intersección

English	Unit/Topic	Español
Corresponding Parts of Congruent Triangles are Congruent (CPCTC) if two or more triangles are proven congruent, then all of their corresponding parts are congruent as well	2/A	**Las partes correspondientes de triángulos congruentes son congruentes (CPCTC)** si se comprueba que dos o más triángulos son congruentes, entonces todas sus partes correspondientes son también congruentes
corresponding sides sides of two figures that lie in the same position relative to the figure. In transformations, the corresponding sides are the preimage and image sides, so $\overline{AB}$ and $\overline{A'B'}$ are corresponding sides and so on.	1/C 2/A 3/A	**lados correspondientes** lados de dos figuras que están en la misma posición relativa a la figura. En las transformaciones, los lados correspondientes son los de preimagen e imagen, entonces $\overline{AB}$ y $\overline{A'B'}$ son los lados correspondientes, etc.
cosecant the reciprocal of the sine ratio; $\csc\theta = \frac{\text{length of hypotenuse}}{\text{length of opposite side}}$ or $\csc\theta = \frac{1}{\sin\theta}$	3/D 3/E	**cosecante** razón inversa del seno; $\csc\theta = \frac{\text{longitud de la hipotenusa}}{\text{longitud del lado opuesto}}$ o $\csc\theta = \frac{1}{\text{sen}\,\theta}$
cosine a trigonometric function of an acute angle in a right triangle that is the ratio of the length of the side adjacent to the length of the hypotenuse; $\cos\theta = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$	3/D	**coseno** función trigonométrica de un ángulo agudo en un triángulo rectángulo que es la proporción de la longitud de lado adyacente a la longitud de la hipotenusa; $\cos\theta = \frac{\text{longitud del lado adyacente}}{\text{longitud de la hipotenusa}}$
cotangent the reciprocal of tangent; $\cot\theta = \frac{\text{length of adjacent side}}{\text{length of opposite side}}$ or $\cot\theta = \frac{1}{\tan\theta}$	3/D 3/E	**cotangente** recíproco de la tangente, $\cot\theta = \frac{\text{longitud del lado adyacente}}{\text{longitud del lado opuesto}}$ o $\cot\theta = \frac{1}{\tan\theta}$
counterclockwise rotating a figure in the opposite direction that the hands on a clock move	1/B 1/C	**en sentido antihorario** rotación de una figura en la dirección opuesta a la que se mueven las agujas de un reloj
cross section the plane figure formed by the intersection of a plane with a solid figure	4/B	**sección transversal** figura del plano formada por la intersección de un plano con una figura sólida

English	Unit/Topic	Español
cylinder a solid or hollow object that has two parallel congruent bases connected by a curved surface; the bases are usually circular	4/A	**cilindro** objeto sólido o hueco que tiene dos bases paralelas conectadas por medio de una superficie curva
	D	
decay factor $1 - r$ in the exponential decay model $f(t) = a(1 - r)^t$, or b in the exponential function $f(t) = ab^t$ if $0 < b < 1$; the multiple by which a quantity decreases over time. The general form of an exponential function modeling decay is $f(t) = a(1 - r)^t$.	7/A	**factor de decaimiento** $1 - r$ en el modelo de decaimiento exponencial $f(t) = a(1 - r)^t$, o b en la función exponencial $f(t) = ab^t$ si $0 < b < 1$; el múltiplo por el que una cantidad disminuye con el tiempo. La forma general de una función exponencial que determina decaimiento es $f(t) = a(1 - r)^t$.
density the amount, number, or other quantity per unit of area or volume of some substance or population being studied	4/B	**densidad** la cantidad, el número u otra cantidad por unidad de área o volumen de alguna sustancia o población que está siendo estudiada
dependent events events that are not independent. The outcome of one event affects the probability of the outcome of another event.	7/B	**eventos dependientes** eventos que no son independientes. El resultado de un evento afecta la probabilidad del resultado de otro.
diagonal a line segment that connects nonconsecutive vertices of a polygon	2/C	**diagonal** línea que conecta vértices no consecutivos de un polígono
diameter a line segment passing through the center of a circle connecting two points on the circle; twice the radius	6/A	**diámetro** línea recta que pasa por el centro de un círculo y conecta dos puntos en el círculo; dos veces el radio
dilation a transformation in which a figure is either enlarged or reduced by a scale factor in relation to a center point	1/C 3/A	**dilatación** transformación en la que una figura se amplía o se reduce por un factor de escala en relación con un punto central
disjoint events events that have no outcomes in common. If A and B are disjoint events, then they cannot both occur. Disjoint events are also called mutually exclusive events.	7/B	**eventos disjuntos** eventos que no tienen resultados en común. Si A y B son eventos disjuntos, entonces no pueden producirse ambos. También se denominan eventos mutuamente excluyentes.
dissection breaking a figure down into its components	4/A	**disección** desglose de una figura en sus componentes

English	Unit/Topic	Español
distance along a line the linear distance between two points on a given line; written as PQ	1/A	**distancia a lo largo de una recta** distancia lineal entre dos puntos de una determinada línea; se expresa como PQ
distance formula a formula that states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	5/A 5/B 5/C	**fórmula de distancia** fórmula que establece la distancia entre los puntos (x_1, y_1) y (x_2, y_2) equivale a $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
dodecagon a 12-sided polygon	4/A	**dodecágono** polígono de 12 lados
	E	
element an item in a set; also called a member	7/B	**elemento** ítem en un conjunto; también se denomina miembro
empty set a set that has no elements, denoted by $\varnothing$. The empty set is also called the null set.	7/B	**conjunto vacío** conjunto que no contiene elementos, indicado con $\varnothing$. También se denomina conjunto nulo.
enlargement a dilation of a figure where the scale factor is greater than 1	3/A	**ampliación** dilatación de una figura en la que el factor de escala es mayor que 1
equal sets sets with all the same elements	7/B	**conjuntos iguales** conjuntos con todos los mismos elementos
equiangular having equal angles	2/B	**equiangular** que tiene ángulos iguales
equidistant the same distance from a reference point	1/C 1/D 6/B	**equidistante** a la misma distancia de un punto de referencia
equilateral triangle a triangle with all three sides equal in length	2/B	**triángulo equilátero** triángulo que tiene los tres lados de la misma longitud
event an outcome or set of outcomes of an experiment. An event is a subset of the sample space.	7/B	**evento** resultado o conjunto de resultados de un experimento. Un evento es un subconjunto del espacio de muestral.
experiment a process or action that has observable results. The results are called outcomes.	7/B	**experimento** proceso o acción con consecuencias observables. Las consecuencias se denominan resultados.
exterior angles angles that lie outside a pair of parallel lines	1/D 2/B	**ángulos exteriores** ángulos que están fuera de un par de líneas paralelas
exterior angle of a polygon an angle formed by one side of a polygon and the extension of another side	2/B	**ángulo exterior de un polígono** ángulo formado por un lado de un polígono y la extensión de otro lado

English	Unit/Topic	Español
	F	
factorial the product of an integer and all preceding positive integers, represented using a ! symbol; $n! = n \bullet (n - 1) \bullet (n - 2) \bullet \ldots \bullet 1$. For example, $5! = 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1$. By definition, $0! = 1$.	7/B	**factorial** producto de un entero y todos los enteros positivos anteriores, que se representa con el símbolo !; $n! = n \bullet (n - 1) \bullet (n - 2) \bullet \ldots \bullet 1$. Por ejemplo, $5! = 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1$. Por definición, $0! = 1$.
first quartile the value that identifies the lower 25% of the data; the median of the lower half of the data set; written as Q_1	7/A	**primer cuartil** valor que identifica el 25% inferior de los datos; mediana de la mitad inferior del conjunto de datos; se expresa Q_1
flow proof a graphical method of presenting the logical steps used to show an argument. In a flow proof, the logical statements are written in boxes and the reason for each statement is written below the box.	3/C	**prueba de flujo** método gráfico para presentar los pasos lógicos utilizados para mostrar un argumento. En una prueba de flujo, las declaraciones lógicas se expresan en casillas y la razón de cada declaración se escribe debajo de la casilla.
	G	
growth factor the multiple by which a quantity increases over time	7/A	**factor de crecimiento** múltiplo por el que una cantidad aumenta con el tiempo
	H	
histogram a frequency plot that shows the number of times a response or range of responses occurred in a data set. Example: 	7/A	**histograma** una diagrama de frecuencia que muestra la cantidad de veces que se produce una respuesta o rango de respuestas en un conjunto de datos. Ejemplo: 

English	Unit/Topic	Español
hypotenuse the side opposite the 90° angle in a right triangle	3/D	**hipotenusa** lado opuesto al ángulo de 90° en un triángulo rectángulo
hypotenuse-leg (HL) if the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg in another right triangle, then the two triangles are congruent	2/A	**hipotenusa de la ida (HL)** si la hipotenusa y una pierna de un triángulo rectángulo son congruentes a la hipotenusa y una pierna en otro triángulo rectángulo, entonces los dos triángulos son congruentes
	I	
identity an equation that is true regardless of what values are chosen for the variables	3/D 3/E	**identidad** ecuación verdadera independientemente de los valores elegidos para las variables
image the new, resulting figure after a transformation	1/A 1/C	**imagen** nueva figura resultante después de una transformación
incenter the intersection of the angle bisectors of a triangle	2/B 6/B	**incentro** intersección de las bisectrices del ángulo de un triángulo
included angle the angle between two sides of a triangle	2/A	**ángulo incluido** ángulo entre dos lados de un triángulo
included side the side between two angles of a triangle	2/A	**lado incluido** lado entre dos ángulos de un triángulo
independent events events such that the outcome of one event does not affect the probability of the outcome of another event	7/B	**eventos independientes** eventos en los que el resultado de un evento no afecta la probabilidad del resultado de otro evento
initial value (of an exponential function) the value that represents the point where the exponential function intersects the y-axis	7/A	**valor inicial (de una función exponencial)** el valor que representa el punto donde la función exponencial se cruza con el eje y
inscribed angle an angle formed by two chords whose vertex is on the circle	6/A	**ángulo inscrito** ángulo formado por dos cuerdas cuyo vértice está en el círculo
inscribed circle a circle that contains one point from each side of a triangle or other polygon	2/B 6/B	**círculo inscrito** círculo que contiene un punto de cada lado de un triángulo o otro polígono
inscribed quadrilateral a quadrilateral whose vertices are on a circle	6/B	**cuadrilátero inscrito** cuadrilátero cuyos vértices están en un círculo
inscribed triangle a triangle whose vertices are on a circle	6/B	**triángulo inscrito** triangulo cuyos vértices están en un círculo

English	Unit/Topic	Español
integers the set of positive and negative whole numbers and 0; the set {… –3, –2, –1, 0, 1, 2, 3, …}	3/D	**enteros** el conjunto de números enteros positivos y negativos y 0; el conjunto {… –3, –2, –1, 0, 1, 2, 3, …}
intercepted arc an arc whose endpoints lie on the sides of an angle and whose other points are in the interior of the angle	6/A	**arco interceptado** un arco cuyos extremos se encuentran en los lados de un ángulo y cuyos otros puntos se sitúan en el interior del ángulo
interior angle of a polygon an angle on the inside of a polygon that is formed by two consecutive sides of the polygon	2/B	**ángulo interior de un polígono** ángulo en el interior de un polígono que está formado por dos lados consecutivos del polígono
interior angles angles that lie between a pair of parallel lines	1/D 2/B	**ángulos interiores** ángulos ubicados entre un par de líneas paralelas
interquartile range the difference between the third and first quartiles; 50% of the data is contained within this range	7/A	**rango intercuartílico** diferencia entre el tercer y primer cuartil; el 50% de los datos está contenido dentro de este rango
intersection a set whose elements are each in both of two other sets. The intersection of sets *A* and *B*, denoted by $A \cap B$, is the set of elements that are in both *A* and *B*.	7/B	**intersección** conjunto cuyos elementos están todos en otros dos conjuntos. La intersección de los conjuntos *A* y *B*, indicada por $A \cap B$, es el conjunto de elementos que se encuentran tanto en *A* como en *B*.
irrational number a number that cannot be written as $\frac{m}{n}$, where *m* and *n* are integers and $n \neq 0$; any number that cannot be written as a decimal that ends or repeats	3/D 4/A	**números irracionales** un número que no pueden expresarse como $\frac{m}{n}$, en los que *m* y *n* son enteros y $n \neq 0$; cualquier número que no puede expresarse como decimal finito o periódico
irreducible radical a radical whose radicand contains no perfect square factors. In other words, the radical cannot be further reduced. For example, $\sqrt{7}$ is an irreducible radical because the radicand, 7, does not have any perfect square factors.	3/D	**radical irredutíble** un radical cuya radicand no contiene factores cuadrados perfectos. En otras palabras, el radical no puede ser más reducido. Por ejemplo, $\sqrt{7}$ es un radical irreducible porque la radicand, 7, no tiene ningún factor cuadrado perfecto.
isometry a transformation in which the preimage and image are congruent	1/A 1/C	**isometría** transformación en la que la preimagen y la imagen son congruentes

English	Unit/Topic	Español
isosceles trapezoid a trapezoid with one pair of opposite parallel sides and congruent legs	2/C	**trapezoide isósceles** trapezoide con un par de líneas paralelas opuestas y catetos congruentes
isosceles triangle a triangle with at least two congruent sides	2/B	**triángulo isósceles** triángulo con al menos dos lados congruentes
	K	
kite a quadrilateral with two distinct pairs of congruent sides that are adjacent	2/C	**cometa** cuadrilátero con dos pares distintos de lados congruentes que son adyacentes
	L	
lateral surface the curved portion of the surface of a cylinder; i.e., the side. This surface does not include the top or bottom of the cylinder.	4/A	**superficie lateral** la porción curva de la superficie de un cilindro; es decir, el lado. Esta superficie no incluye la parte superior o inferior del cilindro.
lateral surface area the area of the curved (lateral) surface of a cylinder; the formula for lateral surface area is $LSA = 2\pi rh$	4/A	**área de superficie lateral** el área de la superficie curva (lateral) de un cilindro; la fórmula para el área de la superficie lateral es $LSA = 2\pi rh$
legs congruent sides of an isosceles triangle, or the two shorter sides of a right triangle	2/B	**catetos** lados congruentes de un triángulo isósceles, o los dos lados más cortos de un triángulo rectángulo
limit the value that a sequence approaches as a calculation becomes more and more accurate	4/A	**límite** valor al que se aproxima una secuencia cuando un cálculo se vuelve cada vez más exacto
line the straight path connecting two points and extending beyond the points in both directions; written as $\overleftrightarrow{PQ}$ or ℓ	1/A	**línea recta** trayectoria recta que conecta dos puntos y que se extiende más allá de los puntos en ambas direcciones; se expresa como $\overleftrightarrow{PQ}$ o ℓ
line of reflection the perpendicular bisector of the segments that connect the corresponding vertices of the preimage and the image	1/C	**línea de reflexión** bisectriz perpendicular de los segmentos que conectan los vértices correspondientes de la preimagen y la imagen
line of symmetry a line separating a figure into two halves that are mirror images	1/A	**línea de simetría** línea que separa una figura en dos mitades que son imágenes en espejo

English	Unit/Topic	Español
line segment a part of a line that is between two endpoints and that includes the endpoints; written as $\overline{PQ}$	1/A 5/B	**segmento de línea** parte de una línea que se encuentra entre dos puntos finales y que incluye los puntos finales; escrito como $\overline{PQ}$
line symmetry e exists for a figure if for every point on one side of the line of symmetry, there is a corresponding point the same distance from the line of symmetry on the other side	1/A	**simetría lineal** la que existe en una figura si para cada punto a un lado de la línea de simetría, hay un punto correspondiente a la misma distancia de la línea de simetría en el otro lado
linear pair a pair of adjacent angles whose non-shared sides form a straight angle	1/D	**par lineal** par de ángulos adyacentes cuyos lados no compartidos forman un ángulo recto
	M	
mean a measure of center in a set of numerical data, computed by adding the values in a data set and then dividing the sum by the number of values in the data set; denoted as the Greek lowercase letter *mu*, μ; given by the formula $\mu = \frac{x_1 + x_2 + \ldots + x_n}{n}$, where each x-value is a data point and n is the total number of data points in the set	7/A	**media** medida del centro en un conjunto de datos numéricos, calculada al sumar los valores en un conjunto de datos y luego al dividir la suma por el número de valores en el conjunto de datos; indicada con la letra griega minúscula *mu*, μ; dada por la fórmula $\mu = \frac{x_1 + x_2 + \ldots + x_n}{n}$, donde cada valor de x es un punto de datos y n es la cantidad total de puntos de datos en el conjunto
mean absolute deviation the average distance between each data point and the mean; found by summing the absolute values of the difference between each data point and the mean, then dividing this sum by the total number of data points	7/A	**desviación media absoluta** distancia promedio entre cada punto de datos y la media; se determina al sumar los valores absolutos de la diferencia entre cada punto de datos y la media y luego dividir esta suma por la cantidad total de puntos de datos
major arc part of a circle's circumference that is larger than its semicircle	6/A	**arco mayor** parte de la circunferencia de un círculo que es mayor que su semicírculo

English	Unit/Topic	Español
measures of center values that describe expected and repeated data values in a data set; the mean and median are two measures of center	7/A	**medidas de centro** valores que describen los valores de datos esperados y repetidos de un conjunto de datos; la media y la mediana son dos medidas de centro
measures of spread a measure that describes the variance of data values, and identifies the diversity of values in a data set	7/A	**medidas de dispersión** medidas que describen la varianza de los valores de datos e identifican la diversidad de valores en un conjunto de datos
median the middle-most value of a data set; 50% of the data is less than this value, and 50% is greater than it	7/A	**mediana** valor máximo-medio de un conjunto de datos; el 50 % de los datos es menor que este valor y el otro 50 % es mayor que él
median of a triangle the segment joining the vertex to the midpoint of the opposite side	2/B	**mediana de un triángulo** segmento que une el vértice con el punto medio del lado opuesto
member an item in a set; also called an element	7/B	**miembro** ítem en un conjunto; también se denomina elemento
midpoint a point on a line segment that divides the segment into two equal parts	2/B 5/B 5/C	**punto medio** punto en un segmento de recta que lo divide en dos partes iguales
midpoint formula a formula that states the midpoint of a segment created by connecting (x_1, y_1) and (x_2, y_2) is given by the formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$	2/B 5/B	**fórmula de punto medio** fórmula que establece el punto medio de un segmento creado al conectar (x_1, y_1) con (x_2, y_2) está dado por la fórmula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
midsegment a line segment joining the midpoints of two sides of a triangle	2/B	**segmento medio** segmento de recta que une los puntos medios de dos lados de una figura
midsegment triangle the triangle formed when all three of the midsegments of a triangle are connected	2/B	**segmento medio de un triángulo** triángulo que se forma cuando los tres segmentos medios de un triángulo están conectados
minor arc part of a circle's circumference that is smaller than its semicircle	6/A	**arco menor** parte de la circunferencia de un círculo que es menor que su semicírculo

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Multiplication Rule the probability of two events, A and B, is $P(A \text{ and } B) = P(A) \bullet P(B\|A) = P(B) \bullet P(A\|B)$; for independent events A and B, the rule is $P(A \text{ and } B) = P(A) \bullet P(B)$.	7/B	**Regla de multiplicación** probabilidad de que dos eventos, A y B, sea $P(A \text{ y } B) = P(A) \bullet P(B\|A) = P(B) \bullet P(A\|B)$; para eventos independientes A y B, la regla es $P(A \text{ y } B) = P(A) \bullet P(B)$.
mutually exclusive events events that have no outcomes in common. If A and B are mutually exclusive events, then they cannot both occur. Mutually exclusive events are also called disjoint events.	7/B	**eventos mutuamente excluyentes** eventos que no tienen resultados en común. Si A y B son eventos mutuamente excluyentes, entonces no pueden producirse ambos. También se denominan eventos disjuntos.
	N	
nonadjacent angles angles that have no common vertex or common side, or have shared interior points	1/D	**ángulos no adyacentes** ángulos que no tienen vértices ni lados comunes, o que tienen puntos interiores compartidos
non-rigid motion a transformation done to a figure that changes the figure's shape and/or size	1/C 2/A 3/A	**movimiento no rígido** transformación hecha a una figura que cambia su forma o tamaño
***n*th root** a number that, when multiplied by itself n times, equals x.	3/D	**raíz enésima** un número que multiplicado por sí mismo n veces da x.
null set a set that has no elements, denoted by $\varnothing$. The null set is also called the empty set.	7/B	**conjunto nulo** conjunto que no tiene elementos, indicado con $\varnothing$. También se denomina conjunto vacío.
	O	
obtuse angle an angle measuring greater than 90° but less than 180°	1/A	**ángulo obtuso** ángulo que mide más de 90˚ pero menos de 180˚
obtuse triangle a triangle with one angle that is obtuse (greater than 90˚)	2/B	**obtuse triangle** a triangle with one angle that is obtuse (greater than 90°)
one-to-one a relationship wherein each input is mapped to exactly one output, and each output is associated with exactly one input	1/A	**unívoca** relación en la que cada entrada se asigna a exactamente una salida, y cada salida está asociada con exactamente una entrada
opposite side the side across from an angle in a triangle	3/D	**lado opuesto** lado al otro lado de un ángulo en un triángulo
orthocenter the intersection of the altitudes of a triangle	2/B	**ortocentro** intersección de las alturas de un triángulo

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outlier 1. a data value that is much greater than or much less than the rest of the data in a data set; mathematically, any data less than $Q_1 - 1.5(IQR)$ or greater than $Q_3 + 1.5(IQR)$ is an outlier 2. a value far above or below other values of a distribution	7/A	**valor atípico** 1. valor de datos que es mucho mayor o mucho menor que el resto de los datos de un conjunto de datos; en matemática, cualquier dato menor que $Q_1 - 1{,}5(IQR)$ o mayor que $Q_3 + 1{,}5(IQR)$ es un valor atípico 2. un valor muy por encima o muy por debajo de otros valores de una distribución
outcome a result of an experiment	7/B	**resultado** consecuencia de un experimento
	P	
paragraph proof statements written out in complete sentences in a logical order to show an argument	3/C	**prueba de párrafo** declaraciones redactadas en oraciones completas en orden lógico para demostrar un argumento
parallel lines lines in a plane that do not share any points and never intersect; written as $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$; line segments and rays can also be parallel	1/A 3/C 5/A	**líneas paralelas** líneas en un plano que no comparten ningún punto y nunca se cortan; se expresan como $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$; segmentos de línea y los rayos también pueden ser paralelos
parallelogram a special type of quadrilateral with two pairs of opposite sides that are parallel; denoted by the symbol ▱	2/C 5/A	**paralelogramo** un tipo especial de cuadrilátero con dos pares de lados opuestos paralelos; se expresa con el símbolo ▱
perimeter the distance around a two-dimensional figure	5/C	**perímetro** distancia alrededor de una figura bidimensional
permutation a selection of objects where the order matters and is found either using n^r, if repetitions are allowed, or by using ${}_nP_r = \frac{n!}{(n-r)!}$, where n is the number of objects to select from and r is the number of objects being selected and ordered.	7/B	**permutación** selección de objetos en la que el orden importa y se encuentra con el uso de n^r, si se permiten las repeticiones, o con ${}_nP_r = \frac{n!}{(n-r)!}$, donde n es la cantidad de objetos de donde seleccionar y r es la cantidad de objetos seleccionados y ordenados.

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perpendicular bisector a line that intersects a segment at its midpoint at a right angle	1/D 6/B	**bisectriz perpendicular** línea que corta un segmento en su punto medio en ángulo recto
perpendicular lines two lines that intersect at a right angle (90°); written as $\overleftrightarrow{AB} \perp \overleftrightarrow{PQ}$; line segments and rays can also be perpendicular	1/A 1/D 5/A	**líneas perpendiculares** dos líneas que se cortan en ángulo recto (90˚); se expresan como $\overleftrightarrow{AB} \perp \overleftrightarrow{PQ}$; segmentos de línea y los rayos también pueden ser perpendicular
phi **(ϕ)** a Greek letter sometimes used to refer to an unknown angle measure	3/D	***fi*** **(ϕ)** letra del alfabeto griego que se utiliza a veces para referirse a la medida desconocida de un ángulo
pi **(π)** the ratio of circumference of a circle to the diameter; equal to approximately 3.14	6/A	***pi*** **(π)** proporción de la circunferencia de un círculo al diámetro; equivale aproximadamente a 3.14
plane a flat, two-dimensional figure without depth that is defined by three non-collinear points and extends infinitely in all directions	1/D 4/B	**plano** figura plana, bidimensional, sin profundidad, que está definido por tres puntos no colineales y se extiende infinitamente en todas direcciones
plane figure a two-dimensional shape on a plane	4/B	**figura plana** forma bidimensional sobre un plano
point an exact position or location in a given plane	1/A	**punto** posición o ubicación exacta en un plano determinado
point of concurrency a single point of intersection of three or more lines	2/B 6/B	**punto de concurrencia** punto único de intersección de tres o más líneas
point of rotation the fixed location that an object is turned around; the point can lie on, inside, or outside the figure	1/C	**punto de rotación** ubicación fija en torno a la que gira un objeto; el punto puede estar encima, dentro o fuera de la figura
point of tangency the only point at which a line and a circle intersect	6/C	**punto de tangencia** punto único de intersección entre una línea y un círculo
polygon two-dimensional figure with at least three sides	5/C	**polígono** figura bidimensional con al menos tres lados
polyhedron a three-dimensional solid that has faces that are polygons	4/A 4/B	**poliedro** un sólido tridimensional que tiene caras que son polígonos
postulate a true statement that does not require a proof	1/D 2/A	**postulado** declaración verdadera que no requiere prueba
preimage the original figure before undergoing a transformation	1/A 1/C	**preimagen** figura original antes de sufrir una transformación

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prism a hollow or solid object with two congruent bases that are polygons; the lateral faces are rectangles or parallelograms	4/A	**prisma** un sólido hueco o sólido con dos bases congruentes que son polígonos; las caras laterales son rectángulos o paralelogramos
probability a number from 0 to 1 inclusive or a percent from 0% to 100% inclusive that indicates how likely an event is to occur	7/B	**probabilidad** número de 0 a 1 inclusivo o porcentaje de 0% a 100% inclusivo que indica cuán probable es que se produzca un evento
probability model a mathematical model for observable facts or occurrences that are assumed to be random; a representation of a random phenomenon	7/B	**modelo de probabilidad** modelo matemático para hechos o sucesos observables que se presumen aleatorios; representación de un fenómeno aleatorio
probability of an event *E* denoted $P(E)$, and is given by $P(E)=\frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$ in a uniform probability model	7/B	**probabilidad de un evento *E*** se expresa como $P(E)$, y está dado por $P(E)=\frac{\text{número de resultados en } E}{\text{número de resultados en el espacio de muestreo}}$ en un modelo de probabilidad uniforme
proof a set of justified statements organized to form a convincing argument that a given statement is true	1/D 3/C	**prueba** conjunto de declaraciones justificadas y organizadas para formar un argumento convincente de que determinada declaracíones verdadera
proportional having a constant ratio to another quantity	3/B	**proporcional** que tiene una proporción constante con otra cantidad
pyramid a solid or hollow polyhedron that has a polygonal base and triangular faces that converge at a single vertex	4/A	**pirámide** un poliedro sólido o hueco que tiene una base poligonal y caras triangulares que convergen en un solo vértice
Pythagorean identities trigonometric identities that are derived from the Pythagorean Theorem: $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, and $1 + \cot^2\theta = \csc^2\theta$	3/E	**identidades Pitagóricas** identidades trigonométricas que se derivan de el teorema de Pitágoras: $\text{sen}^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, and $1 + \cot^2\theta = \csc^2\theta$
Pythagorean Theorem a theorem that relates the length of the hypotenuse of a right triangle (c) to the lengths of its legs (a and b). The theorem states that $a^2 + b^2 = c^2$.	3/E	**Teorema de Pitágoras** teorema que relaciona la longitud de la hipotenusa de un triángulo rectángulo (c) con las longitudes de sus catetos (a y b). El teorema establece que $a^2 + b^2 = c^2$.

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Q		
quadrant the coordinate plane is separated into four sections: • In Quadrant I, x and y are positive. • In Quadrant II, x is negative and y is positive. • In Quadrant III, x and y are negative. • In Quadrant IV, x is positive and y is negative.	1/B	**cuadrante** plano de coordenadas que se divide en cuatro secciones: • En el cuadrante I, x y y son positivos. • En el cuadrante II, x es negativo y y es positivo. • En el cuadrante III, x y y son negativos. • En el cuadrante IV, x es positivo y y es negativo.
quadrilateral a polygon with four sides	2/C 5/A	**cuadrilátero** polígono con cuatro lados
R		
radian the measure of the central angle that intercepts an arc equal in length to the radius of the circle; π radians = 180°	6/D	**radián** medida del ángulo central que intercepta un arco de longitud igual al radio del círculo; π radianes = 180°
radian measure the ratio of the arc intercepted by the central angle to the radius of the circle	6/D	**medida de radián** proporción del arco interceptado por el ángulo central al radio del círculo
radical expression an expression containing a root, such as $\sqrt[5]{9}$	3/D	**expresión radical** expresión que contiene una raíz, tal como $\sqrt[5]{9}$
radicand in a radical expression, the number under the root sign; in the expression $\sqrt{5}$, the radicand is 5	3/D	**radicand** en una expresión radical, el número bajo el signo de la raíz; en la expresión $\sqrt{5}$, la radicand es 5
radius 1. a line segment that extends from the center of a circle to a point on the circle. Its length is half the diameter. 2. the distance from the center to a point on the circle; equal to one-half the diameter	6/A	**radio** 1. segmento de línea que se extiende desde el centro de un círculo hasta un punto de la circunferencia del círculo. Su longitud es la mitad del diámetro. 2. distancia desde el centro a un punto en el círculo; equivale a la mitad del diámetro
ratio the relation between two quantities; can be expressed in words, fractions, decimals, or as a percentage	3/D	**proporción** relación entre dos cantidades; puede expresarse en palabras, fracciones, decimales o como porcentaje

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ratio identities identities that define tangent and cotangent in terms of sine and cosine; the following two identities are ratio identities: $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$	3/E	**identidades de proporciones** identidades que definen tangente y cotangente en términos de seno y el coseno; las dos identidades siguientes son identidades de proporciones: $\tan\theta = \frac{\text{sen}\,\theta}{\cos\theta}$ y $\cot\theta = \frac{\cos\theta}{\text{sen}\,\theta}$
ratio of similitude a ratio of corresponding sides; also known as the scale factor	3/B	**proporción de similitud** proporción de lados correspondientes; se conoce también como factor de escala
rational number a real number that can be written as $\frac{m}{n}$, where both m and n are integers and $n \neq 0$; a terminating or repeating decimal	3/D	**número racional** en los que m y n son enteros y $n \neq 0$; cualquier número que puede escribirse como decimal finito o periódico
ray a part of a line that starts at a point and extends infinitely in one direction; written as $\overrightarrow{PQ}$	1/A	**semirrecta** parte de una línea que comienza en un punto y se extiende infinitamente en una dirección línea con un solo extremo; se expresa como $\overrightarrow{PQ}$
real numbers the set of all rational and irrational numbers	3/D	**números reales** conjunto de todos los números racionales e irracionales
reciprocal a number that, when multiplied by the original number, has a product of 1	3/D	**recíproco** número que multiplicado por el número original tiene producto 1
reciprocal identities trigonometric identities that define cosecant, secant, and cotangent in terms of sine, cosine, and tangent: $\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\cot\theta = \frac{1}{\tan\theta}$, $\sin\theta = \frac{1}{\csc\theta}$, $\cos\theta = \frac{1}{\sec\theta}$, and $\tan\theta = \frac{1}{\cot\theta}$	3/E	**identidades recíprocas** identidades trigonométricas que definen cosecante, secante y cotangente en términos de seno, coseno y tangente: $\csc\theta = \frac{1}{\text{sen}\,\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\cot\theta = \frac{1}{\tan\theta}$, $\text{sen}\,\theta = \frac{1}{\csc\theta}$, $\cos\theta = \frac{1}{\sec\theta}$, and $\tan\theta = \frac{1}{\cot\theta}$
rectangle a special parallelogram with four right angles	2/C 5/A	**rectángulo** paralelogramo especial con cuatro ángulos rectos

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reduction a dilation where the scale factor is between 0 and 1	3/A	**reducción** dilatación en la que el factor de escala está entre 0 y 1
reflection a transformation where a mirror image is created; also called a flip	1/A 1/B	**reflexión** transformación por la cual se crea una imagen en espejo
Reflexive Property of Congruent Segments a segment is congruent to itself; $\overline{AB} \cong \overline{AB}$	3/C	**Propiedad reflexiva de congruencia de segmentos** un segmento es congruente con él mismo; $\overline{AB} \cong \overline{AB}$
regular polygon a closed two-dimensional figure with all sides congruent and all angles congruent	1/A 4/B	**polígono regular** figura bidimensional cerrada con todos los lados y todos los ángulos congruentes
regular polyhedron a polyhedron with faces that are all congruent regular polygons; the angles created by the intersecting faces are congruent	4/B	**poliedro regular** poliedro cuyas caras son todas polígonos regulares congruentes; los ángulos creados por las caras que se cruzan son congruentes
relative frequency (of an event) the number of times an event occurs divided by the number of times an experiment is performed	7/B	**frecuencia relativa (de un evento)** cantidad de veces que un evento se produce dividido por la cantidad de veces que se realiza el experimento
remote interior angles interior angles that are not adjacent to an exterior angle	2/B	**ángulos interiores remotos** ángulos interiores que no son adyacentes al ángulo exterior
rho **(ρ)** a lowercase Greek letter commonly used to represent density	4/B	***rho*** **(ρ)** letra griega minúscula comúnmente utilizada para representar densidad
rhombus a special parallelogram with all four sides congruent	2/C 5/A	**rombo** paralelogramo especial con sus cuatro lados congruentes
right angle an angle measuring 90˚	1/A 1/D	**ángulo recto** ángulo que mide 90˚
right prism a three-dimensional solid that has two congruent bases and rectangular faces that join the two bases at 90° angles	4/B	**prisma recto** un sólido tridimensional que tiene dos bases congruentes y caras rectangulares que se unen las dos bases en ángulos de 90°
right triangle a triangle with one angle that measures 90°	2/B 3/D	**triángulo rectángulo** triángulo con un ángulo que mide 90°
rigid motion a transformation done to a figure that maintains the figure's shape and size or its segment lengths and angle measures	1/C 2/A 3/A	**movimiento rígido** transformación que se realiza a una figura que mantiene su forma y tamaño o las longitudes de sus segmentos y las medidas de ángulos

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rotation 1. a transformation that turns a figure around a fixed center point; also called a turn 2. in three dimensions, a transformation in which a plane figure is revolved about one of its sides or a line that is not located in the plane of the figure, such that a solid figure is produced	1/A 1/B 4/B	**rotación** 1. transformación que convierte una figura alrededor de un punto central fijo; también llamado un giro 2. en tres dimensiones, transformación en la cual una figura plana se girar sobre uno de sus lados o una línea que no está ubicada en el plano de la figura, de manera que se produce una figura sólida
	S	
same-side exterior angles angles that lie on the same side of the transversal and are outside the lines that the transversal intersects; sometimes called consecutive exterior angles	1/D	**ángulos exteriores del mismo lado** ángulos que se ubican en el mismo lado de la transversal y están fuera de las líneas que corta la transversal; a veces se denominan ángulos exteriores consecutivos
same-side interior angles angles that lie on the same side of the transversal and are in between the lines that the transversal intersects; sometimes called consecutive interior angles	1/D	**ángulos interiores del mismo lado** ángulos que se ubican en el mismo lado de la transversal y están en medio de las líneas que corta la transversal; a veces se los denomina ángulos interiores consecutivos
sample space the set of all possible outcomes of an experiment	7/B	**espacio de muestreo** conjunto de todos los resultados posibles de un experimento
scale factor a multiple of the lengths of the sides from one figure to the transformed figure. If the scale factor is larger than 1, then the figure is enlarged. If the scale factor is between 0 and 1, then the figure is reduced.	1/C 3/A 3/D	**factor de escala** múltiplo de las longitudes de los lados de una figura a la figura transformada. Si el factor de escala es mayor que 1, entonces la figura se agranda. Si el factor de escala se encuentra entre 0 y 1, entonces la figura se reduce.
scalene triangle a triangle with no congruent sides	2/B	**triángulo escaleno** triángulo sin lados congruentes
secant the reciprocal of cosine; $\sec\theta = \frac{\text{length of hypotenuse}}{\text{length of adjacent side}}$	3/D 3/E	**secante** recíproco del coseno; $\sec\theta = \frac{\text{longitud de la hipotenusa}}{\text{longitud del lado adyacente}}$
secant line a line that intersects a circle at two points	6/A	**línea secante** recta que corta un círculo en dos puntos

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sector a portion of a circle bounded by two radii and their intercepted arc	6/D	**sector** porción de un círculo limitado por dos radios y el arco que cortan
Segment Addition Postulate If B is a point on line segment $\overline{AC}$, then $AB + BC = AC$. Conversely, if $AB + BC = AC$, then B is on line segment $\overline{AC}$.	3/C	**Segmento postulado de la suma** Si B es un punto sobre el segmento de línea de $\overline{AC}$, a continuación, $AB + BC = AC$. Por el contrario, si $AB + BC = AC$, entonces B es el segmento $\overline{AC}$.
semicircle an arc that is half of a circle	6/A	**semicírculo** arco que es la mitad de un círculo
set a collection or list of items	7/B	**conjunto** colección o lista de elementos
side-angle-side (SAS) if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent	2/A 3/C	**lado-ángulo-lado (SAS)** si dos lados y el ángulo incluido de un triángulo son congruentes con dos lados y el ángulo incluido de otro triángulo, entonces los dos triángulos son congruentes
Side-Angle-Side (SAS) Similarity Statement If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.	3/C	**Criterio de semejanza lado-ángulo-lado (SAS)** Si las medidas de dos lados de un triángulo son proporcionales a las medidas de dos lados correspondientes de otro triángulo y los ángulos incluidos son congruentes, entonces los triángulos son similares.
side-side-side (SSS) if three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent	2/A	**lado-lado-lado (SSS)** si los tres lados de un triángulo son congruentes con los tres lados de otro triángulo, entonces los dos triángulos son congruentes
Side-Side-Side (SSS) Similarity Statement If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.	3/C	**Criterio de semejanza lado-lado-lado (SSS)** Si las medidas de los lados correspondientes de dos triángulos son proporcionales, entonces los triángulos son similares.
similar (or **similar figures**) two figures that are the same shape but not necessarily the same size. Corresponding angles must be congruent and sides must have the same ratio. The symbol for representing similarity is $\sim$.	3/B 3/D 4/B	**similar** (o **figuras similares**) dos figuras que tienen la misma forma pero no necesariamente el mismo tamaño. Ángulos correspondientes deben ser congruentes y los lados deben tener la misma relación. El símbolo para representar similitud entre figuras es $\sim$.

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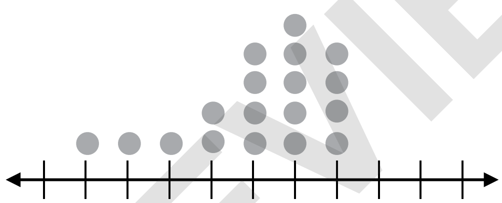
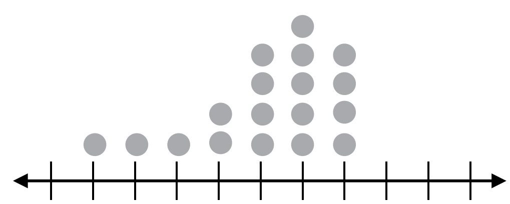
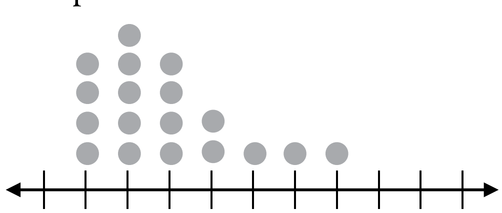
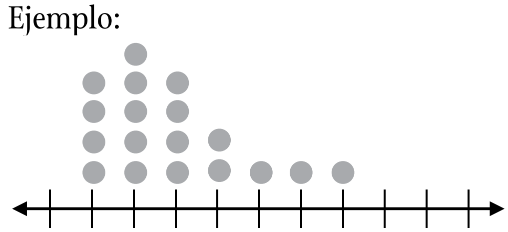
similarity transformation a dilation; a transformation that results in the size of a figure changing, but not the shape	3/B	**transformación de semejanza** dilatación; una transformación que resulta en el tamaño de un cambio de figura, pero no la forma
simple event an event that has only one outcome; sometimes called a single event	7/B	**evento simple** evento que sólo tiene un resultado; a veces se denomina evento único
sine a trigonometric function of an acute angle in a right triangle that is the ratio of the length of the opposite side to the length of the hypotenuse; $\sin\theta = \dfrac{\text{length of opposite side}}{\text{length of hypotenuse}}$	3/D	**seno** función trigonométrica de un ángulo agudo en un triángulo rectángulo que es la proporción de la longitud del lado opuesto a la longitud de la hipotenusa; sen $\theta = \dfrac{\text{longitud del lado opuesto}}{\text{longitud de la hipotenusa}}$
skewed to the left a distribution in which there is a "tail" of isolated, spread-out data points to the left of the median. "Tail" describes the visual appearance of the data points in a histogram. Data that is skewed to the left is also called *negatively skewed*. Example: 	7/A	**sesgado a la izquierda** distribución en la cual existe una "cola" de puntos de datos aislados extendidos hacia la izquierda de la mediana. La "cola" describe la apariencia de los puntos de datos en un histograma. Los datos sesgados a la izquierda también se denominan *negativamente sesgados*. Ejemplo: 
skewed to the right a distribution in which there is a "tail" of isolated, spread-out data points to the right of the median. "Tail" describes the visual appearance of the data points in a histogram. Data that is skewed to the right is also called *positively skewed*. Example: 	7/A	**sesgado a la derecha** distribución en la cual existe una "cola" de puntos de datos aislados extendidos hacia la derecha de la mediana. La "cola" describe la apariencia de los puntos de datos en un histograma. Los datos sesgados a la derecha también se denominan *positivamente sesgados*. Ejemplo: 

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slope the measure of the rate of change of one variable with respect to another variable; slope = $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$; the slope in the equation $y = mx + b$ is m	5/A 7/A	**pendiente** medida de la tasa de cambio de una variable con respecto a otra; pendiente $= m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$; la pendiente en la ecuación $y = mx + b$ es m
solid figure a three-dimensional object that has length, width, and height (depth)	4/B	**figura sólida** objeto tridimensional que tiene largo, ancho y altura (profundidad)
sphere a three-dimensional solid or surface that has all its points equidistant from its center	4/A	**esfera** superficie tridimensional que tiene todos sus puntos equidistante de su centro
square a special parallelogram with four congruent sides and four right angles	2/C 5/A	**cuadrado** paralelogramo especial con cuatro lados congruentes y cuatro ángulos rectos
straight angle an angle with rays in opposite directions; i.e., a straight line; a straight angle measures 180°.	1/D	**ángulo recto** ángulo con semirrectas en direcciones opuestas; es decir, línea recta; ángulo recto mide 180°.
stretch a transformation in which a figure becomes larger; stretches may be horizontal (affecting only horizontal lengths), vertical (affecting only vertical lengths), or both	3/A	**estirar** transformación en la que una figura se hace más grande; estiramientos pueden ser (que afecta sólo a longitudes horizontales) vertical (que afecta sólo a longitudes verticales) horizontal, o ambos
subset a set whose elements are in another set. Set A is a subset of set B, denoted by $A \subset B$, if all the elements of A are also in B.	7/B	**subconjunto** conjunto cuyos elementos están en otro conjunto. El conjunto A es un subconjunto del conjunto B, indicado por $A \subset B$, si todos los elementos de A se encuentran también en B.
supplementary angles two angles whose sum is 180°	1/D	**ángulos suplementarios** anglos cuya suma es de 180°
surface area for a 3-dimensional shape, the surface area is the sum of the areas of each surface	4/A	**área de superficie** para una forma tridimensional, el área de la superficie es la suma de las áreas de cada superficie

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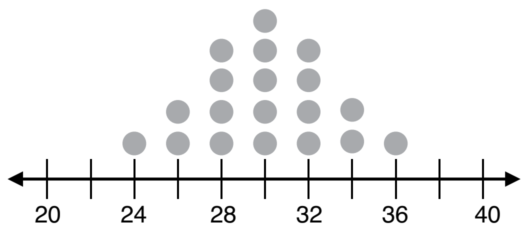
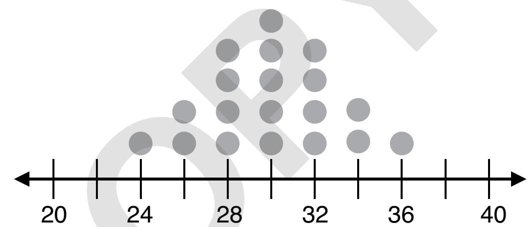
symmetric situation in which data is concentrated toward the middle of the range of data; data values are distributed in the same way above and below the middle of the sample. Example: 	7/A	**simétrico** situación en la que los datos se concentran hacia el medio del rango de datos; los valores de datos se distribuyen de la misma manera por encima y por debajo del medio de la muestra. Ejemplo: 
Symmetric Property of Congruent Segments If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.	3/C	**Propiedad simétrica de los segmentos congruentes** Si $\overline{AB} \cong \overline{CD}$, entonces $\overline{CD} \cong \overline{AB}$.
	T	
tangent a trigonometric function of an acute angle in a right triangle that is the ratio of the length of the opposite side to the length of the adjacent side; $\tan\theta = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$	3/D	**tangente** función trigonométrica de un ángulo agudo en un triángulo rectángulo que es la proporción de la longitud del lado opuesto a la longitud del lado adyacente; $\tan\theta = \dfrac{\text{longitud del lado opuesto}}{\text{longitud del lado adyacente}}$
tangent line a line that intersects a circle at exactly one point and is perpendicular to the radius of the circle at the point of tangency	6/A 6/C	**recta tangente** línea que corta un círculo en exactamente un punto y es perpendicular al radio del círculo en el punto de tangencia
theorem a statement that is shown to be true	3/C	**teorema** declaración que se demuestra que es verdadera
***theta* (*θ*)** a Greek letter commonly used to refer to unknown angle measures	3/D	***teta* (*θ*)** letra griega que se utiliza por lo general para referirse a medidas de ángulos desconocidas
third quartile value that identifies the upper 25% of the data; the median of the upper half of the data set; 75% of all data is less than this value; written as Q_3	7/A	**tercer cuartil** valor que identifica el 25% superior de los datos; mediana de la mitad superior del conjunto de datos; el 75% de los datos es menor que este valor; se expresa como Q_3

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transformation a change in a geometric figure's position, shape, or size	1/A	**transformación** cambio en la posición, la forma o el tamaño de una figura geométrica
Transitive Property of Congruent Segments If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.	3/C	**Propiedad transitiva de congruencia de segmentos** Si $\overline{AB} \cong \overline{CD}$, y $\overline{CD} \cong \overline{EF}$, entonces $\overline{AB} \cong \overline{EF}$.
translation 1. a transformation that moves each point of a figure the same distance in the same direction; also called a slide 2. in three dimensions, the horizontal or vertical movement of a plane figure in a direction that is not in the plane of the figure, such that a solid figure is produced	1/A 1/B 4/B	**traslación** 1. transformación que mueve cada punto de una figura a la misma distancia en la misma dirección; también llamado una diapositiva 2. en tres dimensiones, movimiento horizontal o vertical de una figura plana en una dirección que no está en el plano de la figura, de manera que se produzca una figura sólida
transversal a line that intersects a system of two or more lines	1/D	**transversal** línea que corta un sistema de dos o más líneas
trapezoid a quadrilateral with exactly one pair of opposite parallel sides	2/C	**trapezoide** cuadrilátero con exactamente un par de líneas paralelas opuestas
tree diagram a diagram that shows possible outcomes by listing each in a row or column; as the diagram grows, it resembles the branches of a tree	7/B	**diagrama de árbol** diagrama que presenta resultados posibles en una fila o columna; a medida que el diagrama crece, se ramifica como un árbol
trigonometry the study of triangles and the relationships between their sides and angles	3/D	**trigonometría** estudio de los triángulos y las relaciones entre sus lados y ángulos
two-column proof numbered statements and corresponding reasons that show an argument in a logical order	3/C	**prueba de dos columnas** declaraciones numeradas y las razones correspondientes que muestran el argumento en orden lógico
	U	
uniform probability model a probability model in which all the outcomes of an experiment are assumed to be equally likely	7/B	**modelo de probabilidad uniforme** modelo de probabilidad en el que se presume que todos los resultados de un experimento son igualmente probables

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union a set whose elements are in at least one of two other sets. The union of sets *A* and *B*, denoted by $A \cup B$, is the set of elements that are in either *A* or *B* or both *A* and *B*.	7/B	**unión** conjunto cuyos elementos están al menos en uno de otros dos conjuntos. La unión de los conjuntos *A* y *B*, indicada por $A \cup B$, es el conjunto de elementos que están en *A* o en *B*, o a la vez en *A* y *B*.
universal set a set of all elements that are being considered in a particular situation. In a probability experiment, the universal set is the sample space.	7/B	**conjunto universal** conjunto de todos los elementos que se consideran en una situación particular. En un experimento de probabilidad, el conjunto universal es el espacio de muestreo.
	V	
Venn diagram a diagram that shows how two or more sets in a universal set are related	7/B	**diagrama de Venn** diagrama que muestra cómo se relacionan dos o más conjuntos en un conjunto universal
vertex angle angle formed by the legs of an isosceles triangle	2/B	**ángulo vértice** ángulo formado por los catetos de un triángulo isósceles
vertical angles nonadjacent angles formed by two pairs of opposite rays	1/D	**ángulos verticales** ángulos no adyacentes formados por dos pares de semirrectas opuestas
	Y	
***y*-intercept** the *y*-coordinate of the point where a line or a curve intersects the *y*-axis	7/A	**intersección *y*** la coordenada *y* del punto en que una recta o curva corta el eje *y*