South Carolina CCR Mathematics

Applications and Modeling

Program Overview

TEACHER RESOURCE

BW WALCH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

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Introduction to the Program

Introduction

The *South Carolina CCR Mathematics Standards: Applications and Modeling* program is a complete set of materials developed around the South Carolina College and Career-Ready (SC CCR) Mathematics Standards. Topics are built around accessible core curricula, ensuring that the South Carolina College and Career-Ready Mathematics Standards Algebra I Program is useful for striving students and diverse classrooms.

This program realizes the benefits of exploratory and investigative learning and employs a variety of instructional models to meet the learning needs of students with a range of abilities.

The *South Carolina CCR Mathematics Standards: Applications and Modeling* program includes components that support problem-based learning, instruct and coach as needed, provide practice, and assess students' skills. Instructional tools and strategies are embedded throughout.

The program includes:

- More than 150 hours of lessons addressing the SC CCR Mathematics Standards
- Essential Questions for each instructional topic
- Vocabulary
- Instruction and Guided Practice
- Problem-based Tasks and Coaching questions
- Step-by-step graphing calculator instructions for the TI-Nspire and the TI-83/84
- Station activities to promote collaborative learning and problem-solving skills

Purpose of Materials

The *South Carolina CCR Mathematics Standards: Applications and Modeling* program has been organized to coordinate with the SC CCR Math Standards Algebra I content map and specifications from the SC CCR Mathematics Standards.

Each topic includes activities that offer opportunities for exploration and investigation. These activities incorporate concept and skill development and guided practice, then move on to the application of new skills and concepts in problem-solving situations. Throughout the lessons and activities, problems are contextualized to enhance rigor and relevance.

This program includes all the topics addressed in the South Carolina Applications and Modeling content map. These include:

- Probability
- Matrices
- Finance
- Geometry

The five Mathematical Process Standards are infused throughout:

- **MPS.PS.1:** Make sense of problems and persevere in solving them strategically.
- **MPS.RC.1:** Explain ideas using precise and contextually appropriate mathematical language, tools, and models.
- **MPS.C.1:** Demonstrate a deep and flexible conceptual understanding of mathematical ideas, operations, and relationships while making real-world connections.
- **MPS.AJ.1:** |Use critical thinking skills to reason both abstractly and quantitatively.
- **MPS.SP.1:** Identify and apply regularity in repeated reasoning to make generalizations.

Structure of the Teacher Resource

The *South Carolina Applications and Modeling* program is completely reproducible. Online materials can be provided in your Learning Management System (such as Canvas or Schoolology) or in BW Walch's proprietary course management platform, the Curriculum Engine. The nested folder organization in the Curriculum Engine allows you to access the materials quickly and easily. The digital format also facilitates printing and copying student pages and/or making assignments online.

The Program Overview is the first section. This section helps you to navigate the materials, offers a collection of research-based Instructional Strategies along with their literacy connections and implementation suggestions, and shows the correlation between the South Carolina CCR for Mathematics and the South Carolina Applications and Modeling course.

The remaining materials focus on content, knowledge, and application of the units in the South Carolina Applications and Modeling curriculum: Probability; Matrices; Finance; and Geometry. These units are designed to be flexible so that you can mix and match activities as the needs of your students and your instructional style dictate.

The Station Activities correspond to the content in the units and provide students with the opportunity to apply concepts and skills, while you have a chance to circulate, observe, speak to individuals and small groups, and informally assess and plan.

Each topic ends with a progress assessment, allowing you to assess students' progress as you move from topic to topic, enabling you to gauge how well students have understood the material and to differentiate as appropriate. Each unit includes at least one Conceptual Task or Extension Activity, and concludes with a unit assessment.

Glossary

The Glossary contains vocabulary terms and formulas from throughout the program, organized alphabetically. Each listing provides the term and the definition in both English and Spanish. The listings include the lesson number(s) where the terms can be found in the Words to Know.

Correspondence to NCTM *Principles to Actions* Teaching Practices

How Do Walch Integrated Mathematics Resources Address the NCTM *Principles to Actions* Mathematics Teaching Practices?

Walch's programs for the South Carolina College- and Career Ready Standards were designed by experienced educators and curriculum developers, informed by best-practice research, and refined through an iterative process of implementation and feedback. Together with professional development, these materials support and sustain good teaching practices.

NCTM Mathematics Teaching Practices	Relevant Attributes of Walch Integrated Math Resources
Establish mathematics goals to focus learning.	Each lesson in Walch's programs addresses specified standards which can be used as goals to focus learning. Essential Questions offer further focus.
Implement tasks that promote reasoning and problem solving.	Each lesson in Walch's programs is built around a Problem-Based Task (PBT), set in a meaningful real-world context and designed to promote reasoning and problem solving. The courses include dozens of PBTs as well as warm-up and practice problems.
Use and connect mathematical representations.	Walch's mathematics programs make frequent use of, and connections among and between, equations, tables, and graphs. PBTs often require students to use and connect two or more of these representations, and the representations are modeled through guided practice.
Facilitate meaningful mathematical discourse.	Several features of the programs support mathematical discourse, including warm-up debriefs with connections to upcoming lessons, optional coaching questions for the PBTs, and discussion guides for Performance Tasks. Explanations of PBT solutions are another opportunity for discourse. Please note: Mathematical discourse is an important topic for professional development, in conjunction with implementation of these materials.
Pose purposeful questions.	The coaching questions and discussion guides provide samples of purposeful questions. Note that this is another important topic for professional development.
Build procedural fluency from conceptual understanding.	The programs develop conceptual understanding through modeling, guided practice, and application, and then provide additional opportunities to practice and develop fluency.
Support productive struggle in learning mathematics.	The PBTs require "productive struggle;" coaching questions provide an option for additional support as appropriate, allowing students to proceed through the task and ensuring that the struggle remains productive rather than too frustrating.
Elicit and use evidence of student thinking.	Various discussions and PBTs require students to display their thinking. Coaching sample responses offer specific prompts and suggestions for eliciting and responding to student thinking. Professional development supports teachers in using that evidence to respond in instructionally appropriate ways.

Unit Structure

All of the instructional units have common features. Each unit begins with a list of all the standards addressed in the topics and a list of one or more conceptual activities. Each topic begins with an overview of the standards addressed in the topic; Essential Questions; vocabulary (titled "Words to Know"); and a list of recommended websites to be used as additional resources.

Each lesson begins with a list of identified prerequisite skills that students need to have mastered in order to be successful with the new material in the upcoming lesson. This is followed by an introduction, key concepts, common errors/misconceptions, scaffolded practice problems, guided practice examples, a problem-based task with coaching questions and sample responses, a closure activity, and practice. Each topic ends with a progress assessment to evaluate students' learning.

All of the components are described below and on the following pages for your reference.

South Carolina College and Career-Ready Standards for the Topic

All standards that are addressed in the entire topic are listed.

Essential Questions

These are intended to guide students' thinking as they proceed through the topic. By the end of each topic, students should be able to respond to the questions.

Words to Know

Vocabulary terms and formulas are provided as background information for instruction or to review key concepts that are addressed in the topic.

Recommended Resources

This is a list of websites that can be used as additional resources. Some websites are games; others provide additional examples and/or explanations. (*Note*: Links will be monitored and repaired or replaced as necessary.) Each Recommended Resource is also accessible through BW Walch's cloud-based Curriculum Engine Learning Object Repository as a separate learning object that can be assigned to students.

Conceptual Activities

Conceptual understanding serves as the foundation on which to build deeper understanding of mathematics. In an effort to build conceptual understanding of mathematical ideas and to provide more than procedural fluency and application, links to interactive open education and Desmos resources are included. (*Note*: These website links will be monitored and repaired or replaced as necessary.) These and many other open educational resources (OERs) are also accessible through the Learning Object Repository as separate objects that can be assigned to students.

Warm-Up

Each warm-up takes approximately 5 minutes and addresses either prerequisite and critical-thinking skills or previously taught math concepts.

South Carolina College and Career-Ready Standards for the Lesson

When topics are broken down into lessons, the specific standard or standards that are addressed are presented at the beginning of the instructional portion of the lesson.

Warm-Up Debrief

Each debrief provides the answers to the warm-up questions, and offers suggestions for situations in which students might have difficulties. A section titled Connection to the Lesson is also included in the debrief to help answer students' questions about the relevance of the particular warm-up activity to the upcoming instruction. Warm-Ups with debriefs are also provided in PowerPoint presentations.

Identified Prerequisite Skills

This list cites the skills necessary to be successful with the new material.

Introduction

This brief paragraph gives a description of the concepts about to be presented and often contains some Words to Know.

Key Concepts

Provided in bulleted form, this instruction highlights the important ideas and/or processes for meeting the standard.

Graphing Calculator Directions

Step-by-step instructions for using a TI-Nspire and a TI-83/84 are provided whenever graphing calculators are referenced.

Common Errors/Misconceptions

This is a list of the common errors students make when applying Key Concepts. This list suggests what to watch for when students arrive at an incorrect answer or are struggling with solving the problems.

Scaffolded Practice (Printable Practice)

This set of 10 printable practice problems provides introductory level skill practice for the lesson. This practice set can be used during instruction time.

Guided Practice

This section provides step-by-step examples of applying the Key Concepts. The three to five examples are intended to aid during initial instruction, but are also for individuals needing additional instruction and/or for use during review and test preparation.

Enhanced Instructional PowerPoint (Presentation)

Each lesson includes an instructional PowerPoint presentation with the following components: Warm-Up, Key Concepts, and Guided Practice. Selected Guided Practice examples include links to GeoGebra applets. These instructional PowerPoints are downloadable and editable.

Problem-Based Task

This activity can serve as the centerpiece of a problem-based lesson, or it can be used to walk students through the application of the standard, prior to traditional instruction or at the end of instruction. The task makes use of critical-thinking skills.

Optional Problem-Based Task Coaching Questions with Sample Responses

These questions scaffold the task and guide students to solving the problem(s) presented in the task. They should be used at the discretion of the teacher for students requiring additional support. The Coaching Questions are followed by answers and suggested appropriate responses to the coaching questions. In some cases answers may vary, but a sample answer is given for each question.

Recommended Closure Activity

Students are given the opportunity to synthesize and reflect on the lesson through a journal entry or discussion of one or more of the Essential Questions.

Printable Practice (Sets A and B) and Interactive Practice (Set A)

Each lesson includes two sets of practice problems to support students' achievement of the learning objectives. They can be used in any combination of teacher-led instruction, cooperative learning, or independent application of knowledge. Each Practice A is also available as an interactive Learnosity activity with Technology-Enhanced Items.

Progress Assessment

Each lesson ends with 10 multiple-choice questions, as well as one extended-response question that incorporates critical thinking and writing components. This can be used to document the extent to which students grasp the concepts and skills addressed during instruction.

Unit Assessment

Each unit ends with 12 multiple-choice questions and three extended-response questions that incorporate critical thinking and writing components. This can be used to document the extent to which students grasped the concepts and skills of each unit.

Answer Key

Answers for all of the Warm-Ups and practice problems are provided at the end of each unit.

Conceptual Tasks and/or Extension Activities

These engaging tasks provide opportunities for students to deepen their understanding and develop their conceptual knowledge of math concepts. These tasks provide multiple entry points and are accessible for ALL learners.

Standards Correlations

Each lesson in this program was written specifically to address the South Carolina College and Career-Ready Standards for Mathematics. Each topic lists the standards covered in all the lessons, and each lesson lists the standards addressed in that particular lesson. In this section, you'll find a comprehensive list mapping the lessons to the SC CCR.

Unit 1: Probability			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Theoretical and Empirical Probability**		
	1.1	Comparing Theoretical to Empirical Probability	AM.DPSR.2.1 AM.DPSR.2.2
	1.2	Assessing Theoretical Probability Models	AM.DPSR.2.1 AM.DPSR.2.2
Topic B	**Probability Distributions**		
	1.3	Creating Graphs of Probability Distributions	AM.DPSR.2.2
	1.4	Expected Value	AM.DPSR.2.3
	1.5	Developing Probability Distributions	AM.DPSR.2.2
	1.6	Using Probability Distributions to Evaluate Outcomes	AM.DPSR.2.2

Unit 2: Matrices			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Manipulating Matrices**		
	2.1	Performing Operations on Matrices	AM.MGSR.5.2
	2.2	Using Operations on Matrices	AM.MGSR.5.2
Topic B	**Vectors**		
	2.3	Representing and Modeling with Vector Quantities	AM.MGSR.5.1
	2.4	Performing Operations on Vectors	AM.MGSR.5.1
	2.5	Determinants and Vectors	AM.MGSR.5.1
Topic C	**Using Matrices to Solve Systems of Equations**		
	2.6	Representing a System of Linear Equations as a Single Matrix	AM.MGSR.5.2
	2.7	Finding the Inverse of a Matrix and Using It to Solve a System of Equations	AM.MGSR.5.2 AM.PAFR.3.1
	2.8	Linear Programming	AM.PAFR.3.1

Unit 3: Finance			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Income**		
	3.1	Creating Equations and Inequalities–Gross Pay	AM.DPSR.1.1
	3.2	Creating Equations in Context–Net Pay	AM.DPSR.1.1
	3.3	Income and Constraints	AM.DPSR.1.1, AM.DPSR.1.2
Topic B	**Credit**		
	3.4	Solving Linear Equations–Simple Interest	AM.DPSR.1.1, AM.DPSR.1.3
	3.5	Analyzing Credit Offers with Linear and Exponential Equations	AM.DPSR.1.1, AM.DPSR.1.3
Topic C	**Loans and Financed Purchases**		
	3.6	Recursion and Sequences–Payment Plans	AM.DPSR.1.1, AM.DPSR.1.3
	3.7	Finite Geometric Series—Amortized Loans	AM.DPSR.1.1, AM.DPSR.1.3
Topic D	**Banking**		
	3.8	Interpreting Complicated Expressions–Bank Statements and Savings Accounts	AM.DPSR.1.1, AM.DPSR.1.3
	3.9	Analyzing Savings Account Options Using Equations and Inequalities	AM.DPSR.1.1, AM.DPSR.1.3
Topic E	**Investing**		
	3.10	Interpreting Expressions and Equations–Stocks and Shares	AM.DPSR.1.3,
	3.11	Interpreting Stock Parameters	AM.DPSR.1.1
	3.12	Reading Stock Reports	AM.DPSR.1.4, AM.DPSR.1.3,
Topic F	**Planning for Retirement**		
	3.13	Planning for Retirement	AM.DPSR.1.2, AM.DPSR.1.1
Topic G	**Insurance**		
	3.14	Insurance	AM.DPSR.1.3

Unit 4: Geometry			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Trigonometry of General Angles**		
	4.1	Proving the Law of Sines	AM.MGSR.1.1
	4.2	Proving the Law of Cosines	AM.MGSR.1.1
	4.3	Applying the Laws of Sines and Cosines	AM.MGSR.1.1
Topic B	**Geometric Modeling**		
	4.4	Density	AM.MGSR.3.1, AM.MGSR.3.3
	4.5	Design	AM.MGSR.3.1, AM.MGSR.3.3
Topic C	**Creating and Using Geometric Models**		
	4.6	Scaling Across Dimensions	AM.MGSR.3.2, AM.MGSR.3.3
	4.7	Modeling in Three Dimensions	AM.MGSR.2.2, AM.MGSR.2.3
Topic D	**Scale and Dimension in Art**		
	4.8	Proportions and Perspective	AM.MGSR.2.2, AM.MGSR.3.3
	4.9	Dimension in Art	AM.MGSR.2.1, AM.MGSR.4.1 AM.MGSR.4.2

Conceptual Activities

Use these interactive open education and/or Desmos resources to build conceptual understanding of mathematical ideas. (*Note*: Activity links will be monitored and repaired or replaced as necessary.)

Unit 1

- Desmos. "Binomial Distribution."

  http://www.walch.com/ca/20007

  This resource is an interactive plot of the binomial distribution. There are sliders for the probability of the favorable outcome and the number of trials. Moving the slider changes the plot.

- Desmos. "Chance Experiments."

  http://www.walch.com/ca/20006

  This activity uses spinners as a chance event to predict empirical probabilities from theoretical probabilities. There are no calculations involved; probabilities are visually interpreted from the segments of the spinners.

- Desmos. "Geometric Distribution."

  http://www.walch.com/ca/20008

  This resource is an interactive plot of the geometric distribution. There is a slider for the probability of the favorable outcome. Moving the slider changes the plot.

Unit 2

- Humberstone, Bryn. "Linear Programming."

  https://www.walch.com/ca/01078

  This Desmos activity goes through the process of creating and then solving a linear programming problem from context, step by step. Care is taken to explain how the graph of the inequalities leads to solutions to the linear programming problem.

Unit 3

- Amortization-Calc.com. "Amortization Schedule Calculator."

  http://www.walch.com/rr/00172

  This site shows a real-world example of sums of finite series. Users can see exactly how much they can expect to pay for a home when taking out a mortgage. Enter values for the loan amount, number of years, interest rate, ZIP code, and the starting month of the loan, then click "Calculate." The next screen reveals the monthly payment, the total amount paid over the life of the loan, the total amount of interest paid, and the pay-off date.

- MathIsFun.com. "Compound Interest."

  http://www.walch.com/rr/00146

  This site provides an overview of compound interest and shows how to derive the compound interest formula.

Unit 4

- Desmos. "Special Right Triangles."

  http://www.walch.com/ca/10018

  In this activity, students work with the side length ratios of 45°–45°–90° and 30°–60°–90° right triangles.

- Desmos. "Working with Dilations."

  http://www.walch.com/ca/10019

  This activity is a basic introduction to dilations.

- Illustrative Mathematics. "Mt. Whitney to Death Valley."

  http://www.walch.com/ca/10020

  In this task, students will apply trigonometric ratios to solve a real-life problem.

- Illuminations. "Trigonometry Square."

  http://www.walch.com/ca/10021

  This activity allows students to practice evaluating trigonometric ratios for specific values.

Digital Enhancements Guide

Introduction

With this program, you have access to the following digital components, described here with guidelines and suggestions for implementation.

Digital Instruction

These optional versions of the Warm-Ups, Warm-Up Debriefs, Introductions, Key Concepts, and Guided Practices for each lesson run on PowerPoint. (*Please note*: Computers may render PowerPoint images differently. For best viewing and display, use a PowerPoint Viewer and adjust your settings to optimize images and text.)

Each PowerPoint begins with the lesson's Warm-Up and is followed by the Warm-Up Debrief, which reveals the answers to the Warm-Up questions.

In the notes section of the last Warm-Up slide, you will find the "Connections to the Lesson," which describes concepts students will glean or skills they will need in the upcoming lesson. The "Connections" help transition from the Warm-Up to instruction.

GeoGebra Applets

In addition to the hard copy materials found in the Teacher Resource, two interactive GeoGebra applets are provided for most lessons. The applets model the mathematics in two of the Guided Practice examples for these lessons. Links to these applets are embedded within the Instructional PowerPoints. With an Internet connection, simply click on the "Play" button slide that follows selected examples.

Once you've accessed the GeoGebra applet, please adjust your view to maximize the image. Each applet illustrates the specific problem addressed in the Guided Practice example. The applets allow you to walk through the solution by visually demonstrating the steps, such as defining points and drawing lines. Variable components of the applets (usually fill-in boxes or sliders) allow you to substitute different values in order to explore the mathematics. For example, "What happens to the line when we increase the amount of time?" or "What if we cut the number of students in half?" This experimentation and discussion supports development of conceptual understanding.

GeoGebra for PC/MAC

GeoGebra is not required for using the applets, but can be downloaded for free for further exploration at the following link:

https://www.geogebra.org/download

Curriculum Engine Learning Object Repository

BW Walch's Curriculum Engine comes loaded with thousands of curated learning objects that can be used to build formative and summative assessments, as well as practice worksheets, instructional components, and an item bank. District leaders and teachers can search for items by standard and create assessments or worksheets in minutes using the three-step assessment builder.

For more information about the Curriculum Engine, or for additional support, please contact us at (207) 828-8800 or customerservice@bwwalch.com.

SC CCR Mathematical Process Standards Implementation Guide

Introduction

The five Mathematical Process Standards describe features of lesson design, teaching pedagogy, and student actions that will lead to a true conceptual understanding of the mathematics standards. The Walch lessons, practice problems, and Problem-Based Tasks lend themselves to teaching through this framework. When the Walch resources are combined with high-level questioning and engaging teacher decisions in the classroom, it will lead to high-level math instruction and student achievement.

Here is a brief description of the MPSs and how they can be applied in the classroom:

MPS.PS.1: Make sense of problems and persevere in solving them strategically.

Students understand there are multiple entry points that can identify and explain a problem. Using prior knowledge, a variety of methods, and continual self-reflection, students can check for reasonable solutions. Students can monitor progress and confidently change course if necessary to plan a solution pathway. Teacher prompts that can enhance this standard include:

- What do you already know that might help you solve the problem? -
- What is the problem asking you to solve?
- What are some different strategies you could use to solve this problem?
- How can you explain your strategy to someone else?
- Compare your answer with a classmate's answer. Who is correct? Why?
- How can you check your solution for reasonableness and accuracy?
- Using the context of the problem, is your solution reasonable?

MPS.RC.1: Explain ideas using precise and contextually appropriate mathematical language, tools, and models.

Students can consider the available and relevant tools that are helpful to explore, model, and deepen their understanding of concepts. They can use precise mathematical language to model, explain, and justify valid solutions. Students can engage in constructive dialogue individually and collaboratively through writing, speaking, and listening. Teacher prompts that can enhance this standard include:

- Can you graph this equation in the calculator to see a relationship?

- What formula or strategy might help you determine the answer to this question?
- How can you represent the situation using handheld tools (rulers, protractors, etc.) to determine an answer?
- Can you represent this situation with a visual model?
- How will it help you solve the problem?
- What information is needed to solve this problem?
- Is there another way to solve this problem?

MPS.C.1: Demonstrate a deep and flexible conceptual understanding of mathematical ideas, operations, and relationships while making real-world connections.

Students can make connections between different areas of mathematics, other content areas, and real-world context. They can identify applicable quantities, interpret mathematical models, and describe their relationships in the context of relevant situations. Teacher prompts that can enhance this standard include:

- How can this concept be applied in a real-world context?
- Is your answer reasonable based on your initial estimate?
- While working to solve this problem, what misconceptions might someone have with this?
- What do the characteristics of the graph tell us about the situation?
- What do each of the variables and numbers in the equation/formula represent?
- How are these situations the same and different based on their representations?
- How does this concept relate to concepts we have learned previously?
- What connections can you make between this concept and other areas of mathematics?
- What units of measure help describe your numerical answer?

MPS.AJ.1: Use critical thinking skills to reason both abstractly and quantitatively.

Students can construct arguments using multiple representations (objects, symbols, drawings, and actions). They can recognize and explain bias and errors in an argument. Mathematical students can listen to and read the arguments of others to critique whether they make sense and ask questions for clarification. Students can use reasoning to make and explore the truth of conjectures. Teacher prompts that can enhance this standard include:

- How can you represent this problem with diagrams and models?

- What evidence supports your argument? Can you justify your reasoning?
- Will your strategy work for any number?
- How does your solution compare with your classmates?
- For which categories of numbers (negative integers, all real numbers, etc.) will your strategy work?
- How did you determine your answer?
- Why did you choose that strategy?

MPS.SP.1: Identify and apply regularity in repeated reasoning to make generalizations.

Students can make and test conjectures, express regularities as generalizations about relationships, and then use the generalizations to solve problems. They can recognize complex mathematical objects and situations as being composed of multiple parts. Teacher prompts that can enhance this standard include:

- Can you identify any patterns or regularities in the data or problem?
- How can you express your observations about the problems as a rule?
- What relationship do you notice in the graph/table/numbers?
- Why did you choose to use this process to solve this word problem/equation?
- How can you apply this process in other situations?
- How can you generalize your reasoning to apply to similar problems?
- How might this generalization help you solve similar problems?

Source

- South Carolina Department of Education. (n.d.)." 2023 South Carolina College- and Career-Ready Mathematics Standards." Accessed June 4, 2024. https://www.walch.com/SCCCR/00001

Instructional Strategies

Ensuring Access for All Students

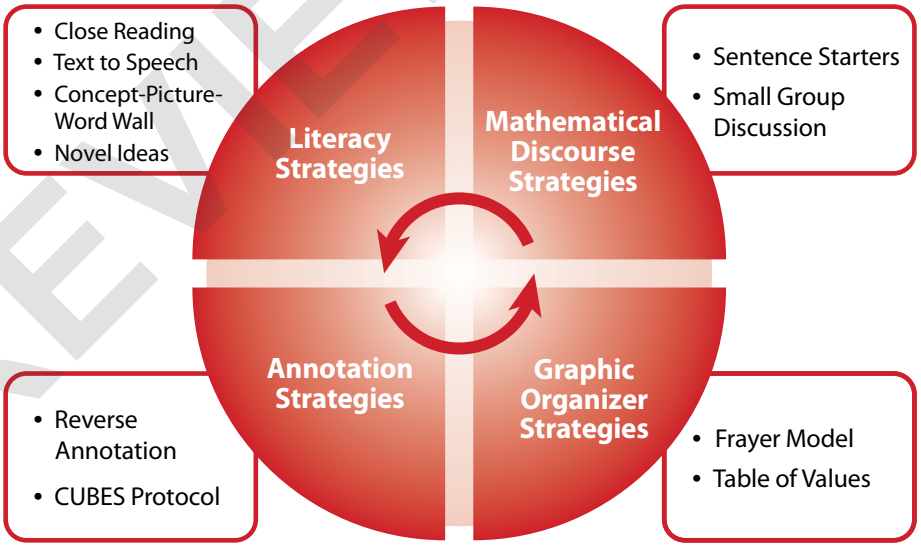
Introduction

The increased focus on literacy in math instruction can help some students navigate mathematical contexts, but for struggling readers, it can further complicate calculations. English language learners struggle to master difficult mathematical concepts while simultaneously processing a new language. Students with learning and behavioral disabilities struggle with the math concepts in their own contexts. This is where teachers and the strategies they select for their classrooms become essential.

The strategies presented here can help all students succeed in math, literacy, school, and, ultimately, in life. These instructional strategies provide teachers with a wide range of instructional support to aid English as a Second Language (ESL) students, students with disabilities (SWD), and struggling readers. These strategies provide support for the Mathematics Standards and the Mathematical Process Standards (MPS), English Language Development (ELD) Standards, English Language Arts Standards, and WIDA English Language Development Standards.

Within each lesson throughout this course, you will find suggested instructional strategies. These instructional strategies are research-based strategies and best practices that work well for all students.

The instructional strategies detailed here fall into four main categories: Literacy, Mathematical Discourse, Annotation, and Graphic Organizers. These strategies provide teachers with research-based strategies to address the needs of all students.

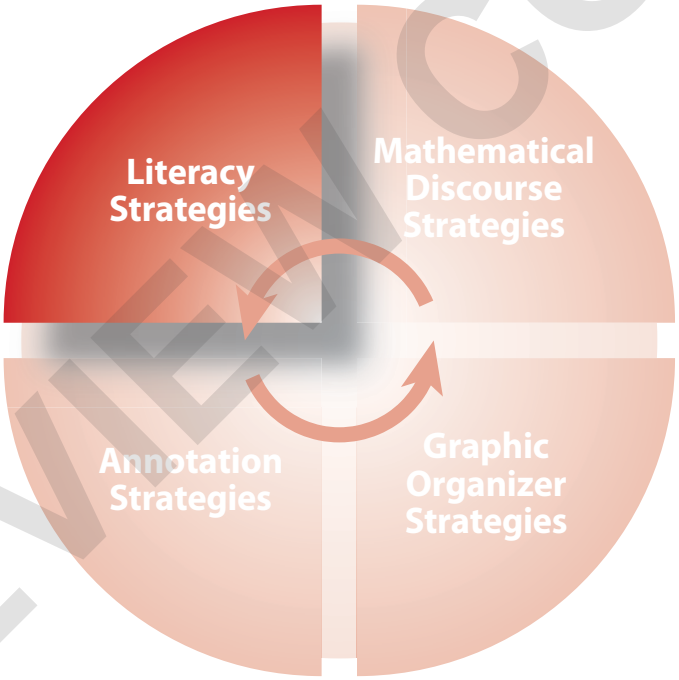


Source

- WIDA: https://www.walch.com/rr/09052

Understanding the Language of Mathematics: Literacy

Mathematics has its own language consisting of words, notations, formulas, and visuals. In education, the language of mathematics is often regarded solely in the context of word problems and articles. This neglects the vocabulary and other mathematical representations students must be able to interpret. The strategies presented here help students navigate the language of mathematics so that they can understand text and feel confident speaking in and listening to mathematical discussions. For students with disabilities, the stress on repetition and different representations in this approach is essential to their ability to grasp the math concepts. For ESL students, repetition and different representations can strip out some of the English language barriers to understanding the language of mathematics, as well as provide multiple means of accessing the content. Literacy strategies include Close Reading, Text-to-Speech, Concept-Picture-Word Walls, and Novel Ideas.



Literacy Strategies

Close Reading with Guiding Questions

What is Close Reading with Guiding Questions?

Close Reading with Guiding Questions is a process that allows students to preview mathematical reading and problems by answering questions related to the text in advance and reviewing their responses during and/or after reading. Multiple reading protocols can be used in conjunction with guiding questions to enhance their effectiveness.

How do you implement Close Reading with Guiding Questions in the classroom?

When utilizing a textbook, task, or article in a math class, literacy struggles are often a strong barrier to entry into the mathematical ideas. Asking students to answer accessible questions before and/or as they read can lead them to the key information.

Prior to implementation, the teacher should determine the most important information students need to obtain from a text, whether it is a math problem to solve, a task to complete, or an informational lesson or article to read. Then, the teacher should come up with some questions to guide students before they read. These questions can:

- assess and relate prior knowledge
- define key vocabulary words
- discuss non-mathematical concepts in the text

The teacher should also prepare some questions to guide students as they read. These questions can:

- point out key concepts within the text
- relate the text and concepts to future learning
- assist students in identifying key facts in the text
- highlight the importance of text features (graphics, headings, etc.) in the text

To ensure the questions are accessible for students and to encourage reflection and debate after reading, many of these questions should be designed as either "True/False" or "Always True/Sometimes True/Never True." Students can represent their reasoning for their answer in writing, numbers, or graphic/pictorial representations. Students should complete the guiding questions and reading individually, with discussion to follow.

After students complete the reading, they should be given some time to individually evaluate their initial answers. Then, in partners or in groups, they can discuss their answers and come to final conclusions that will help them find the important information initially identified by the teacher. After deciphering the text through close reading, students will be able to complete the given activity.

When would I use Close Reading with Guiding Questions in the classroom?

Close Reading with Guiding Questions can be used for any activity in which literacy could be a barrier to learning or demonstrating mastery of mathematical concepts. The number of questions and length of the discussions can be altered based on the length, importance, and difficulty of the text and concept. As students become more accustomed to mathematical literacy, the text complexity can be increased, but the adherence to close reading strategies must be maintained to ensure students can access the mathematical concepts. The length of time spent on the literacy aspect can be shortened as students become more skilled, but the questioning and discussions must occur to ensure students are properly interpreting the text in the mathematical context.

How can I use Close Reading with Guiding Questions with students needing additional support?

For struggling readers, including ESLs, Close Reading with Guiding Questions can help make an intimidating lesson, word problem, or task much more accessible. Questions focusing more on Tier 2 and Tier 3 vocabulary, text features, and real-world concepts can help struggling readers relate to the text and learn how to decipher the text in context. Discussions around the questions will help students grasp the math concepts.

Allowing struggling readers to explain their answers using words, numbers, or graphics/pictures ensures that they can express their opinion and rationale despite a potential lack of vocabulary. Through these representations and the ensuing discussion, students will begin to learn the necessary vocabulary to be successful.

What other standards does Close Reading with Guiding Questions address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E1.AOR.2
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1
- ELA.E1.AOR.8.1

Sources

- Anne Adams, Jerine Pegg, and Melissa Case. "Anticipation Guides: Reading for Mathematics Understanding."

  https://www.walch.com/rr/09053

- Diane Staehr Fenner and Sydney Snyder. "Creating Text Dependent Questions for ELLs: Examples for 6th to 8th Grade."

  https://www.walch.com/rr/09054

Literacy Strategies

Text-to-Speech Technology

What is Text-to-Speech Technology?

Text-to-Speech Technology is an adaptive technology that reads text aloud from a text source for students. It is usually accessed through an application or program on a computer, smartphone, or tablet. Some new programs utilize Mathematical Markup Language (MathML) to read mathematical notation in a common, understandable manner for students. Many programs also highlight the words and notation on the screen as the audio plays, which helps students relate the written representation to the words they hear. The use of Text-to-Speech Technology allows students who struggle with literacy to hear the words and notation and access the text in a different way.

How do you implement Text-to-Speech Technology?

A classroom community focused on everyone's learning and a growth mindset is the first step in implementing Text-to-Speech Technology. One of the main barriers to implementation is encouraging students to use the program. Once they do, they will realize how the audio can help them understand the difficult mathematical texts and interpret the math content within them. After students realize the benefits of Text-to-Speech Technology, it can become part of the regular routine for group and independent work.

The use of headphones can be very important for effective use of Text-to-Speech Technology. Students can use the technology to listen to lessons and texts at their own pace. Extra noise from other students working or other students listening at different paces can confuse students attempting to use Text-to-Speech Technology, and headphones can help mitigate these distractions. Many teachers are nervous about the potential disruption headphones can cause in class. However, well-managed use of headphones can help students successfully utilize the technology to learn.

When would I use Text-to-Speech Technology in the classroom?

Text-to-Speech Technology can be used at any time throughout the year, and if the program speaks in MathML, it can be used with any lesson. Without MathML, effective use could be limited to word problems without unusual notation. For example, if x^2 is read as "x-two" instead of "x-squared" or "x to the second power," that could confuse students more.

During a lesson or small group discussion, Text-to-Speech Technology could detract from students' ability to listen, question, and process information. However, during warm-ups, independent work, or assessments, Text-to-Speech Technology can help students process the information and access the activity. It can become a routine for students to automatically listen to the question, problem, or directions first, and then attempt the activity.

How can I use Text-to-Speech Technology with students needing additional support?

Text-to-Speech Technology is an important adaptation and accommodation for struggling readers. Students who have read-aloud accommodations sometimes don't receive them because they are either embarrassed to accept them or because of staffing restrictions. These students can use Text-to-Speech Technology to supplement their math instruction by having text automatically read to them in a manner in which they can process it.

Additionally, for ESL students, hearing the English mathematical language, especially referring to mathematical representations and notation, can help put English words to the ideas they see. Some Text-to-Speech Technology can translate written and mathematical text into other languages, so students can hear the text in their natural language and see the English highlighted on the screen as they hear it. In this way, students are learning English vocabulary as well as learning the mathematical content in a language they can understand.

What other standards does Text-to-Speech Technology address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E1.AOR.2
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1
- ELA.E1.AOR.8.1

Source

- Steve Noble. "Using Mathematics eText in the Classroom: What the Research Tells Us." https://www.walch.com/rr/09055

Literacy Strategies

Concept-Picture-Word Wall

What is a Concept-Picture-Word Wall?

A Concept-Picture-Word Wall is a classroom display, often a bulletin board or a set of posters, that exposes students to important vocabulary words they will use in math class.

Posting vocabulary words in class helps reinforce the words students will see in textbooks, videos, websites, and test questions on math concepts. These Tier 3 vocabulary words are often not used in everyday language, and the exposure to the words visually through Concept-Picture-Word Walls can help students connect them to the math content.

How do you implement Concept-Picture-Word Walls in the classroom?

Just seeing the vocabulary on a Concept-Picture-Word Wall by itself will help students; more importantly, referring to the words as the teacher uses them in class helps students connect the visual to the application. A simple gesture to the wall makes a very explicit reference to the word as it is used and allows students to connect the unfamiliar word to its meaning in context. Additionally, students can be taught to refer to the wall as they use the words in class, and they can be asked to make sure they say at least 3 words from the wall during each class period in small-group discourse or as answers to whole-class questions. The comfort gained from using these Tier 3 words will help students to use appropriate math vocabulary while solving problems and will help students connect concepts more explicitly.

Postings on the Concept-Picture-Word Wall can be arranged strategically to connect concepts, units of study, or groups of words where appropriate. Having three sections of the Concept-Picture-Word Wall—for example, an "In the Future" section, a "Live in the Present" section, and a "Remember the Past" section——can help students see and remember the vocabulary throughout the entire course. Even without regular use of some words, just seeing the words before a unit can help instill a familiarity with the vocabulary. Leaving the words on the Concept-Picture-Word Wall after a unit is taught can help students connect "old" concepts to the current lesson and ensure that students still have access to the vocabulary.

When would I use Concept-Picture-Word Walls in the classroom?

Concept-Picture-Word Walls can be used for the entire year. The actual words might have to change, or at least be moved to different areas of the Concept-Picture-Word wall. The more exposure students have to the words, the more familiar and comfortable they will become. The constant exposure to the math context is beneficial for students throughout the entire course, especially for words with multiple meanings (bias, tangent, etc.) that could exist as Tier 2 words in everyday conversation but are Tier 3 words in the math classroom.

How can I use Concept-Picture-Word Walls with students needing additional support?

For all students learning mathematics, knowing and using the math vocabulary is often a major barrier. This is a problem especially for ESL students, who are learning the English language along with math content. If teachers try to simplify the words too much for students, it does them a disservice as they seek out information from other teachers, textbooks, and online sources that use the proper vocabulary. Most tests, especially state tests, will expect students to have knowledge of the Tier 3, math-specific vocabulary. The more students see these words, the more familiarity they will have when they apply them.

Concept-Picture-Word Walls can also be written in multiple languages. Especially for students who are on-grade-level in their native language, a multi-lingual Concept-Picture-Word Wall can help students connect the content they already know in another language to the English vocabulary necessary for success on English-language math activities and tests.

This website can help you get started on an English-Spanish Concept-Picture-Word Wall: https://www.walch.com/rr/09056

What other standards do Concept-Picture-Word Walls address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E1.AOR.2
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1
- ELA.E1.AOR.8.1

Source

- Janis M. Harmon, Karen D. Wood, Wanda B. Hedrick, Jean Vintinner, and Terri Willeford. "Interactive Word Walls: More Than Just Reading the Writing on the Walls."

  https://www.walch.com/rr/09057

Literacy Strategies

Novel Ideas

What is Novel Ideas?

Novel Ideas is a classroom activity that explores students' understanding of important Tier 2 vocabulary words they will use in math class. Instead of asking students to look up vocabulary words in the dictionary, Novel Ideas allows students to have conversations with their peers about vocabulary words in class. This reinforces the mathematical vocabulary students will see in textbooks, videos, websites, and test questions. These Tier 2 vocabulary words are often used in everyday language, but have specific meaning in mathematics. Exposure to the words through Novel Ideas can help students connect them to the math content.

How do you implement Novel Ideas in the classroom?

While building a rich representation of math content words and connecting the words to other words and concepts has inherent merit, it is more important to consider that pre-teaching the words before they are used in class helps students connect to the application. The understanding gained from discussing these Tier 2 words will help students apply them in a mathematical context to solve problems and connect concepts.

Here is a step-by-step process for implementing Novel Ideas:

1. Students separate into groups of four.
2. Students copy the teacher generated prompt/sentence starters and number their papers 1–8.
3. One student offers an idea, another echoes it, and all write it down.
4. After three minutes, students draw a line under the last item in the list.
5. All students stand, and the teacher calls one student from a group to read the group's list.
6. The student starts by reading the prompt/sentence starters, "We think a _____ called _____ may be about ... ," and then adds whatever ideas the team has agreed on.
7. The rest of the class must pay attention because after the first group has presented all their ideas, the teacher asks them to sit down and calls on a student from another team to add that team's "novel ideas only." Ideas that have already been presented cannot be repeated.
8. As teams complete their turns and sit down, each seated student should record novel ideas from other groups below the line that marks the end of his or her team's ideas.

When would I use Novel Ideas in the classroom?

Novel Ideas can be used for the entire year. The more students are exposed to mathematical vocabulary, the more familiar and comfortable they become, leading to increased usage of these math terms in their conversation and writing. Using math vocabulary in context is beneficial for students throughout the entire course, especially for words with multiple meanings (bias, tangent, etc.) that could exist as Tier 2 words in everyday conversation but are Tier 3 words in the math classroom.

How can I use Novel Ideas with students needing additional support?

Most tests, especially state tests, will expect students to have knowledge of the Tier 3, math-specific vocabulary. The more students use these words in conversation, the more familiarity they will have when they apply them. Understanding Tier 2 words also helps students avoid misconceptions in mathematics. Twice a week before the start of a lesson, allow students to use sentence starters in small groups that include all students. Prepare the sentence starter "When I hear the word _____, I think about _____" to share out with whole class. This will allow students who know the vocabulary words to share their knowledge, and will allow other students to hear the meaning of the vocabulary words. This strategy is particularly helpful for ESL students.

What other standards does Novel Ideas address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E1.AOR.2
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1
- ELA.E1.AOR.8.1

Sources

- Colorín Colorado. "Selecting Vocabulary Words to Teach English Language Learners."

  https://www.walch.com/rr/09058

- Elsa Billings and Peggy Mueller, WestEd. "Quality Student Interactions: Why Are They Crucial to Language Learning and How Can We Support Them?"

  https://www.walch.com/rr/09059

Novel Ideas Sentence Starters

Slope

- When I hear the word climb, I think about ...
- When I hear the word steep, I think about ...

Volume

- When I hear the word filling, I think about ...

Equations

- When I hear the word balance, I think about ...
- When I hear the word equal, I think about ...

Graphing

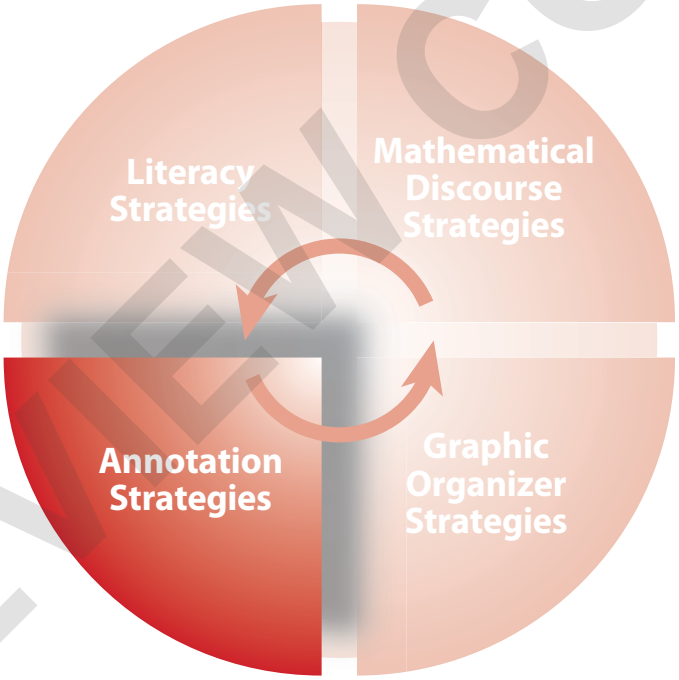
- When I hear the word grid, I think about ...
- When I hear the word graph, I think about ...

Scatter Plots

- When I hear the word scattered, I think about ...

Understanding Mathematical Content: Annotation

Understanding mathematical content is an extremely important skill, both in the math classroom and in life. When students read word problems, articles, charts, graphs, equations, tables, or other forms of mathematical text, they must be able to decode and extract meaning from the text. Annotation can help. The strategies presented here help students identify and focus on key characteristics and facts from various forms of text while ignoring the non-essential information. For students with disabilities, many of whom struggle with the distractions inherent in many high-school level texts, making notes and drawing pictures to explain a problem can help them focus. ESL students will be pointed to certain Tier 3 vocabulary words and determine which Tier 2 vocabulary words they must learn to be proficient in math class and in the English language. Annotation strategies include Reverse Annotation and CUBES protocol.



Annotation Strategies

Reverse Annotation Protocol

What is Reverse Annotation?

Reverse Annotation is a strategy that asks students to identify and write down key information from math problems. This is especially helpful for problems given on a computer or tablet, where students can't annotate directly on the problem. A template is given at the end of this section.

How do you implement Reverse Annotation in the classroom?

Many annotation strategies ask students to write, underline, or mark directly on the text of a problem. While those forms of annotation are also beneficial, they are not always possible with technology. Whether the problem is given on paper or using technology, having students write the answers to these questions will ensure that they are thinking strategically and specifically about the strategies and information needed to solve the problem.

The three questions at the top of the Reverse Annotation template are the key to understanding mathematical problems. For every problem given in class, ask students:

1. What is the problem asking us to solve?
2. What key words tell us the mathematical steps we need to perform?
3. What information in the problem can help us figure it out?

After answering the initial questions, students should make a guess, or estimate, of what they think the answer will be. This helps grow their number sense, and provides an initial, reasonable solution to guide their work. Students can then use the strategies they selected to solve the problem and evaluate their solution using the questions at the bottom of the template.

When students first begin to use Reverse Annotation, the teacher should walk them through the steps individually to ensure they can accurately identify the question, key words, and important information. Teachers can also lead students through the estimation process, making a game out of which student has the closest estimate.

Work through each step individually for several "easy" problems first, so that difficult math doesn't interfere with the process. Increase the problem difficulty incrementally as students begin to master the process. This may seem like a long process at first, but the ultimate result is worth the time investment.

When would I use Reverse Annotation in the classroom?

Reverse Annotation can be used to solve any math problem, and is especially helpful for word problems. When Reverse Annotation is initially implemented, the steps should be discussed in detail. As students become accustomed to Reverse Annotation and begin thinking about problems in

this manner automatically, the individual steps become less important and can be scaffolded out to improve efficiency. Students should reach the point where they immediately ask themselves the three initial questions when they first see a problem. However, the teacher should ensure that students are truly evaluating all the key information before routine discussions of the individual steps are removed.

How can I use Reverse Annotation with students needing additional support?

Annotation strategies can help students identify key information, even when certain vocabulary words are not known. As teachers introduce the content-specific Tier 3 vocabulary to their classes, annotation strategies such as reverse annotation can help students use these words to apply appropriate strategies while problem solving. Answering the three initial questions can help students organize the key facts and vocabulary, and the identification of key information can simplify the problem. This strategy is especially beneficial for ESL students.

Using reverse annotation with graphic organizers benefits ESL students by removing a lot of the confusing wording and allowing them to focus on the important pieces of a problem. When using Reverse Annotation, all students, including ESL students, will begin to think about problem solving in a way that encourages them to use the appropriate information to find a solution.

What other standards does the Reverse Annotation Protocol address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.RC.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E2.C.7.1
- ELA.E4.C.9.1
- ELA.E1.C.2.1
- ELA.E1.AOR.7.1

Source

- Alliance for Excellent Education. "Six Key Strategies for Teachers of English Language Learners." https://www.walch.com/rr/09060

Reverse Annotation Template

Name: _____ Problem/Assignment: _____

Analyze the Problem

What is the problem asking us to solve?	
What key words will tell us the mathematical steps we need to perform?	
What information in the problem can help us figure it out?	

Initial estimate of solution:

Work Space

Remember to box in your solution!

Name: _____ Problem/Assignment: _____

Check It Over

How close was your estimate?	
Does your answer make sense? Is it reasonable? How do you know?	
Did you perform the calculations correctly?	
What does your answer mean in context?	

Annotation Strategies

CUBES Protocol

What is the annotation strategy CUBES?

CUBES is an annotation strategy in which students use different written designs to highlight the key aspects of word problems. It can help them choose the correct mathematical strategy to solve the problem accurately.

How do you implement CUBES in the classroom?

The steps for CUBES are:

1. **C**: **C**ircle all the key numbers.
2. **U**: **U**nderline the question.
3. **B**: **B**ox in the key words that will determine the operation(s) necessary and write the mathematical symbol for the operation(s).
4. **E**: **E**valuate the information given to determine the strategy needed. **E**liminate any unnecessary information.
5. **S**: **S**olve the problem, **s**how your work, and check your answer.

As students learn to use CUBES, walk them through the steps individually to ensure they can accurately identify the key numbers, question, key words, unnecessary information, and strategy. Work through each step individually for several "easy" problems first, so that difficult math doesn't interfere with the process. Increase the problem difficulty incrementally as students begin to master the process. This may seem like a long process at first, but the ultimate result is worth the time investment.

A graphic organizer can help students master the process, especially when problems are given on a computer or tablet where students can't always annotate directly on the problem. Students can write down the key numbers and circle them, write down the question and underline it, and so on. This will encourage students to truly think about the different pieces of the problem they are identifying, and how these pieces will guide the strategy and affect the solution.

When would I use CUBES in the classroom?

CUBES can be used to solve any math problem, and is especially helpful for word problems. When CUBES is initially implemented, the steps should be discussed in detail. As students become accustomed to using CUBES and begin thinking about problems in this manner automatically, the individual steps become less important and can be scaffolded out to improve efficiency. However, the teacher should ensure that students are truly evaluating all the key information before routine discussions of the individual steps are removed.

How can I use CUBES with students needing additional support?

Design features can help students identify key words and features, even when certain vocabulary words are not known. As teachers introduce the content-specific Tier 3 vocabulary to their classes, annotation strategies such as CUBES can help students use these words to apply appropriate strategies while problem solving. Using circles, underlines, and boxes can help students organize the key facts and vocabulary, and the elimination of unnecessary information can simplify the problem. This strategy is especially beneficial for ESL students.

Combining CUBES with graphic organizers also benefits ESL students by removing a lot of the confusing wording and allowing them to focus on the important facts of a problem. When using CUBES with a graphic organizer, all students, including ESL students, will begin to think about problem solving in a way that helps encourage them to use the appropriate information to find a solution.

What other standards does the CUBES Protocol address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.RC.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E2.C.7.1
- ELA.E4.C.9.1
- ELA.E1.C.2.1
- ELA.E1.AOR.7.1

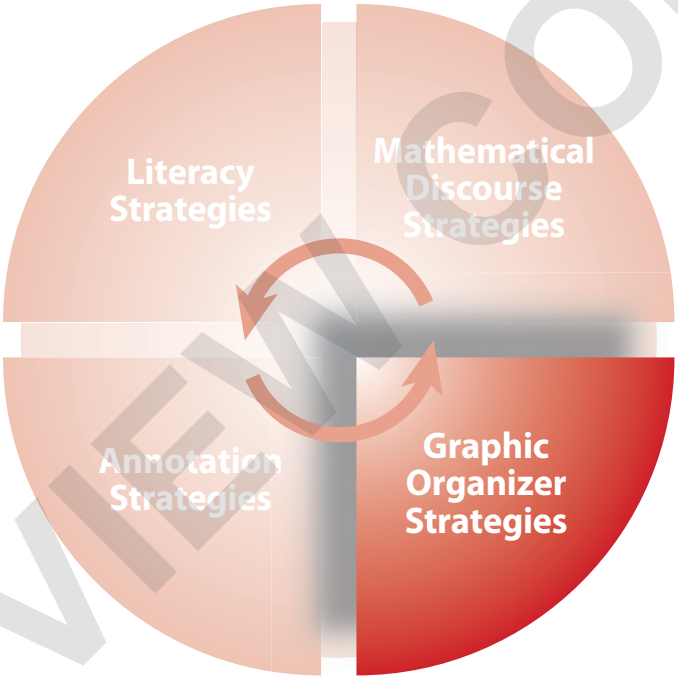
Source

- Margaret Tibbett. "Comparing the effectiveness of two verbal problem solving strategies: Solve It! and CUBES."

  https://www.walch.com/rr/09061

Organizing Mathematical Content: Graphic Organizers

Organizing mathematical content is a crucial skill for problem solving, exploring other possible methods for finding solutions, and managing math content. All students need strategies for organizing content to build conceptual understanding. For students with disabilities, visual representations and graphic organizers can help them clarify their thoughts and focus on the math. ESL students also benefit from visual representations and graphic organizers. Organizing mathematical knowledge with visuals can help ESL students navigate math content while learning the language. Graphic organizers include Frayer Models and Tables of Values.



Graphic Organizers

Frayer Models

What is a Frayer Model?

A Frayer Model is a graphic organizer that can help students understand new vocabulary words and concepts by exploring their characteristics. A Frayer model lists the definition of a word or concept, describes some key facts, and gives examples and non-examples. Examples and non-examples can come from a mathematical or real-world context.

How do you implement Frayer Models in the classroom?

Students can learn to create Frayer Models the first week of school, and the process can be used throughout the year each time students experience a new word or concept.

While it is important for teachers to give students precise mathematical definitions with appropriate content vocabulary, it is maybe more important for students to understand the application of mathematical words and concepts in their own context. As students learn new information, small group discussions and think-pair-share activities are great ways for students to formulate their own definitions, review the characteristics and facts they have learned, and discuss examples and non-examples.

Discussions of the examples and non-examples can help lead to the mathematical definition. For example, if students use a Frayer Model to define a quadratic function, they would notice that all examples have a highest exponent of 2, and all non-examples would not have a highest exponent of 2. All examples would have parabolic graphs, and all non-examples would have other graphs. Through these comparisons, students will understand the definition of quadratics using different representations, and they will be able to apply it in different contexts.

When would I use Frayer Models in the classroom?

Frayer Models can be used at different points during instruction. They are appropriate as introductions to new concepts, summaries to ensure understanding of new concepts, or as note-organizers throughout the lesson for students to fill in as they learn new concepts. At first, students might need help figuring out how to list and differentiate between the definition, facts and characteristics, examples, and non-examples. As students adapt to the process, they will be able to categorize information on their own or in small groups. As they compare newer Frayer Models to previous models, they will also be able to see how concepts build upon each other.

How can I use Frayer Models with students needing additional support?

Frayer Models can be a point of reference for students as they progress throughout the year. As students determine their own definitions for math-specific words and concepts, and use the examples and non-examples to determine the key facts, they will be able to put them in their own context and apply them to solve complicated problems. As math concepts build upon each other both within a unit and throughout the year, the use of Frayer Models to remind students of their initial definitions of words or concepts can help solidify their understanding. Using Frayer Models as part of a Word Wall or Concept Wall, or having a consistent notebook process to reference past Frayer models, can help consistently reinforce learning.

What other standards do Frayer Models address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E2.C.1.1
- ELA.E1.C.8.1
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1

Source

- Deborah K. Reed. "Building Vocabulary and Conceptual Knowledge Using the Frayer Model." https://www.walch.com/rr/09062

Frayer Model

Definition	Characteristics
WORD	
Examples from Life	Non-Examples

Graphic Organizers

Tables of Values

What is a Table of Values?

A Table of Values is an organized way to list numbers that represent different categories of values. These values can be represented as ordered pairs, graphs, word problems, or lists. Tables can help students see and compare values in a different way.

How do you implement Tables of Values in the classroom?

Tables can be used throughout the year to support various mathematical standards. Some standards mention tables specifically, and in others, tables can be an effective support to help students organize and understand the meaning and application of values.

Tables can be set up with numerical values in rows or columns. The key to understanding the values lies in the headings. The headings must be specific enough to show students the meaning and/or application of the numerical values, but not so wordy that they interfere with the clarity of the numbers in the table. For example:

x **(year)**	***y*** **(population in millions)**
1960	219
1970	230
1980	258
1990	312
2000	342

Mean (statistical average)	50	45
Median (middle value)	52	43
Quartile 1 (median of the lower 50%)	40	38
Quartile 3 (median of the upper 50%)	72	80
Range (difference of max and min values)	80	61
Interquartile Range (difference of quartiles)	32	42
Standard Deviation (measure of spread of data)	7.24	10.23

When would I use Tables of Values in the classroom?

Various mathematical topics can be represented by tables. For example:

- An (x, y) table of values to represent coordinates on a graph or independent and dependent variables for a given context
- A table to represent coefficients and/or constants in an equation
- A table to show different statistical measures when comparing sets of data
- A table to compare output values for the same input given different functions

Each time numbers or values are being listed, compared, or graphed, a table can help students differentiate between the values. Tables are easy to create, and students can be encouraged to create them as another representation to clarify and compare numbers for nearly any topic.

How can I use Tables of Values with students needing additional support?

Tables of Values can help students focus on numerical values and their meaning in context without distraction. They clarify what each number represents, what numbers can be compared, and what ordered pairs can be graphed to give a visual representation. Additionally, headings can be used to either highlight the relevant facts from a context or to describe mathematical vocabulary.

In general, graphic organizers benefit students by removing much of the confusing wording and focusing on the important facts and numbers of a problem.

What other standards do Tables of Values address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

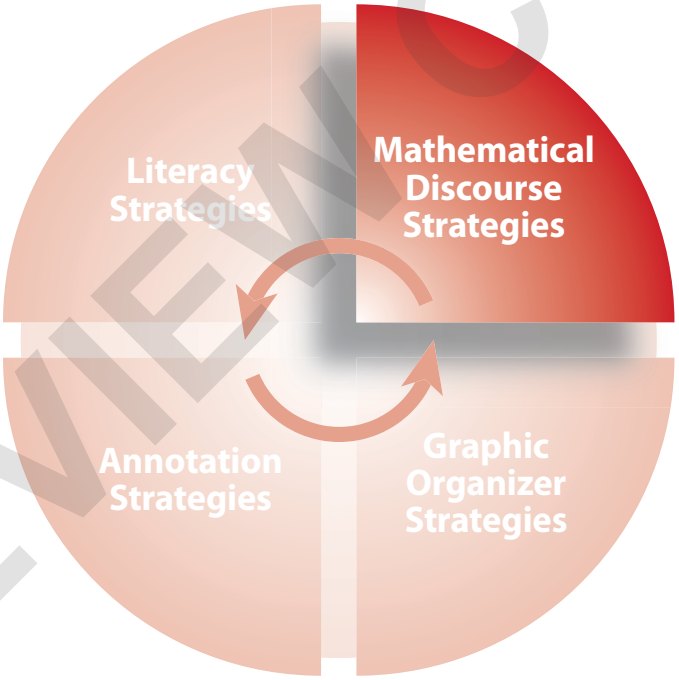
- ELA.E1.C.1.1
- ELA.E2.C.1.1
- ELA.E1.C.8.1
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1

Source

- Alliance for Excellent Education. "Six Key Strategies for Teachers of English Language Learners." https://www.walch.com/rr/09060

Communicating Mathematical Content: Mathematical Discourse

Reading, writing, speaking, and listening are all important ways to learn and express information, but the last two ways are often slighted in the math classroom. The mathematical discourse strategies presented here promote speaking and listening in a math-focused literacy context. Working these strategies into the daily routine of a classroom can help students become comfortable speaking and listening in a mathematical context, which will help them become comfortable with the mathematical content. Routines and structures are essential to support students with disabilities, as they often benefit from following a routine. This can lead to developing capability in their mathematical skills. These strategies also remove the barrier to entry for many ESL students, as structure and routine can help them focus on the math content rather than English language deficiencies. Mathematical Discourse strategies include Sentence Starters and Small Group Discussion.



Mathematical Discourse Strategies

Sentence Starters

What is a Sentence Starter?

A Sentence Starter is a common phrase or mathematical sentence frame that can help students begin and sustain academic conversations around mathematical content. It helps guide students through the discussion and bring out pertinent ideas that can lead to greater understanding.

How do you implement Sentence Starters in the classroom?

Many people view math class as a place to calculate solutions to math problems. However, to ensure the conceptual understanding and proper application of a math concept, students need to be able to explain the concepts and reasoning behind a solution to a problem. As many students are not accustomed to having academic conversations about math, sentence starters can help begin and continue these conversations in a productive manner.

There are two main types of sentence starters for mathematical discussions: discourse starters and math starters. For example, a poster with these or other sentence starters can be displayed from the beginning of the year, and the expectation can be set that any answer to a question or comment in a discussion should be framed using one of these starters. As students become accustomed to framing mathematical conversations in this way, they can expand on the given sentence starters and create some of their own. They will begin to realize how these statements ensure that their conversations revolve around math, enhance understanding of the concept, and force them not only to state, but also to explain their thinking. They will gain confidence from the ability to engage, as the first step has already been taken for them.

When would I use Sentence Starters in the classroom?

Sentence Starters can be used throughout the entire school year with any concept. However, they are most important to use at the beginning of the school year to build a mathematical community in the classroom centered on a comfort with mathematical discourse. Especially at the beginning of the year, students should be encouraged to use these sentence starters for every math statement. Appropriate settings include during small group discussion, while responding to whole class questions, and when writing explanations for problem solutions.

Modifications can be introduced so that students must use certain mathematical vocabulary within the sentences, or must use certain sentence starters at different points in conversations or for different conversation types and situations. However the starters are implemented, it is important for students to realize that these are intended to enhance and focus their conversations, not limit them.

How can I use Sentence Starters with students needing additional support?

Often, students are reluctant to talk about math concepts because they either lack confidence in their knowledge, are afraid to be "wrong," or don't know how to start or continue the conversation. Sentence starters can help students overcome this reluctance. The non-threatening, easy-to-interpret sentence starters remove the barrier to entry for students who don't know how to engage, and the respectful, mathematical focus promoted by sentence starters can help build confidence and provide a structure so that students will not fear being wrong.

For ESL students specifically, sentence starters can provide the English language support to help students engage with and discuss the math. The support of sentence structure removes language barriers to entry for students who don't fully understand English sentence structure.

Discourse Starters	Math Starters
I agree/disagree with ... because ...	My answer was ... because ...
I understand/don't understand ...	The next step is ... because ...
First/Next/Finally I ... because ...	I used (insert formula/equation/concept) because ...
I noticed that ...	
I wonder ...	My answer is right/reasonable because ...

What other standards do Sentence Starters address?

WIDA English Language Development Standards

- ELD Standard 3

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.RC.1

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E2.C.1.1
- ELA.E1.C.8.1
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1

Source

- AVID. "Sentence Starters." https://www.walch.com/rr/09064

Mathematical Discourse Strategies

Small Group Discussion

What is Small Group Discussion?

Small Group Discussion is a structured way for students to verbalize their mathematical thinking in a comfortable setting to solve a problem, build conceptual understanding, or summarize a concept.

How do you implement Small Group Discussion?

Small Group Discussion in math class depends on a trusting relationship between the teacher and the students. From there, students can build trusting relationships among themselves. Once this trust has been built, students will feel free to explore mathematical topics in groups, take risks, and engage in a productive struggle toward understanding or a solution.

Once these relationships have been established, certain structures should be established for Small Group Discussion to be effective. Discussion norms can be set by the class to ensure discussions are respectful and productive, and discussions should have predetermined time limits. The group composition is also important and should be based on instructional measures. For different activities, homogeneous groups, heterogeneous groups, or groups based on specific data by standard could be appropriate. Students should always be aware that the groups were chosen to maximize their learning.

Another structure that can be effective for Small Group Discussion is assigning group roles. These roles can include group leader, note taker, timekeeper, resource manager, culture keeper, or other roles determined to be appropriate for the classroom context. During the discussion, assigning each student a letter within the group (A, B, C, D, etc.) can help structure the discussion. Different roles can specify certain time limits for talk, which sentence starters to use, or other structured aspects of the discussion.

When implementing a Small Group Discussion, the question or task should inspire students to think in different ways about a concept. Through the structured format of the discussion, students will compare their ideas and arrive at an answer or explanation of the concept. Within the trusting framework of the class and group, students can focus on the common goal of the discussion and develop their thinking around the math concept. These rich discussions will enhance their understanding.

When would I use Small Group Discussion in the classroom?

Small Group Discussion can be used for nearly any topic, and it can be used at a variety of times in the classroom. The questions and tasks may need to change depending on when it is used. Opening activities for lessons can be Small Group Discussions where students explore properties of new math concepts or review/build upon their prior learning. Turn and talks throughout the lesson can be structured as Small Group Discussions if a consistent framework is in place. At the end of class, a Small Group Discussion can be used to come to a common understanding about an essential question from the lesson.

Depending on when the Small Group Discussion is used in class, and what the goal of the discussion is, the discussion reporting may vary. For a warm-up, each group might be asked to share their thinking. For a guided practice, recording answers on chart paper and a gallery walk could be appropriate. For a closing activity, individual written responses to a question could be appropriate.

How can I use Small Group Discussion with students needing additional support?

As discussed in other Mathematical Discourse strategies, struggling students are reluctant to talk about math concepts because they lack confidence in their knowledge and don't always have the needed vocabulary in their toolbox. Structured discussions with effective grouping can help students through these barriers. After a trusting and respectful classroom environment has been established, struggling students often feel more comfortable sharing their ideas with just a few classmates rather than the whole class. Additionally, adding structure can help students engage by providing the expectation that they participate in the process.

The intentional grouping of students can also help them succeed using Small Group Discussion. At times, heterogeneous groups could be appropriate so that stronger students can help struggling students, and at other times, homogeneous groups could be appropriate so the teacher can work with an entire group of struggling students. ESL students can be grouped with other students with the same dominant language to help remove the language barrier from the conversation.

What other standards does Small Group Discussion address?

WIDA English Language Development Standards:

- ELD Standard 3

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.RC.1

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E2.C.1.1
- ELA.E1.C.8.1
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1

Source

- Jessie C. Store. "Developing Mathematical Practices: Small Group Discussions." https://www.walch.com/rr/09065

Modeling Strategies

Mathematical Modeling

What is Mathematical Modeling?

Mathematical modeling is generally understood as the process of applying mathematics to a real-world problem with a view of understanding the connection. According to the CCSSM, mathematical modeling is the ability to apply concepts learned in class to real-world applications and to use the model to analyze a situation, draw conclusions, and make predictions.

How do you implement Mathematical Modeling in the classroom?

Modeling can be implemented by demonstrating how to make or generate mathematical representations or models, how to validate them, and how to use them to solve real-world problems. There are many ways to show understanding in a math classroom, such as using words, drawings or sketches, physical models, computer programs, or math formulas.

The following is a list of questions and answers suggested in order to create a mathematical modeling classroom environment:

- **Why?** What are we looking for? Identify the need for the model.
- **Find?** What do we want to know? List the data we are seeking.
- **Given?** What do we know? Identify the available relevant data.
- **Assume?** What can we assume? Identify the circumstances that apply.
- **How?** How should we look at this model? Identify the parameters.
- **Predict?** What will our model predict? Identify the equations that will be used, the calculations that will be made, and the answers that will result.
- **Valid?** Are the predictions valid? Identify tests that can be made to validate the model; i.e., is it consistent with its principles and assumptions?
- **Verified?** Are the predictions good? Identify tests that can be made to verify the model; i.e., is it useful in terms of the initial reason it was done? *(inspired by Carson and Cobelli, 2001)*

Teachers should expect these questions to recur often during the modeling process, and should regard this list as a fairly general approach to ways of thinking about mathematical modeling.

In a classroom where mathematical modeling is the expectation, teachers will need to establish that students are responsible for coming up with methods for solving the problems presented and that the teacher will only assist and facilitate.

When would I use Mathematical Modeling in the classroom?

It should come as no surprise that many students find mathematics boring. The most common question posed to any mathematics teacher is "When will I ever need to use this?" Often teachers fail to find problems in which students are interested or to even take student interest into account when planning a lesson. Problems that spark students' interest and curiosity will increase their attention and desire to learn. These types of real-world problems provide students an opportunity to think and respond as a mathematician. Students should be exposed to rigorous learning tasks that allow opportunities for mathematical modeling in the classroom.

How can I use Mathematical Modeling with struggling students?

When struggling readers, which includes ELLs and students with learning disabilities, are exposed to rigorous math learning tasks, there must be a level of scaffolding that includes coaching and guided questions that help to make a word problem or learning task much more accessible. Teachers should come up with questions to guide the students before and during the engagement of the task. Teachers should also:

- assess prior knowledge;
- define Tier 2 and 3 vocabulary words;
- discuss non-mathematical concepts in the task; and
- assist students in identifying key concepts and facts within the tasks.

Allowing struggling readers to explain their answers using words, numbers, or graphics/pictures ensures that they can express their opinion and rationale despite a potential lack of vocabulary. Through these representations and the ensuing discussion, students will begin to learn the necessary math concepts to be successful.

What other standards does Mathematical Modeling address?

WIDA English Language Development Standards:

- ELD Standard 3

Mathematical Process Standards:

- MPS.PS.1
- MPS.C.1
- MPS.AJ.1
- MPS.RC.1

English Language Development for Mathematics:

- ELD–A.9–12: Explain (Interpretive)
- ELD–MA.9–12: Explain (Expressive)

SC English Language Arts standards:

- ELA.E1.C.2.1
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1
- ELA.E1.AOR.8.1
- ELA.E1.C.1.1
- ELA.E1.AOR.2

Sources

- Carson, Ewart, and Cobelli, Claudio. *Modeling Methodology for Physiology and Medicine*. Academic Press, San Diego, CA, 2001.
- Dym, Clive L. *Principles of Mathematical Modeling*. Amsterdam: Elsevier Academic Press, 2008.
- MathisFun.com. "Mathematical Models." https://www.walch.com/rr/09066
- Oswalt, Selena. "Mathematical Modeling in the High School Classroom." LSU Digital Commons. Accessed February 25, 2021. https://www.walch.com/rr/09067

PROGRAM OVERVIEW

Mathematical Modeling Implementation Guide

Introduction

BW Walch resources support the framework of the Mathematical Process Standards (MPS) and the NCTM Principles of Teaching Practices. Implementing strategies and support from both practices lead to true conceptual understanding of the math standards. One of which includes mathematical modeling, the process of designing and revising representations to solve a problem.

Mathematical modeling is essential to building a deep conceptual understanding of math concepts for students. Teaching students to model boosts engagement, builds student confidence in math concepts, helps them to make sense of problems, and allows them to make connections to the world around them for better understanding. Students then make decisions about the information, create models, interpret the results, and form conclusions.

A Mathematical Modeling Framework

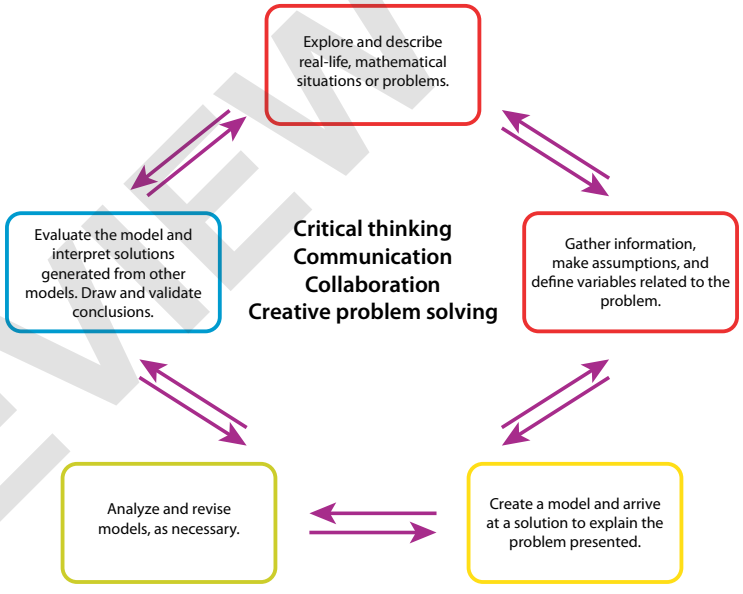


Image adapted from: Suh, Matson, Seshaiyer, 2017

The following is a brief description of how this framework can be applied in the classroom.

Critical Thinking

Students will explore and describe real-life mathematical situations or problems. We want students to discover new ways of thinking and ideas in mathematics. Students do this by developing questions to ask, gathering information, and coming up with solutions. Fostering critical thinking in the classroom

not only makes students better at math, but also prepares them for the real world. Below are some ideas and probing questions teachers may use to implement critical thinking.

- Allow for pair-share and small group discussions.
- Encourage students to think and form their own conclusions.
- Encourage the revision of their own thinking and the thinking of others.
- Ask students to think out loud as they work.
- Create a classroom environment that embraces and values student ideas.

Ask students:

- What is the problem asking you to solve?
- Can you think of other strategies you could use to solve this problem?
- What conclusions can you make from this particular problem?
- Will this strategy work in all problems like this? Why or why not? How can we test that?
- Explain how you got to your answer.
- Explain your reasoning.
- How would you respond to a different answer to the same problem?

Communication

When students gather information, make assumptions, and define variables related to the problem, communication allows for them to show their understanding of the math content. Encourage discourse by allowing students to explain their thinking and challenge each other. This encourages students to justify their reasoning. If students communicate their thinking in various ways (including written and oral responses) while doing math, it will improve their understanding of math concepts.

Teachers can do the following to foster communication in the classroom:

- Ask open-ended questions.
- Encourage oral and visual (written and pictorial) communication through journal writing.
- Provide students with detailed feedback.

Ask students:

- Can you explain your thinking?
- How did you get your answer?
- What strategies did you use?
- What information was necessary for you to solve this problem?

Collaboration

Collaboration is an essential component of student success. It allows students to rely on each other during their problem solving. During collaboration, students work in groups, share ideas, ask questions, and discuss math concepts and additional solution strategies while supporting and defending their thinking. Collaboration is most beneficial to students with the use of effective grouping strategies such as assigning students to heterogeneous groups or random grouping.

The following procedures and probing questions can help you implement collaboration in your classroom.

- Establish a classroom culture where all ideas are valued.
- Establish expectations and routines of collaborative learning.
- Discuss "math talk" passages with students.
- Allow students to teach each other.
- Incorporate an accountability piece for students.
- Arrange student seating to support collaboration (group seating).
- Create heterogeneous student groups with varying skill levels.
- Randomize student groups.
- Keep group sizes between 3 and 6 students.
- Assign group roles.

Ask students:

- Come up with as many strategies to solve the problem as you can.
- Explain how you made your calculations.
- Why did you choose that strategy? Why did that strategy work?
- Describe in your own words how your peer-solved the problem.
- Can you make any connections between your strategies?
- Were there any methods that were better than others when solving this problem? Why or why not?
- What did you learn from your group?
- Defend your reasoning behind that solution.

Creative Problem Solving

Creative problem solving is the ability for students to perform math tasks that allow for challenges that increase their conceptual understanding. While performing these tasks, we want students to use mathematical modeling. We want students to evaluate their models and to interpret solutions from other models.

In creative problem solving, students solve problems using different approaches and models, draw on prior knowledge, and justify their thinking. This results in students becoming better problem solvers and increases their understanding of math concepts. Problem solving should be integrated into their math learning and should not be separated.

Here are some tips for implementing creative problem solving.

- Encourage students to challenge different approaches and strategies from their peers as well as the teacher.
- Encourage discourse.
- Allow appropriate wait time for student responses.
- Refrain from telling students how to solve the problem. Instead, allow students to engage and come to their own solutions.
- Allow students to struggle productively.

Ask students:

- How is the information in the problem important to determining the solution?
- How did you go about solving this?
- Can you explain why you chose that model and strategy?
- Are there other ways to model this particular problem? Can you model the problem another way?
- Why did you make that calculation?
- Justify your solution.
- What generalizations can you make about the math concepts based on this particular problem?

Recommended Resource

- Georgia Department of Education. "Scaffolding Instruction for English Learners: A Georgia Mathematics Instructional Resource Guide."

  https://www.walch.com/rr/09047

  The purpose of this document is to provide mathematics teachers and leaders with evidence-based, pragmatic scaffolds and supports for English Learners (ELs). This guide is a useful tool to help teachers provide high-quality instruction aligned to Georgia's K-12 Mathematics Standards.

Source

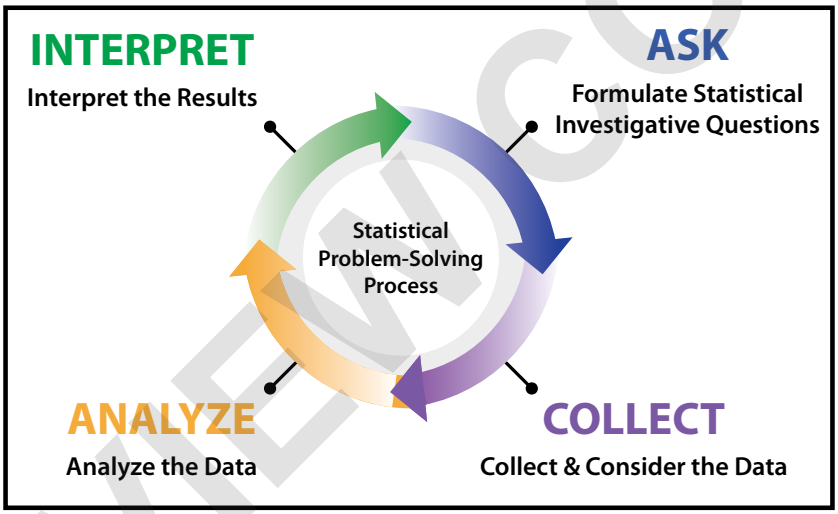
National Council of Teachers of Mathematics. "Problem Solving." Accessed January 11, 2023. https://www.walch.com/rr/09048.

PROGRAM OVERVIEW

Statistical Reasoning Implementation Guide

Introduction

Statistical reasoning allows students to make sense of ideas, information, and the changing world through questioning and exploration. It provides the foundation necessary for students to fully understand the concept. Statistical reasoning is a continuous cycle consisting of students asking questions, collecting, analyzing, and interpreting data. In order to guide students in this sense-making process, BW Walch resources support this four-step statistical problem-solving strategy to help students develop their understanding in statistical reasoning.



Source: Georgia Department of Education

Here is a brief description of how this framework can be applied in the classroom.

Formulate Statistical Investigative Questions

Students will form and ask investigative questions that allow for various answers. These questions will clarify the problem and lead to questions that can be answered with the data. Best practices and teacher prompts that can foster this framework include:

- Using a student-centered approach.
- Having students prepare ahead of time with an assigned reading to familiarize themselves with words and techniques.

Ask students:

- What do you think?
- What do you notice? What do you wonder?
- What criteria need to be met in order for the question to be statistical?
- How did you determine your question?
- What changes would you make to the question?

Collect and Consider the Data

Students will collect data by creating a plan in order to collect real and relevant data. Making sure the data is relevant to students will increase engagement and lead to more math talk and discussion. Strategies include:

- Refraining from presenting students with procedures.
- Allowing students to use real data sets and to generate their own data.
- Encouraging students to discuss the questions and possible ideas.

Ask students:

- What do you notice about the data?
- In what other ways can the data be collected?
- What are some other methods you can use to collect the data? How do these different methods affect your data collection?
- How can you represent your data? Can you represent it with a visual?
- Are there representations better fit for particular findings? Justify your answer.

Analyze the Data

Students will analyze the data by selecting methods that are appropriate. Exploration of various methods will allow for students to make connections and draw conclusions based on the data. This will deepen their understanding of statistical reasoning. Strategies include:

- Allowing students to use technology tools to explore and analyze their findings.
- Refraining from giving students all the information. Allow students to form their own analysis of the data.
- Creating a classroom environment in which student ideas are valued.

Ask students:

- What conclusions can you draw from the data?
- Do you notice any trends in the data? How can you tell?
- What is the relationship between the data points?
- What evidence may help you distinguish between results?
- Do you agree or disagree? Justify your thinking.
- How can we test that conclusion?
- What do you do about outliers in your data? What do they tell you?
- If extreme values are removed, what happens to the data representation?
- Compare your data with a classmate's. What do you notice?

Interpret the Results

Students will interpret and discuss the results by relating all findings to the original question. Students will discuss these findings and justify their reasoning. Best practices and teacher prompts include:

- Encouraging discourse. Encourage students to present their ideas, answer classmates' questions, and support their responses.
- Focusing on key ideas instead of procedures and calculated answers.
- Making sure students have answered their "I wonder" questions.

Ask students:

- What do the results tell you about the original question?
- Have your "I wonder" questions been answered?
- What conclusions can you make from the results?
- Compare your interpretations to those of your classmates. What connections can you make?
- What do your interpretations represent in a real-world context?

Source

Garfield, Joan and Ben-Zvi, Dani. "Helping Students Develop Statistical Reasoning: Implementing a Statistical Reasoning Learning Environment." Accessed Jan. 11, 2023. https://www.walch.com/rr/09049

Graphic Organizers

Overview

Graphic organizers can be a versatile tool in your classroom. Organizers offer an easy, straightforward way to visually present a wide range of material. Research suggests that graphic organizers support learning in the classroom for all levels of learners. Gifted students, students on grade level, and students with learning difficulties all benefit from the use of graphic organizers. They reduce the cognitive demand on students by helping them access information quickly and easily. Using graphic organizers, learners can understand content more clearly and can take concise notes. Ultimately, learners find it easier to retain and apply what they've learned.

Graphic organizers help foster higher-level thinking skills. They help students identify main ideas and details in their reading. They make it easier for students to see patterns such as cause and effect, comparing and contrasting, and chronological order. Organizers also help students master critical-thinking skills by asking them to recall, evaluate, synthesize, analyze, and apply what they've learned. Research suggests that graphic organizers contribute to better test scores because they help students understand relationships between key ideas, and enable them to be more focused as they study.

Types of Graphic Organizers

There are four main purposes for using graphic organizers in mathematics and a variety of tools within each category:

Purpose 1: Organizing, Categorizing, and Classifying	**Purpose 2: Problem Solving**	**Purpose 3: Understanding Mathematical Information**	**Purpose 4: Communicating Mathematical Information**
Tables Flowcharts Webs Venn Diagrams	Number Lines Geometric Drawings Factor Trees Attribute Tables Cause and Effect Maps Coordinate Plane Probability Trees	Frayer Model Semantic Map/ Concept Map Compare-and-Contrast Diagram	Line Graphs Bar Charts

Tables

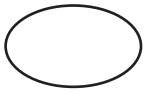
A table is simply a grid with rows and columns. Tables are useful because information stored in a table is easy to find—much easier than the same information embedded in text.

Usually, a table has a row (horizontal) for each item being listed. The columns (vertical) provide places for details about the listed items—the things they have in common. The places where the rows and columns meet are called cells. In each cell, we write information that fits both the topic of the row (the thing being listed) and the topic of the column (the aspect being examined). To create a table, we make rows and columns to fit the number of items and attributes.

Flowcharts

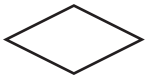
Flowcharts are graphic organizers that show the steps in a process. Flowcharts can be very simple—just a series of boxes with one step in each box. However, there is also a more formal type of flowchart. These flowcharts use special symbols to show different things, such as starting and stopping points, or points where decisions must be made. These symbols make flowcharts especially useful for showing complicated processes.

Each step in a flowchart is written in a box. The boxes are connected by arrows to show the sequence of steps. The boxes aren't all rectangular; different shapes are used to indicate different actions. The shapes and symbols are a kind of visual shorthand. Whenever a certain symbol is used, it always has the same meaning.



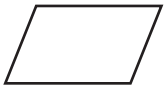
- Circles and ovals show starting and stopping points. They often contain the words start or stop. The "start" circle or oval has no arrows in and one arrow out. The "stop" circle or oval has one arrow in and no arrows out.

- Arrows show the direction in which the process is moving.



- Diamonds show points where a decision must be made or a question must be answered. The question can usually be answered either "yes" or "no."

- Rectangles and squares show steps where a process or an operation takes place.



- Parallelograms show input or output, such as writing or printing a result or solution.

Webs

Webs are graphic organizers that help take notes, identify important ideas, and show relationships between and among pieces of information. In a web, the main idea is written in the center circle. Details are recorded in other circles with lines to connect related topics. Circles or lines can be added or deleted as necessary.

Number Lines

In its simplest form, a number line is any line that uses equally spaced marks to show numbers. Number lines are used to visualize equalities and inequalities, positive and negative numbers, and measurements of all kinds. They can "map" math problems, especially ones that involve negative numbers or distances.

Geometric Drawings

A geometric drawing is a representation on paper (or some other surface) of a geometric figure. The geometric drawings we make can never be as perfect as the geometric figures they represent, but as long as they are reasonably accurate, they can help us visualize the figures. In fact, it's often impossible to solve a geometry problem without making a drawing.

Factor Trees

There are several ways to find factors. One that helps to visually keep track of all the factors is called a factor tree. This is a diagram with a tree-like shape. It uses "branches" to show the factors of a number.

All whole numbers other than 1 can be written as the product of factors. A prime number is a number that has only two factors, itself and 1. An example of a prime number is 13. Its only factors are 13 and 1. A composite number is a number that has more than two factors. An example of a composite number is 6. Its factors include 6, 3, 2, and 1. Prime factors are factors that are also prime numbers. The greatest common factor (GCF) of two numbers is the largest number that is a factor of both numbers.

Coordinate Plane

This is the plane determined by a horizontal number line, called the x-axis, and a vertical number line, called the y-axis, intersecting at a point called the origin. A coordinate plane can be used to illustrate locations and relationships using ordered pairs of numbers.

Venn Diagrams

A set is a list of objects in no particular order. Items in a set can be numbers, but they can also be letters or words. Venn diagrams are a visual way of showing how sets of things can include one another, overlap, or be distinct from one another.

Venn diagrams are often used to compare and contrast things. But they are also a useful tool to sort and classify information. You can use Venn diagrams to take notes on material that shows relationships between things or ideas. You can also use them to solve certain types of word problems. When a word problem names two or three different categories and asks you how many items fall into each category, a Venn diagram can be a useful problem-solving tool.

A Venn diagram begins with a rectangle representing the universal set. Then each set in the problem is represented by a circle. Circles can be separate, overlapping, or one within another. When two circles overlap, it means that the two sets intersect. Some members of one set are also members of the other set.

Venn Diagrams AND **Compare-and-Contrast Diagrams**

The Venn diagram is an organizing device for planning comparisons and contrasts. A completed Venn diagram helps students categorize and organize similarities and differences, and provides a blueprint for a comparison-and-contrast exercise. The compare-and-contrast diagram provides a structure to identify or list similarities and differences between two objects.

Attribute Tables

To solve logic problems, you need a way to keep track of the subjects and which attributes they have or don't have. An attribute table can help. This is a table with a row for each subject in the problem, and a column for each attribute. The rows and columns meet to form cells. Because the attributes in logic problems are usually exclusive, you can use Xs or check marks (✔) to show which attribute belongs to which subject.

Cause and Effect Maps

Cause and effect maps help you work through information to make sense of it. Write each cause in the oval. Write all its effects in the boxes. Add or delete ovals and boxes as needed.

Frayer Model

The Frayer Model is a word categorization activity that helps learners to develop their understanding of concepts. Using this model, students provide a definition, list characteristics, and provide examples and non-examples of the concept.

Semantic Map

A semantic word map allows students to conceptually explore their knowledge of a new term or concept by mapping it with other related words, concepts, or phrases that are similar in meaning. Semantic maps portray the schematic relations that compose a concept. It assumes that there are multiple relations between a concept and the knowledge that is associated with the concept.

Line Graphs

Line graphs are often used to show how things change over time. They clearly show trends in data and can let you make predictions about future trends, too. Line graphs use two number lines, one horizontal and one vertical. The horizontal number line is called the *x*-axis. The vertical line is called the *y*-axis. The *x*-axis often shows the passage of time. The *y*-axis often shows a quantity of some kind, such as height, speed, cost, and so forth.

Bar Charts

Bar charts are useful when you want to compare things or to show how one thing changes over time. They are a good way to show overall trends. Bar charts use horizontal or vertical bars to represent data. Longer bars represent higher values. Different colors can be used to show different variables. When you look at a bar chart, it's easy to see which element has the greatest value—the one with the longest bar.

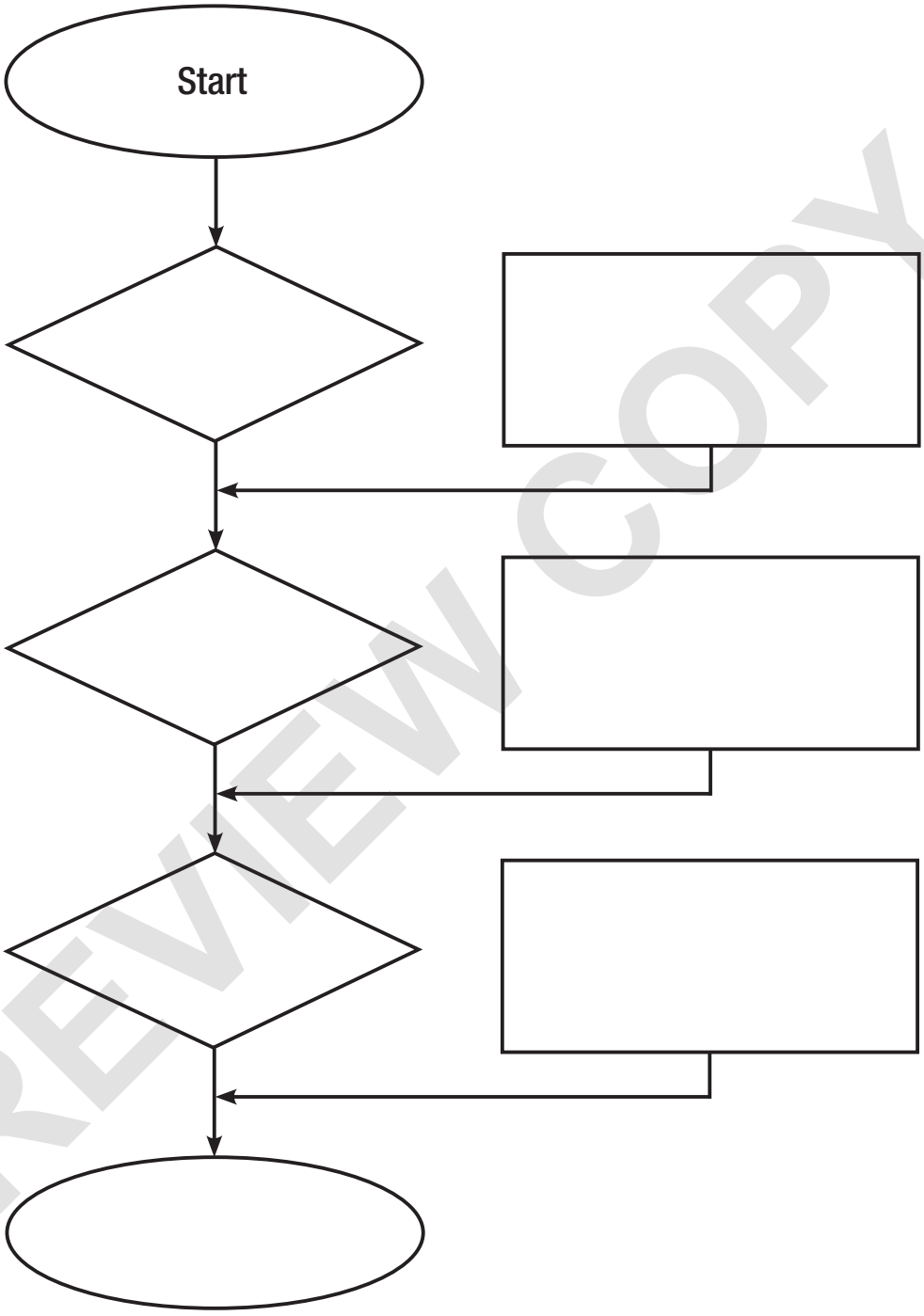
Bar charts have an *x*-axis (horizontal) and a *y*-axis (vertical). If the graph is being used to show how something changes over time, the *x*-axis has numbers for the time period. If the graph is being used to compare things, the *x*-axis shows which things are being compared. The *y*-axis has numbers that show how much of each thing there is.

Probability Trees

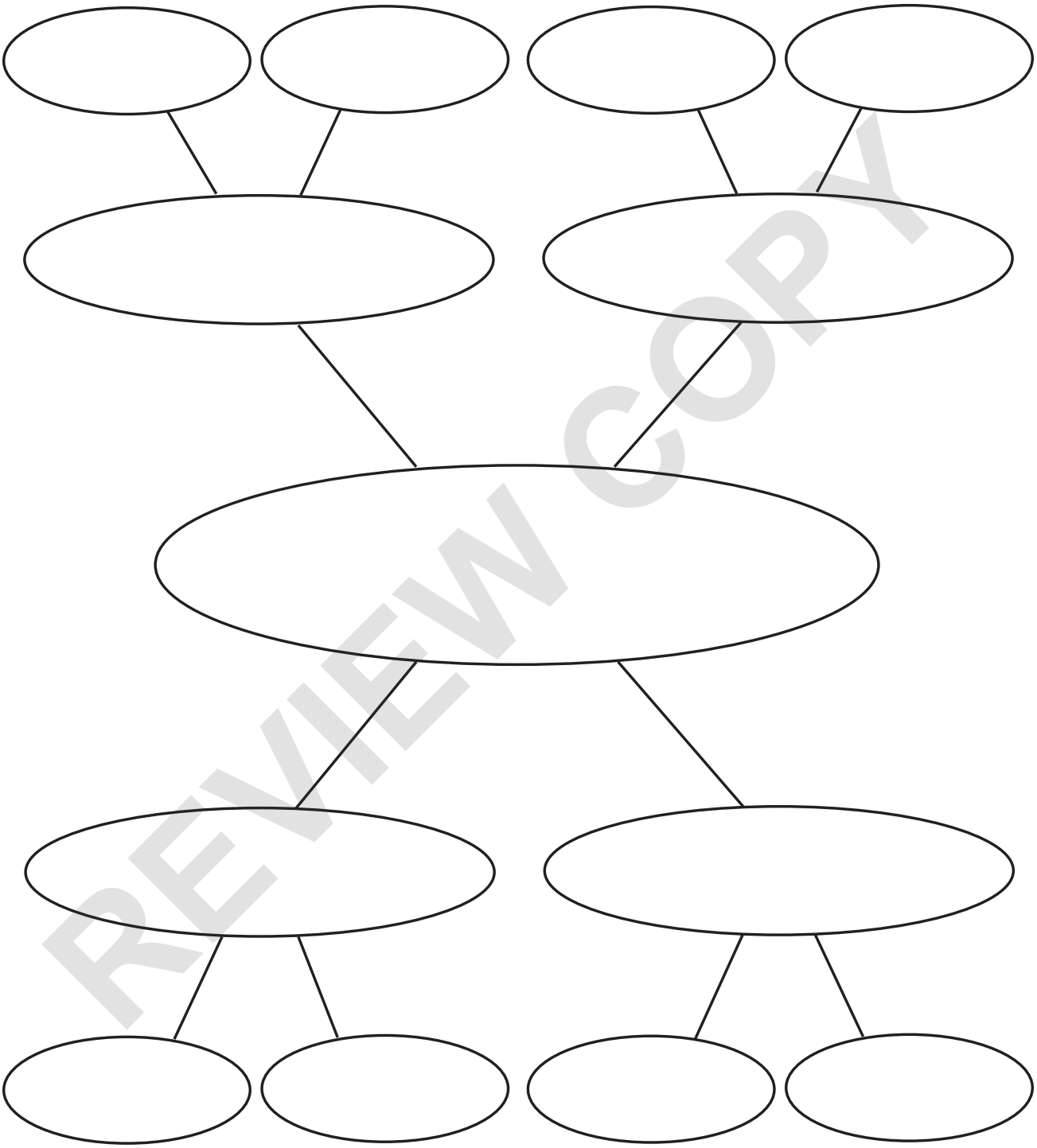
When we have probability problems with many possible outcomes, or events that depend on one another, probability trees can help. Probability trees show all the possible outcomes of an event. Whenever a problem calls for figuring out how many possible outcomes there are, and the probability that any one of them will happen, a probability tree can be useful.

Table

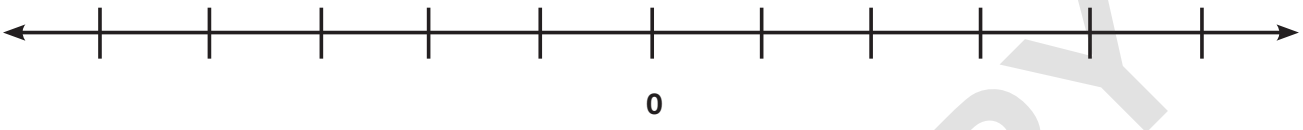
Flowchart



Web



Number Line



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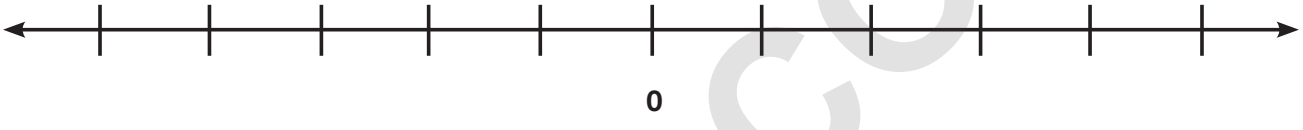

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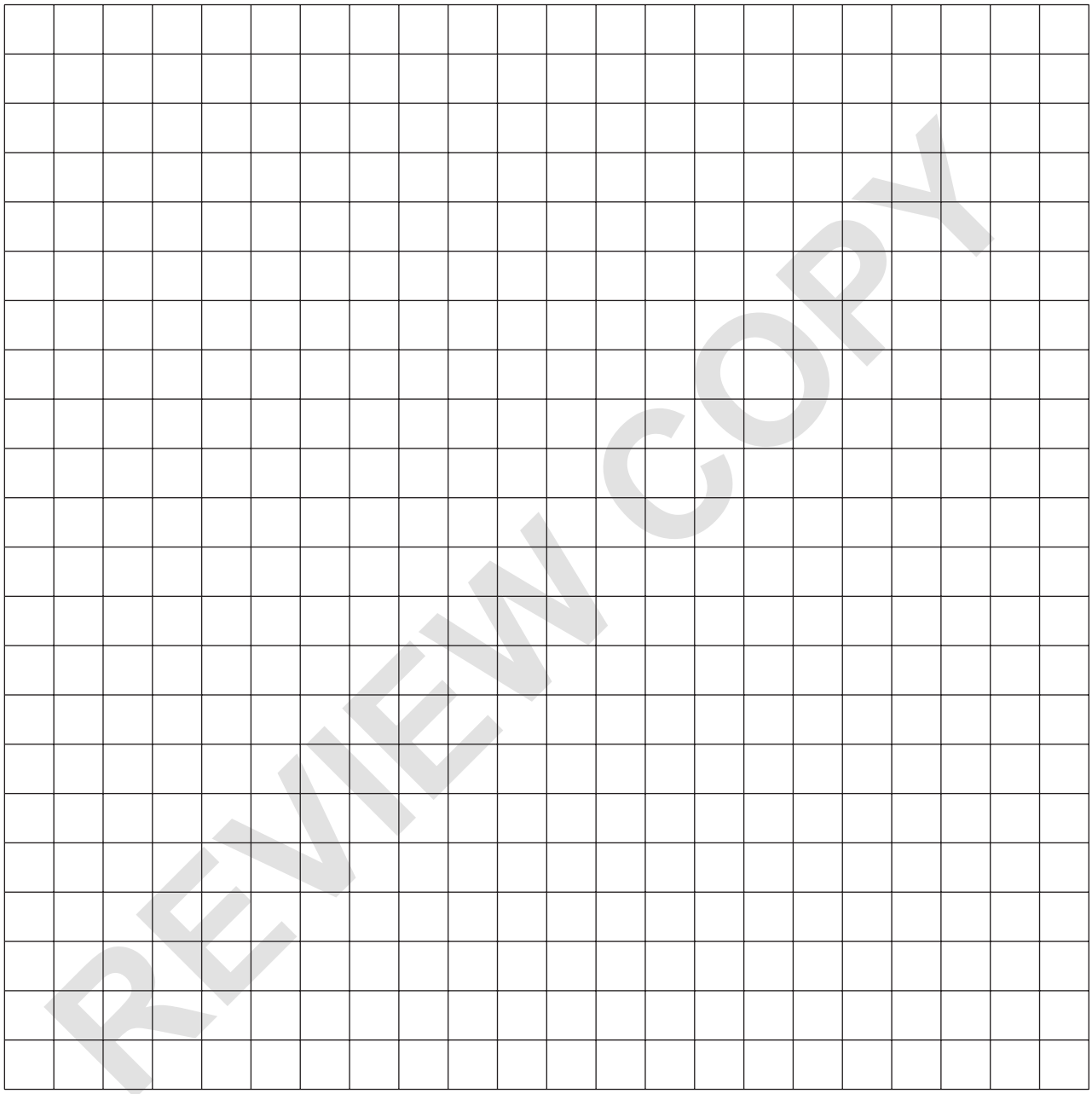
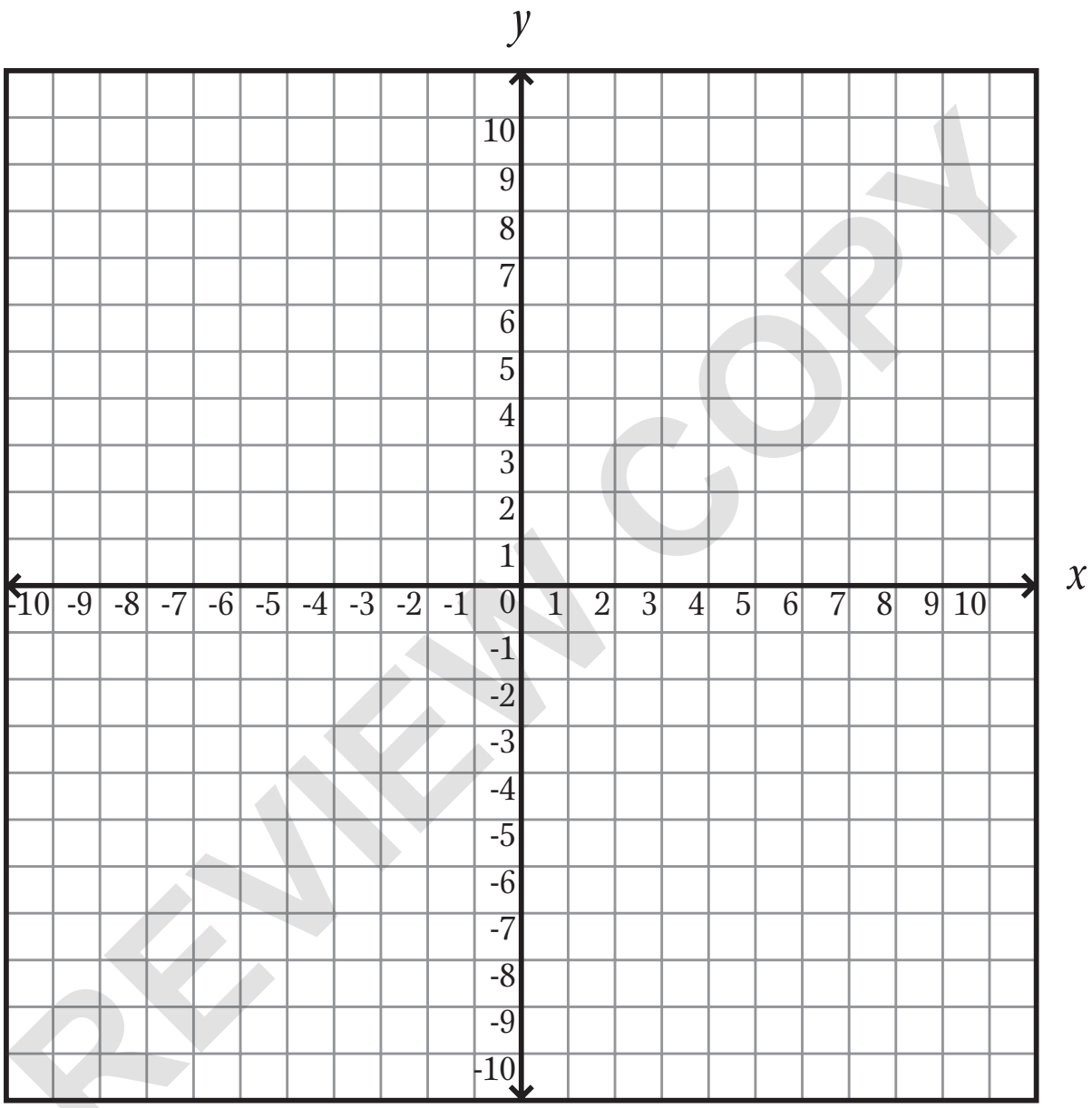
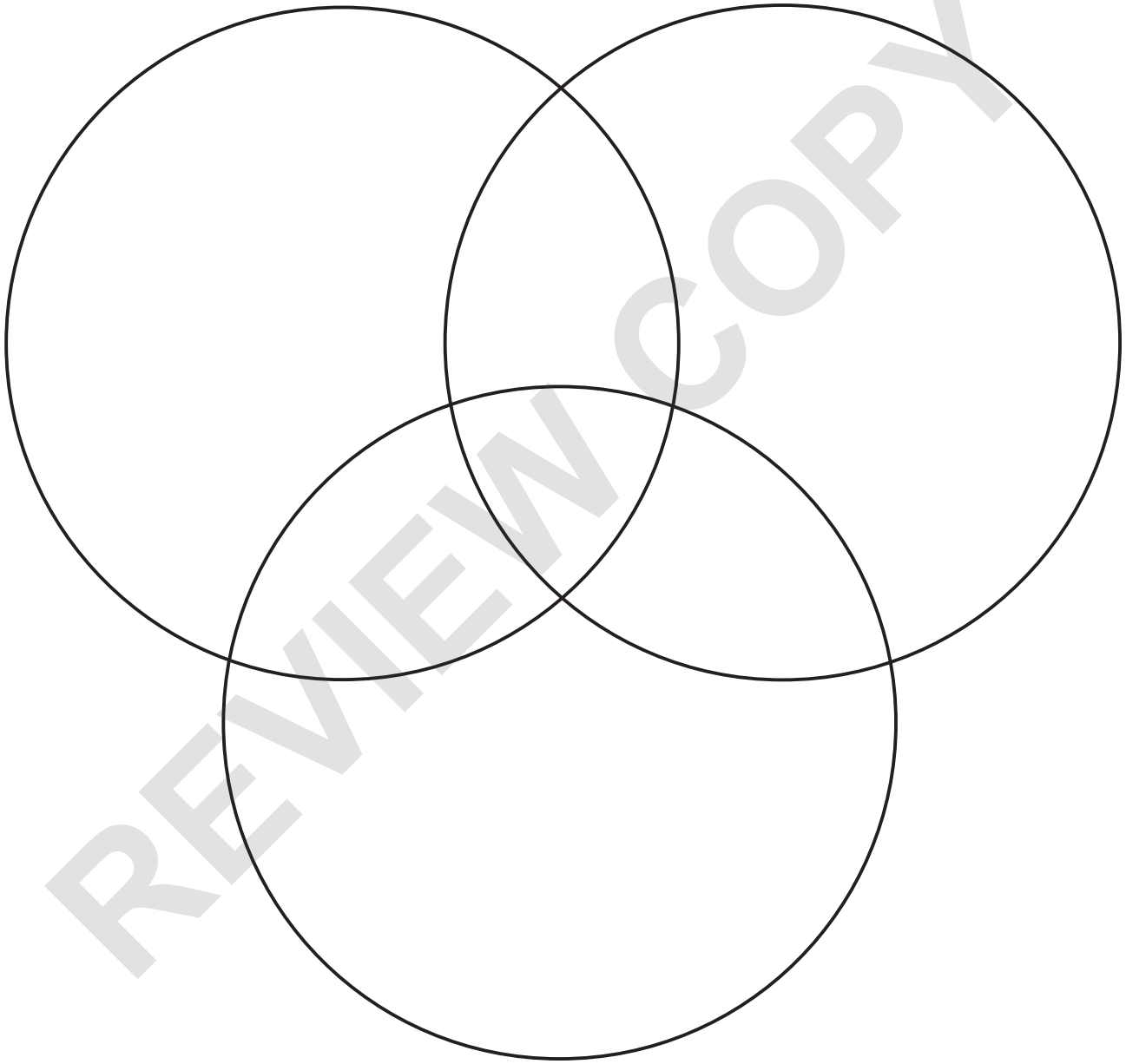
Geometric Drawing



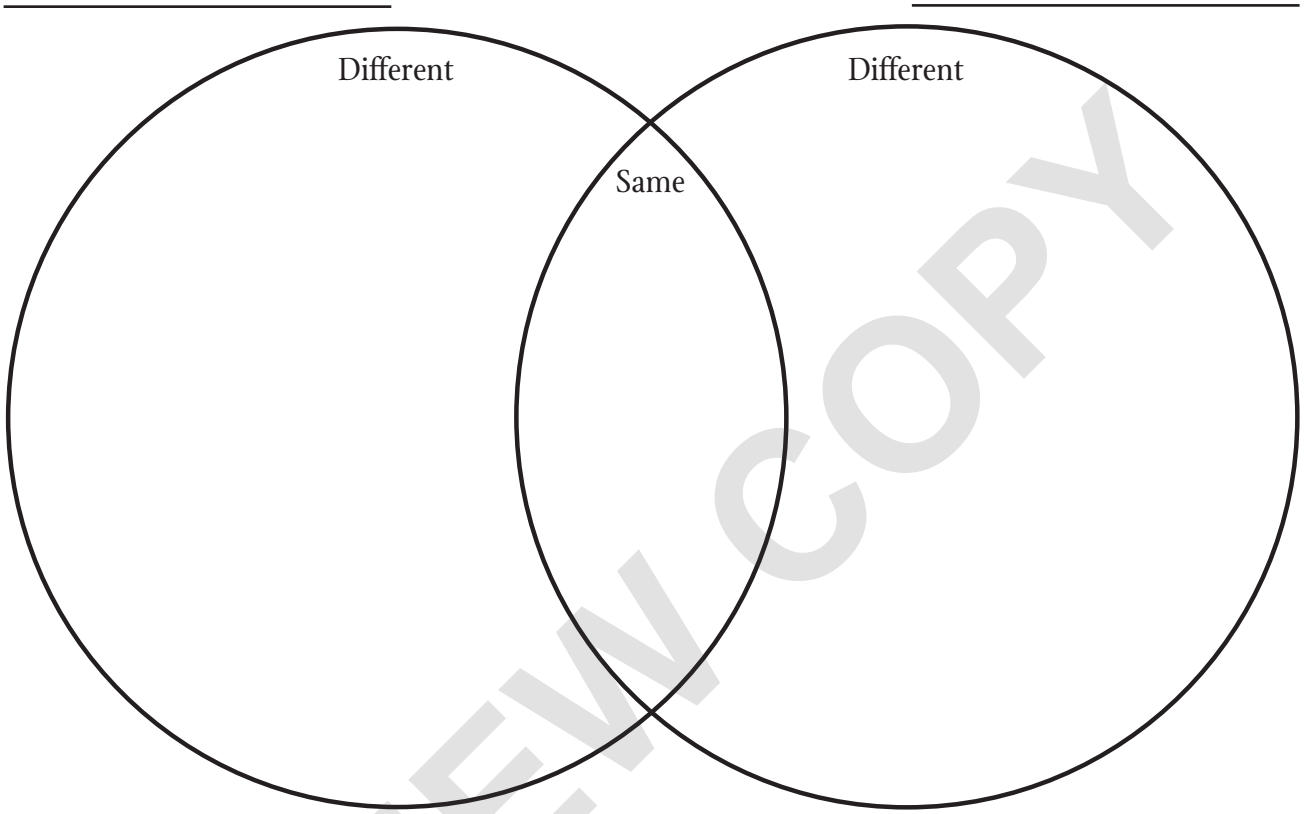
Coordinate Plane



Venn Diagram

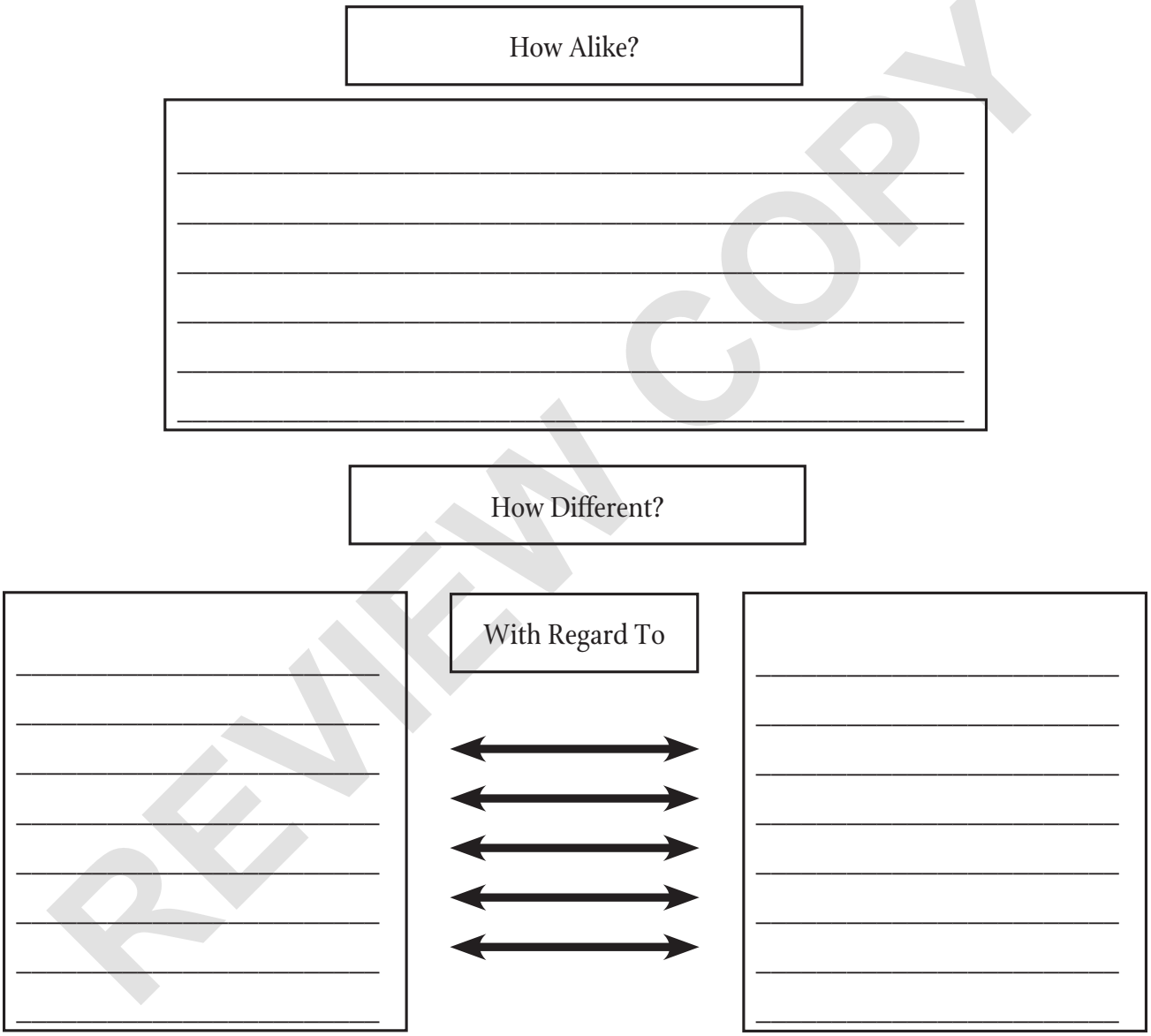


Venn Diagram

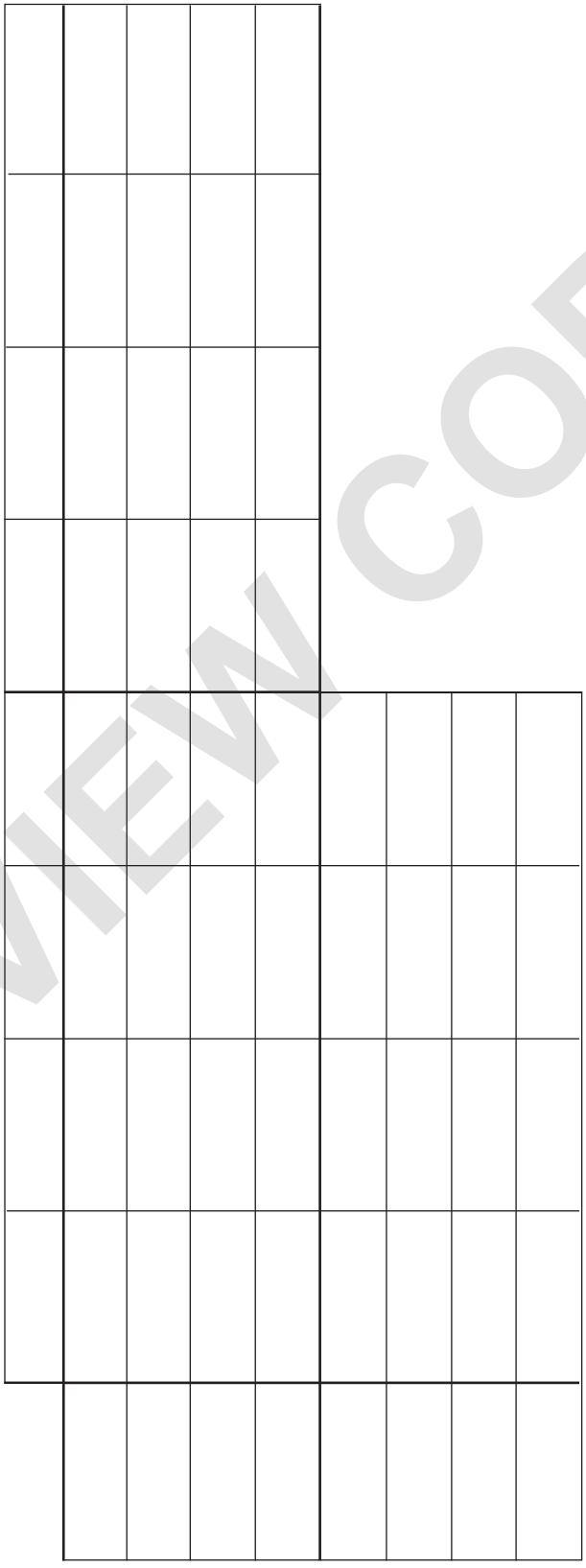


Compare-and-Contrast Diagram

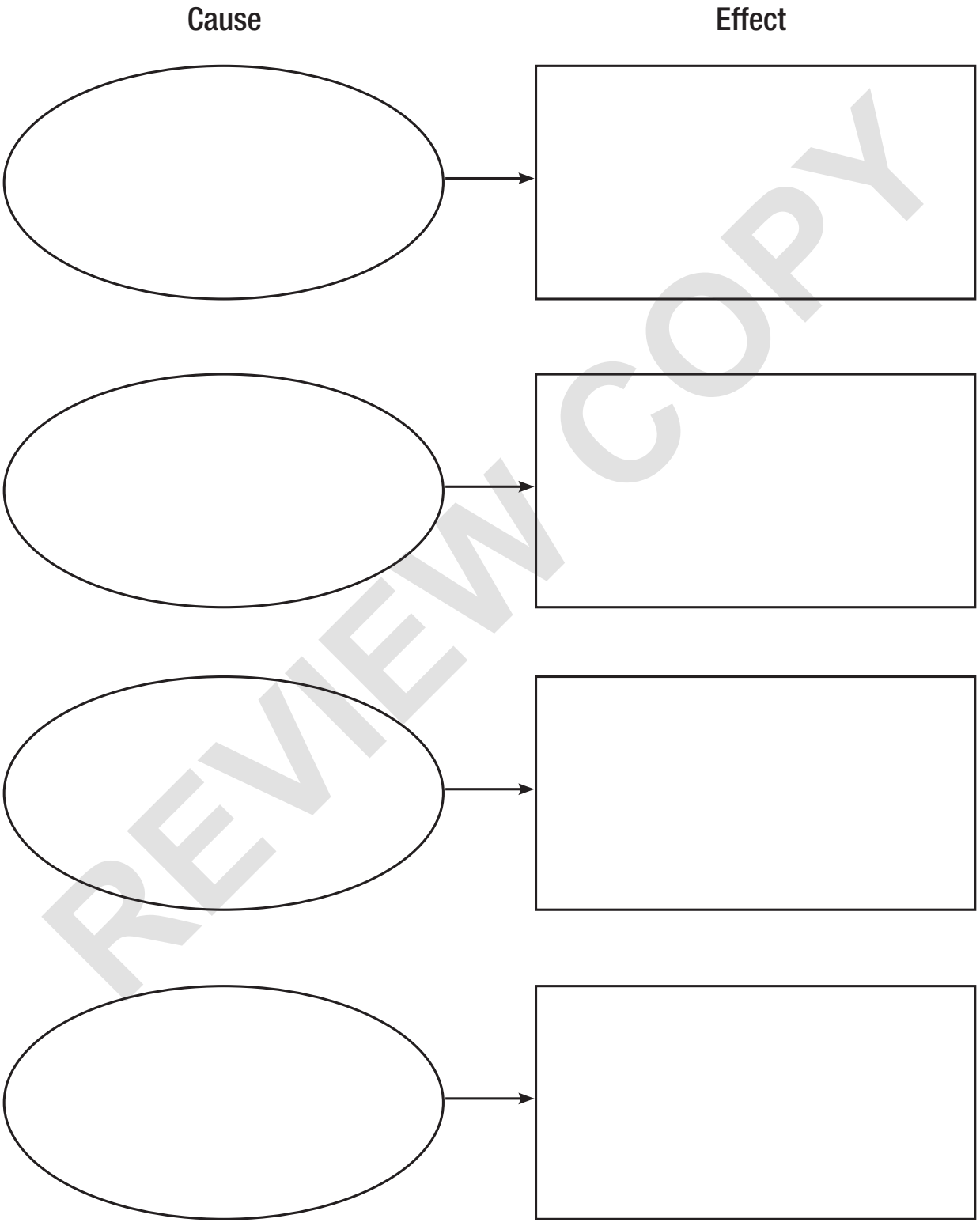
Item 1 _____ Item 2 _____



Attribute Table



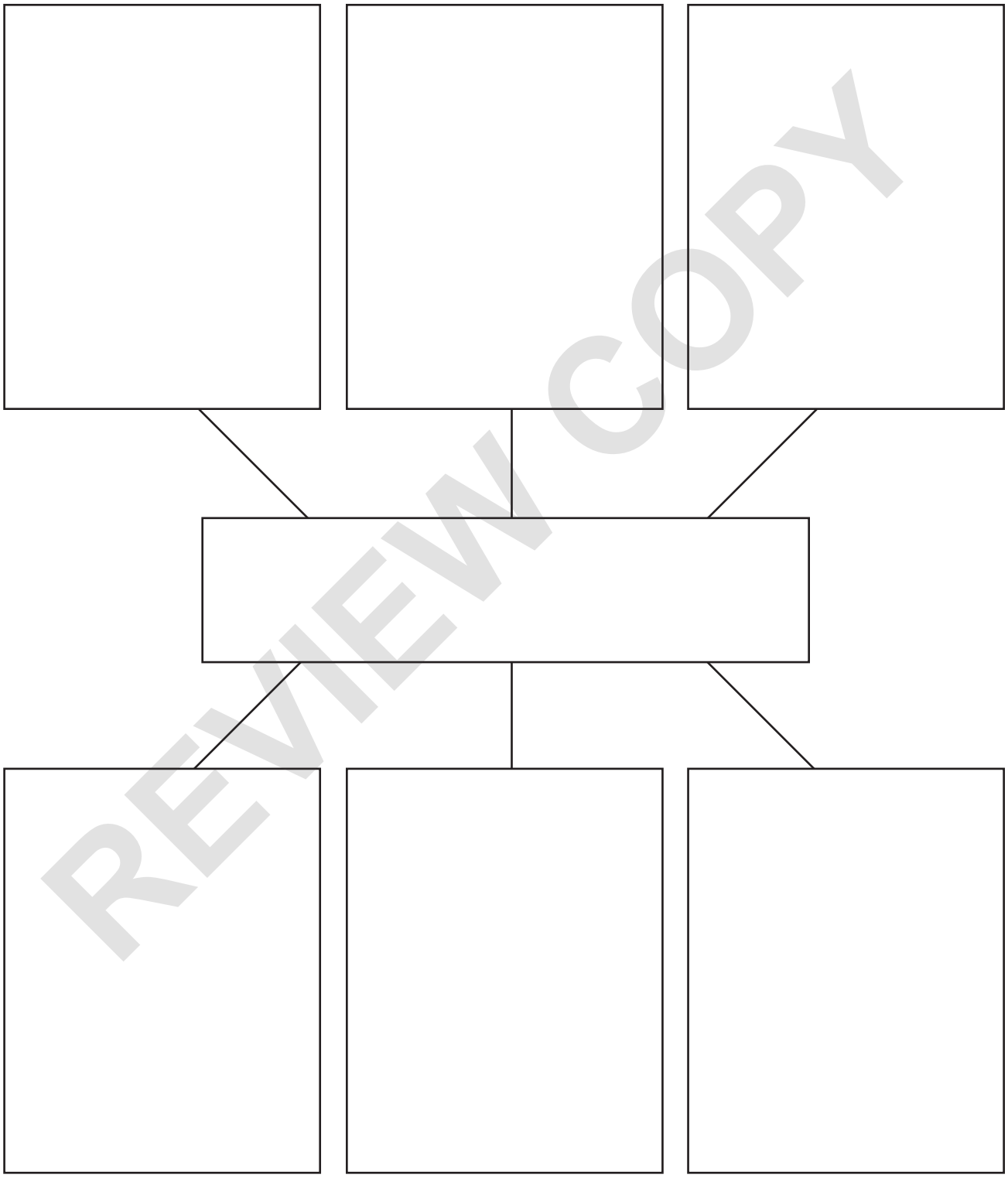
Cause and Effect Map



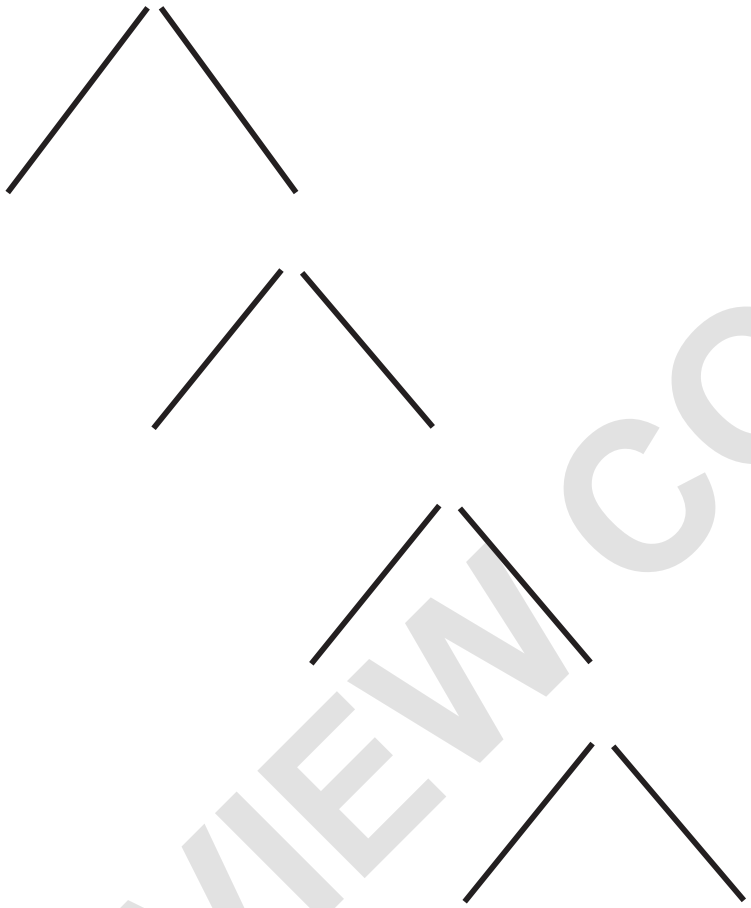
Frayer Model

Definition	Characteristics
WORD	
Examples from Life	Non-Examples

Semantic Map/Concept Map

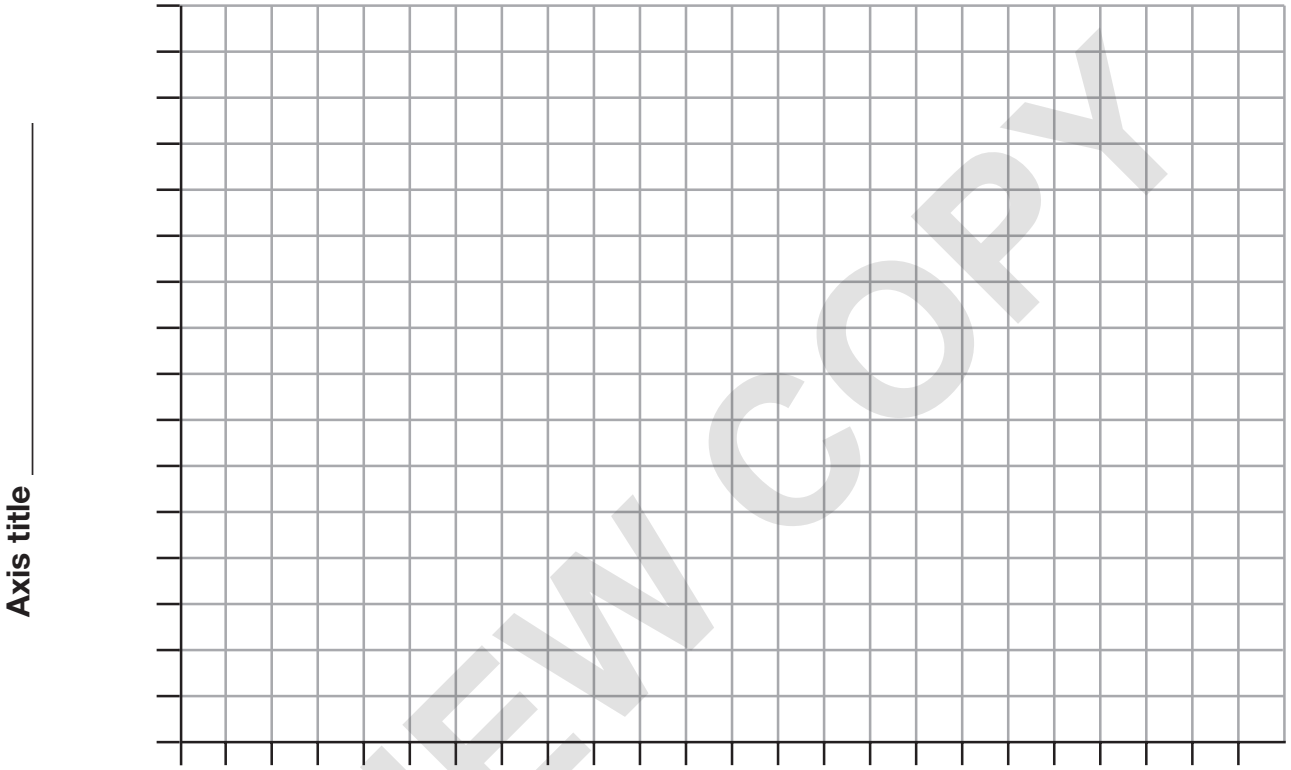


Factor Tree



Line Graph

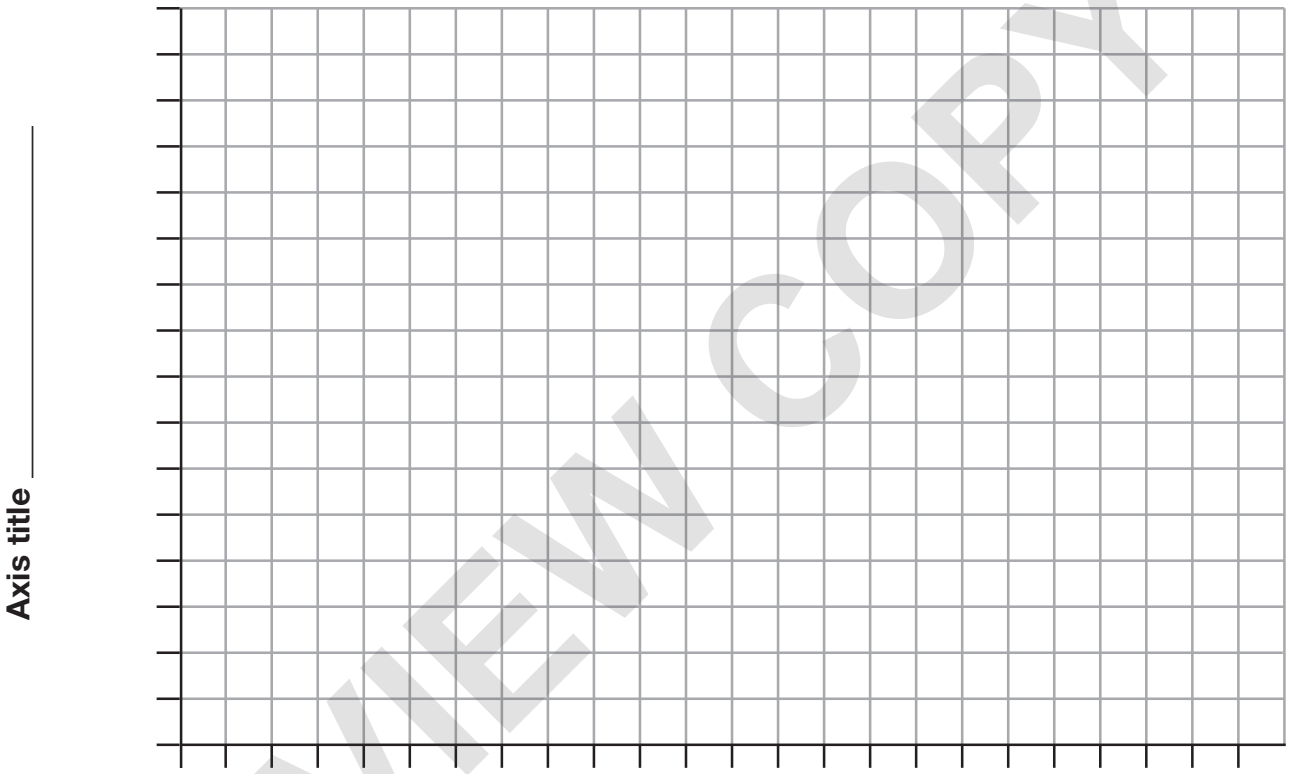
Graph title _____



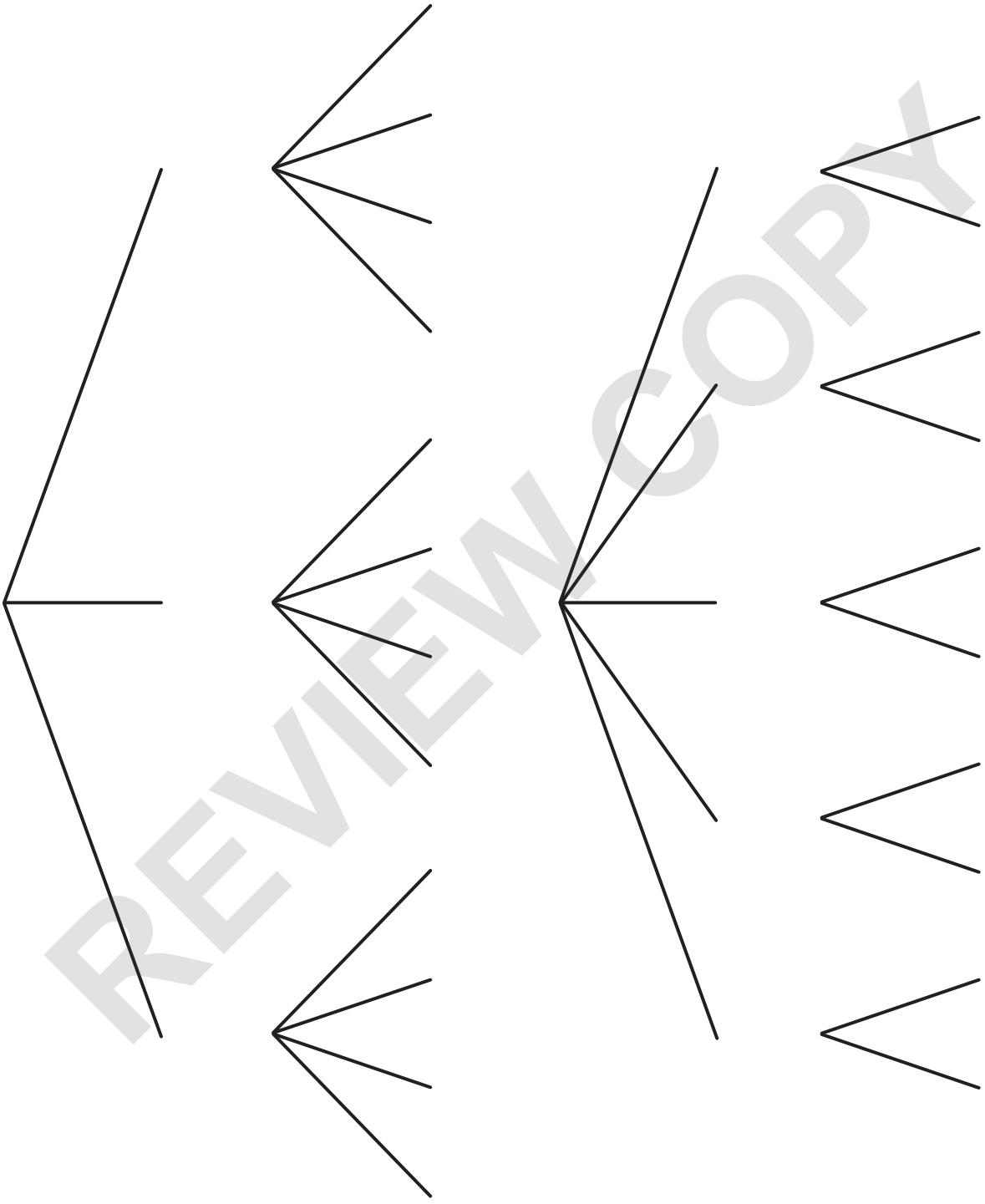
Axis title _____

Bar Chart/Histogram

Graph title _____



Probability Trees



ALGEBRA

Functions	
$f(x)$	Function notation, "f of x"
$f^{-1}(x)$	Inverse function notation
$f(x) = mx + b$	Linear function
$f(x) = b^x + k$	Exponential function
$(f+g)(x) = f(x) + g(x)$	Addition
$(f-g)(x) = f(x) - g(x)$	Subtraction
$(f \bullet g)(x) = f(x) \bullet g(x)$	Multiplication
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	Division
$\frac{f(b)-f(a)}{b-a}$	Average rate of change
$f(-x) = -f(x)$	Odd function
$f(-x) = f(x)$	Even function
$f(x) = \lfloor x \rfloor$	Floor/greatest integer function
$f(x) = \lceil x \rceil$	Ceiling/least integer function
$f(x) = a\sqrt[3]{(x-h)} + k$	Cube root function
$f(x) = a\sqrt[n]{(x-h)} + k$	Radical function
$f(x) = a\lvert x-h \rvert + k$	Absolute value function
$f(x) = \frac{p(x)}{q(x)};\ q(x) \neq 0$	Rational function

Symbols	
$\approx$	Approximately equal to
$\neq$	Is not equal to
$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to
$\lvert a \rvert$	Absolute value of a
$\sqrt{a}$	Square root of a
∞	Infinity
[	Inclusive on the lower bound
]	Inclusive on the upper bound
(	Non-inclusive on the lower bound
)	Non-inclusive on the upper bound

Linear Equations	
$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$y = mx + b$	Slope-intercept form
$ax + by = c$	Standard form
$y - y_1 = m(x - x_1)$	Point-slope form

Quadratic Functions and Equations	
$x=\frac{-b}{2a}$	Axis of symmetry
$x=\frac{p+q}{2}$	Axis of symmetry using the midpoint of the x-intercepts
$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$	Vertex
$f(x) = ax^2 + bx + c$	Standard form
$f(x) = a(x - h)^2 + k$	Vertex form, with the vertex given as (h, k)
$f(x) = a(x - p)(x - q)$	Factored/intercept form
$b^2 - 4ac$	Discriminant
$x^2+bx+\left(\frac{b}{2}\right)^2$	Perfect square trinomial
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	Quadratic formula
$(ax)^2-b^2=(ax+b)(ax-b)$	Difference of squares
$(x - h)^2 = 4p(y - k)$	Standard form for a parabola that opens up or down
$(y - k)^2 = 4p(x - h)$	Standard form for a parabola that opens right or left
$F(h, k + p)$	Focus for a parabola that opens up or down
$F(h + p, k)$	Focus for a parabola that opens right or left
$y = k - p$	Directrix for a parabola that opens up or down
$x = h - p$	Directrix for a parabola that opens right or left

Sequences	
$a_n = a_1 + (n - 1)d$	Explicit formula for the nth term of an arithmetic sequence
$a_n = a_1 \bullet r^{n-1}$	Explicit formula for the nth term of a geometric sequence
$a_n = a_{n-1} + d$	Recursive formula for the nth term of an arithmetic sequence
$a_n = a_{n-1} \bullet r$	Recursive formula for the nth term of a geometric sequence

Exponential Functions	
$1 + r$	Growth factor
$1 - r$	Decay factor
$f(t) = a(1+r)^t$	Exponential growth function
$f(t) = a(1-r)^t$	Exponential decay function
$f(x) = ab^x$	Exponential function in general form

General	
(x, y)	Ordered pair
$(x, 0)$	x-intercept
$(0, y)$	y-intercept

Equations of Circles	
$(x-h)^2 + (y-k)^2 = r^2$	Standard form
$x^2 + y^2 = r^2$	Center at (0, 0)
$Ax^2 + By^2 + Cx + Dy + E = 0$	General form

Properties of Exponents	
Property	General rule
Zero Exponent	$a^0 = 1$
Negative Exponent	$b^{-\frac{m}{n}} = \frac{1}{b^{\frac{m}{n}}}$
Product of Powers	$a^m \bullet a^n = a^{m+n}$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power	$\left(b^m\right)^n = b^{mn}$
Power of a Product	$(bc)^n = b^n c^n$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Imaginary Numbers	
$i = \sqrt{-1}$	$i^3 = -i$
$i^2 = -1$	$i^4 = 1$

Multiplication of Complex Conjugates
$(a + bi)(a - bi) = a^2 + b^2$

Properties of Radicals	
Addition property	$a\sqrt{m} + b\sqrt{m} = (a+b)\sqrt{m}$
Product property	$\sqrt{m \bullet n} = \sqrt{m} \bullet \sqrt{n}$
Quotient property	$\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}, n \neq 0$
Power reduction property	$\sqrt{n^{2a}} = n^a$
Rational denominator property	$\frac{m}{\sqrt{n}} = \frac{m}{\sqrt{n}} \bullet \frac{\sqrt{n}}{\sqrt{n}} = \frac{m\sqrt{n}}{n}$

Radicals to Rational Exponents
$\sqrt[n]{a} = a^{\frac{1}{n}}$
$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Exponential Equations	
$A = P\left(1 + \frac{r}{n}\right)^{nt}$	Compounded interest formula
Compounded...	n (number of times per year)
Yearly/annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

DATA ANALYSIS

Rules and Equations	
$P(E)=\frac{\#\text{ of outcomes in } E}{\#\text{ of outcomes in sample space}}$	Probability of event E
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$	Addition rule
$P(\overline{A})=1-P(A)$	Complement rule
$P\left(B\middle\vert A\right)=\frac{P(A\cap B)}{P(A)}$	Conditional probability
$P(A\cap B)=P(A)\bullet P\left(B\middle\vert A\right)$	Multiplication rule
$P(A\cap B)=P(A)\bullet P(B)$	Multiplication rule if A and B are independent
${}_nC_r=\frac{n!}{(n-r)!r!}$	Combination
${}_nP_r=\frac{n!}{(n-r)!}$	Permutation
$n!=n\bullet(n-1)\bullet(n-2)\bullet\cdots\bullet 1$	Factorial

Symbols	
$\varnothing$	Empty/null set
$\cap$	Intersection, “and”
$\cup$	Union, “or”
$\subset$	Subset
$\overline{A}$	Complement of Set A
!	Factorial
${}_nC_r$	Combination
${}_nP_r$	Permutation

GEOMETRY

Symbols	
$\overset{\frown}{ABC}$	Major arc length
$\overset{\frown}{AB}$	Minor arc length
$\angle$	Angle
$\odot$	Circle
$\cong$	Congruent
$\overleftrightarrow{PQ}$	Line
$\overline{PQ}$	Line segment
$\overrightarrow{PQ}$	Ray
$\parallel$	Parallel
$\perp$	Perpendicular
$\bullet$	Point
$\triangle$	Triangle
$\square$	Parallelogram
A'	Prime
$^\circ$	Degrees
θ	Theta
ϕ	Phi
π	Pi

Area	
$A = lw$	Rectangle
$A = \frac{1}{2}bh$	Triangle
$A = \pi r^2$	Circle
$A = \frac{1}{2}(b_1 + b_2)h$	Trapezoid

Trigonometric Ratios		
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$	$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$	$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

Trigonometric Identities
$\sin\theta = \cos(90° - \theta)$
$\cos\theta = \sin(90° - \theta)$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\csc\theta = \frac{1}{\sin\theta}$
$\sec\theta = \frac{1}{\cos\theta}$
$\cot\theta = \frac{1}{\tan\theta}$
$\cot\theta = \frac{\cos\theta}{\sin\theta}$
$\sin^2\theta + \cos^2\theta = 1$

Pythagorean Theorem
$a^2 + b^2 = c^2$

Volume	
$V = lwh$	Rectangular prism
$V = Bh$	Prism
$V = \frac{1}{3}\pi r^2 h$	Cone
$V = \frac{1}{3}Bh$	Pyramid
$V = \pi r^2 h$	Cylinder
$V = \frac{4}{3}\pi r^3$	Sphere

Distance Formula
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Dilation
$D_k(x, y) = (kx, ky)$

Pi Defined
$\pi = \frac{\text{circumference}}{\text{diameter}} = \frac{\text{circumference}}{2 \bullet \text{radius}}$

Perimeter of a Quadrilateral
$2l + 2w$

Perimeter of a Triangle
$P = a + b + c$

Circumference of a Circle	
$C = 2\pi r$	Circumference given the radius
$C = \pi d$	Circumference given the diameter

Converting Between Degrees and Radians
$\frac{\text{radian measure}}{\pi} = \frac{\text{degree measure}}{180}$

Midpoint Formula
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Inverse Trigonometric Functions
Arcsin $\theta = \sin^{-1}\theta$
Arccos $\theta = \cos^{-1}\theta$
Arctan $\theta = \tan^{-1}\theta$

Arc Length	
$s = \theta r$	Arc length (θ in radians)

MEASUREMENTS

Length
Metric
1 kilometer (km) = 1000 meters (m)
1 meter (m) = 100 centimeters (cm)
1 centimeter (cm) = 10 millimeters (mm)
Customary
1 mile (mi) = 1760 yards (yd)
1 mile (mi) = 5280 feet (ft)
1 yard (yd) = 3 feet (ft)
1 foot (ft) = 12 inches (in)

Volume and Capacity
Metric
1 liter (L) = 1000 milliliters (mL)
Customary
1 gallon (gal) = 4 quarts (qt)
1 quart (qt) = 2 pints (pt)
1 pint (pt) = 2 cups (c)
1 cup (c) = 8 fluid ounces (fl oz)

Weight and Mass
Metric
1 kilogram (kg) = 1000 grams (g)
1 gram (g) = 1000 milligrams (mg)
1 metric ton (MT) = 1000 kilograms
Customary
1 ton (T) = 2000 pounds (lb)
1 pound (lb) = 16 ounces (oz)

Glossary

English	Español
#	
68–95–99.7 rule a rule that states percentages of data under the normal curve are as follows: $\mu \pm 1\sigma \approx 68\%$, $\mu \pm 2\sigma \approx 95\%$, and $\mu \pm 3\sigma \approx 99.7\%$; also known as the *Empirical Rule*	**regla 68–95–99,7** regla que establece los siguientes porcentajes de datos bajo la curva normal: $\mu \pm 1\sigma \approx 68\%$, $\mu \pm 2\sigma \approx 95\%$ y $\mu \pm 3\sigma \approx 99,7\%$; también se la conoce como *Regla Empírica*
A	
absolute value equation an equation of the form $y = \lvert ax + b \rvert + c$, where x is the independent variable, y is the dependent variable, and a, b, and c are real numbers	**ecuación de valor absoluto** ecuación de la forma $y = \lvert ax + b \rvert + c$, donde x es la variable independiente, y es la variable dependiente y a, b y c son números reales
absolute value function a function of the form $f(x) = \lvert ax + b \rvert + c$, where x is the independent variable and a, b, and c are real numbers	**función de valor absoluto** función de la forma $f(x) = \lvert ax + b \rvert + c$, donde x es la variable independiente a, b y c son números reales
accuracy closeness of a measurement to the actual value of the dimension being measured. For example, a measurement of 1.99999 cm for an object that is 2 cm wide has a high level of accuracy.	**exactitud** la proximidad de una medida al valor real de la dimensión que se está midiendo. Por ejemplo, una medida de 1.99999 cm para un objeto de 2 cm de ancho tiene un alto nivel de precisión.
Addition Rule If A and B are any two events, then the probability of A or B, denoted $P(A \text{ or } B)$, is given by: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. Using set notation, the rule is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.	**Regla de la suma** Si A y B son dos eventos cualquiera, entonces la probabilidad de A o B, que se indicacon P (A o B), está dada por: $P(A \text{ o } B) = P(A) + P(B) - P(A \text{ y } B)$. Con el uso de notación de conjuntos, la regla es $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
addition rule for mutually exclusive events If events A and B are mutually exclusive, then the probability that A or B will occur is the sum of the probability of each event; $P(A \text{ or } B) = P(A) + P(B)$.	**regla de adición para eventos mutuamente excluyentes** si los eventos A y B son mutuamente excluyentes, la probabilidad de que A o B suceda es la suma de la probabilidad de cada evento; $P(A \text{ or } B) = P(A) + P(B)$.

Glossary

English

additive identity the element of a set whose addition does not change other elements; the additive identity of the real numbers is 0

additive identity matrix a matrix for which all entries are 0; the matrix whose addition does not change other matrices; also known as the *zero matrix*

additive inverse matrix the matrix which, when added to matrix A, yields the zero matrix; also called the *opposite matrix*. The additive inverse of A is $-A$.

algorithm the steps or rules to be followed in making calculations or other problem solving operations

alternative hypothesis any hypothesis that differs from the null hypothesis; that is, a statement that indicates there is a difference in the data from two treatments; represented by H_a

amplitude the coefficient a or c of the sine or cosine term in a function of the form $f(x) = a \sin bx$ or $g(x) = c \cos dx$; on a graph of the cosine or sine function, the vertical distance from the y-coordinate of the maximum point on the graph to the midline of the cosine or sine curve

annual percentage rate (APR) the yearly interest rate for a loan

Español

identidad de la suma es el elemento de un conjunto cuya sumatoria no cambia a los demás elementos; la identidad de la suma en los números reales es el 0

matriz identidad de la suma matriz cuyas entradas son todas 0; es la matriz cuya sumatoria no cambia a las demás matrices; también se la conoce como la *matriz cero*

matriz inversa aditiva es la matriz que, cuando se la suma a una matriz A, el resultado es la matriz cero; también se la llama *matriz opuesta*. La inversa aditiva de A es $-A$.

algoritmo los pasos o reglas que se deben seguir para realizar cálculos u otras operaciones de resolución de problemas

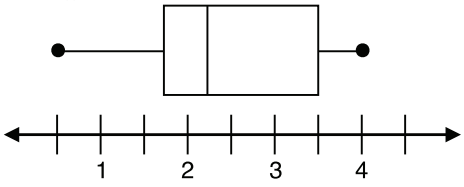
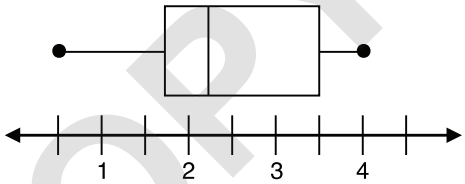
hipótesis alternativa toda hipótesis que difiera de la hipótesis nula; es decir, una afirmación que indica que existe una diferencia en los datos de dos tratamientos; representada por H_a

amplitud el coeficiente a o c del término de seno o coseno en una función de la forma $f(x) = a \sin bx$ o $g(x) = c \cos dx$; en un gráfico de la función seno o coseno, la distancia vertical desde la coordenada y del punto máximo en la gráfica hasta la línea media de la curva de seno o coseno

tasa efectiva annual la tasa de interés anual de un préstamo

English	Español
argument the result of raising the base of a logarithm to the power of the logarithm, so that b is the argument of the logarithm $\log_a b = c$; the term $cx + d$ in a cosine or sine function of the form $f(x) = a + b \sin(cx + d)$ or $g(x) = a + b \cos(cx + d)$	**argumento** el resultado de elevar la base de un logaritmo a la potencia del logaritmo, de manera que b es el argumento del logaritmo $\log_a b = c$; el término $cx + d$ en una función coseno o seno de la forma $f(x) = a + b \sin(cx + d)$ o $g(x) = a + b \cos(cx + d)$
arithmetic sequence a linear function with a domain of positive consecutive integers in which the difference between any two consecutive terms is equal	**secuencia aritmética** función lineal con dominio de enteros consecutivos positivos, en la que la diferencia entre dos términos consecutivos es equivalente
Associative Property of Multiplication When quantities are multiplied, the way they are grouped does not affect the product. For example, $a \bullet (b \bullet c) = (a \bullet b) \bullet c$.	**propiedad asociativa de la multiplicación** Cuando se multiplican cantidades, la manera en que se agrupan no afecta el producto. Por ejemplo, $a \bullet (b \bullet c) = (a \bullet b) \bullet c$.
ATM (automated teller machine) **a** device that allows people to access funds in their accounts without having to go into the bank	**cajero automático** un dispositivo que permite a las personas acceder a los fondos de sus cuentas sin tener que entrar al banco
augmented matrix a matrix in which each row represents an equation of a linear system and the coefficients are separated from the constants by a vertical line	**matriz aumentada** matriz en la que cada fila representa una ecuación de un sistema lineal y los coeficientes se separan de las constantes por una línea vertical
B	
bank statement a summary of the activity in your account for a period of time, usually a month	**extracto de cuenta** un resumen de la actividad en su cuenta durante un período de tiempo, generalmente un mes
base the factor being multiplied together in an exponential expression; in the expression a^b, a is the base	**base** factor que se multiplica en forma conjunta en una expresión exponencial; en la expresión a^b, a es la base
bias leaning toward one result over another; having a lack of neutrality	**sesgo** inclinación por un resultado sobre otro; carecer de neutralidad
biased sample a sample in which some members of the population have a better chance of inclusion in the sample than others	**muestra sesgada** muestra en la cual algunos miembros de la población tienen una mayor posibilidad de ser incluidos en la muestra que otros

English	Español
binomial coefficient the number of combinations of r items that can be chosen from a set of n items, notated as ${}_nC_r$	**coeficiente binomial** cantidad de combinaciones de r elementos que se pueden elegir de un conjunto de n elementos; se nota ${}_nC_r$
binomial distribution a statistical distribution that gives the probability of obtaining a specified number of successes and failures in a repeated event. The probability of success is the same for each trial.	**distribución binomial** distribución estadística que da la probabilidad de obtener una cantidad especificada de éxitos y fracasos cuando se repite un evento. La probabilidad de éxito es la misma en cada intento.
binomial experiment an experiment in which there are a fixed number of trials, each trial is independent of the others, there are only two possible outcomes (success or failure), and the probability of each outcome is constant from trial to trial	**experimento binomial** experimento en el que existe un número fijo de pruebas, cada prueba es independiente de las demás, existen dos resultados posibles (éxito o fracaso) y la probabilidad de cada resultado es constante de prueba a prueba
binomial probability distribution formula the distribution of the probability, P, of exactly x successes out of n trials, if the probability of success is p and the probability of failure is q; given by the formula $P = \binom{n}{x} p^x q^{n-x}$	**fórmula de distribución binomial de probabilidad** la distribución de la probabilidad, P, de exactamente x éxitos entre n pruebas, si la probabilidad de éxito es p y la probabilidad de fracaso es q; dada por la fórmula $P = \binom{n}{x} p^x q^{n-x}$
bounced check A check is said to "bounce" if the funds in a checking account are insufficient to cover a written check.	**cheque sin fondos** Se dice que un cheque "rebota" si los fondos en un recuento de cheques son insuficientes para cubrir un cheque escrito.

English	Español
box plot a plot showing the minimum, maximum, first quartile, median, and third quartile of a data set; the middle 50% of the data is indicated by a box. Example: 	**diagrama de caja** diagrama que muestra el mínimo, máximo, primer cuartil, mediana y tercer cuartil de un conjunto de datos; se indica con una caja el 50% medio de los datos. Ejemplo: 
brute force algorithm an algorithm that explores all possible solutions to a problem and selects the optimal solution; also known as an *exhaustive algorithm*	**algoritmo de fuerza bruta** un algoritmo que explora todas las posibles soluciones a un problema y selecciona la solución óptima; también conocido como *algoritmo exhaustivo*
budget 1. a set amount of money that is to be used for a certain purpose 2. a financial plan for income and expenses over a set period of time	**presupuesto** 1. una cantidad fija de dinero que se utilizará para un propósito determinado 2. un plan financiero para ingresos y gastos durante un período de tiempo determinado
C	
candlestick chart financial chart used to describe price movements of a security	**gráfico de velas** gráfico financiero utilizado para describir los movimientos de precios de un valor
capital wealth in the form of money or other assets owned by a person or organization	**capital** riqueza en forma de dinero u otros activos propiedad de una persona u organización
causation a relationship between two events where a change in one event is responsible for a change in the second event	**causalidad** relación entre dos eventos en la que un cambio en un evento es responsable por un cambio en el segundo evento

English	Español
chance variation a measure showing how precisely a sample reflects the population, with smaller sampling errors resulting from large samples and/or when the data clusters closely around the mean; also called *sampling error*	**variación aleatoria** medida que muestra cómo una muestra refleja con precisión la población, con errores de muestreo más pequeños que resultan de muestras grandes y/o cuando los datos se agrupan estrechamente alrededor de la media; también llamada *error de muestreo*
checking account an account at a bank from which checks can be drawn by the account depositor	**cuenta corriente** una cuenta en un banco sobre la cual el depositante de la cuenta puede girar cheques
closing price the price per share of a stock at the end of a day	**precio de cierre** el precio por acción de una acción al final de un día
cluster sample a sample in which naturally occurring groups of population members are chosen for a sample	**muestreo en grupos** muestra en la cual se eligen para una muestra grupos naturalmente ya formados de miembros de la población
column an arrangement of data in a vertical line	**columna** arreglo de datos en línea vertical
column vector the representation of a vector as a 2×1 matrix, which has a single column. The x-component of the vector $\vec{v} = \langle a,b \rangle$ appears in the first row of the column vector $\begin{bmatrix} a \\ b \end{bmatrix}$, and the y-component of the vector $\vec{v} = \langle a,b \rangle$ appears in the second row of the column vector $\begin{bmatrix} a \\ b \end{bmatrix}$.	**vector columna** es la representación de un vector como matriz de 2×1, la cual tiene una sola columna. El componente x del vector $\vec{v} = \langle a,b \rangle$ aparece en la primera fila del vector columna $\begin{bmatrix} a \\ b \end{bmatrix}$, y el componente y del vector $\vec{v} = \langle a,b \rangle$ aparece en la segunda fila del vector columna $\begin{bmatrix} a \\ b \end{bmatrix}$.

English

combination a subset of a group of objects taken from a larger group of objects; the order of the objects does not matter. A combination of size r from a group of n objects can be represented using the notation ${}_nC_r$, where ${}_nC_r = \frac{n!}{(n-r)!r!}$.

commission
1. a payment to a sales agent based on the sale of a good or service

2. a fee charged by a broker when both buying and selling stock

common difference the number added to each consecutive term in an arithmetic sequence

common logarithm a base-10 logarithm which is usually written without the number 10, such as $\log x = \log_{10} x$

common ratio the number that each consecutive term is multiplied by in a geometric sequence

commutative An operation on two objects is commutative if changing the order of the objects does not change the result.

Commutative Property of Multiplication The order in which quantities are multiplied does not affect the product. For example, $a \bullet b = b \bullet a$.

Español

combinación subconjunto de un grupo de objetos tomado de un grupo más grande de objetos; el orden de los objetos no importa. Se puede representar una combinación de tamaño r de un grupo de n objetos con la notación ${}_nC_r$, donde ${}_nC_r = \frac{n!}{(n-r)!r!}$.

commission
1. un pago a un agente de ventas basado en la venta de un bien o servicio

2. un arancel cobrada por un corredor al comprar y vender acciones

diferencia común número sumado a cada término consecutivo en una secuencia aritmética

logaritmo común logaritmo de base 10 que se escribe normalmente sin el número 10, como $\log x = \log_{10} x$

proporción constante el número por el que cada término está multiplicado en una secuencia geométrica

conmutativa Una operación sobre dos objetos es conmutativa si al cambiar el orden de los objetos no se modifica el resultado.

propiedad conmutativa de la multiplicación el orden en el que se multiplican las cantidades no afecta el producto. Por ejemplo, $a \bullet b = b \bullet a$.

English

complement a set whose elements are not in another set, but are in some universal set being considered. The complement of set A, denoted by $\overline{A}$, is the set of elements that are in the universal set, but not in A. The event does not occur. The probability of an event not occurring is 1 minus the probability of the event occurring, $P\left(\overline{A}\right)=1-P(A)$.

components of a vector for a given vector $\vec{v}=\langle a,b\rangle$, the x-component is a and the y-component is b

component-wise addition addition of vectors performed such that the x-components are added to obtain the new x-component, and the y-components are added to obtain the new y-component

component-wise scalar multiplication multiplication of vectors performed such that the x-component and y-component of a vector are each multiplied by the scalar to obtain the new x-component and new y-component

compound event the combination of two or more simple events

compound interest interest earned on both the initial amount and on previously earned interest

compound probability the probability of compound events

Español

complemento conjunto cuyos elementos no se encuentran en otro conjunto, pero están en algún conjunto universal que se considera. El complemento del conjunto A, que se indica con $\overline{A}$, es el conjunto de elementos que se encuentran en el conjunto universal, pero no en A. El evento no se produce. La probabilidad de que un evento no se produzca es 1 menos la probabilidad de que se produzca, $P\left(\overline{A}\right)=1-P(A)$.

componentes de un vector dado un vector $\vec{v}=\langle a,b\rangle$, el componente x es a y el componente y es b

suma componente a componente suma de vectores realizada de manera tal que se suman los componentes x para obtener el nuevo componente x, y se suman los componentes y para obtener el nuevo componente y

multiplicación por un escalar multiplicación de vectores realizada de manera tal que el componente x y el componente y de un vector se multiplican cada uno por el escalar para obtener el nuevo componente x y el nuevo componente y

evento compuesto combinación de dos o más eventos simples

interés compuesto interés devengado tanto de la cantidad inicial como del interés previamente devengado

probabilidad compuesta probabilidad de eventos compuestos

English

conditional probability of *B* given *A* the probability that event *B* occurs, given that event *A* has already occurred. If *A* and *B* are two events from a sample space with $P(A) \neq 0$, then the conditional probability of *B* given *A*, denoted $P(B|A)$, has two equivalent expressions: $P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{\text{number of outcomes in } (A \text{ and } B)}{\text{number of outcomes in } A}$.

conditional relative frequency a joint frequency compared to either the sum of all frequencies or the sum of the frequencies for a category of the given data

conditional statement a logical (true or false) condition on which a decision is made

confidence interval an interval of numbers within which it can be claimed that repeated samples will result in the calculated parameter; generally calculated using the estimate plus or minus the margin of error

confidence level the probability that a parameter's value can be found in a specified interval; also called *level of confidence*

Español

probabilidad condicional de *B* dado *A* la probabilidad de que el evento *B* se produzca, dado que el evento *A* ya se ha producido. Si *A* y *B* son dos eventos de un espacio muestral con $P(A) \neq 0$, entonces la probabilidad condicional de *B* dado *A*, indicado $P(B|A)$ tiene dos expresiones equivalentes: $P(B|A) = \frac{P(A \text{ y } B)}{P(A)} = \frac{\text{numero de resultados en } (A \text{ y } B)}{\text{numero de resultados en } A}$.

frecuencia condicional relativa frecuencia conjunta comparada con la suma de todas las frecuencias o la suma de las frecuencias para una categoría de los datos dados

sentencia condicional una condición lógica (verdadera o falsa) en la que se toma una decisión

intervalo de confianza intervalo de números dentro del cual se puede afirmar que las muestras repetidas tendrán como resultado el parámetro calculado; generalmente se calcula usando la estimación más o menos el margen de error

nivel de confianza probabilidad de que se pueda encontrar el valor de un parámetro en un intervalo específico; también llamado *grado de confianza*

English	Español
confounding variable an ignored or unknown variable that influences the result of an experiment, survey, or study	**variable de confusión** una variable ignorada o desconocida que influye sobre el resultado de un experimento, encuesta o estudio
constraint a restriction or limitation on any of the variables in an equation or inequality	**limitación** una restricción o limitación de cualquiera de las variables en una ecuación o desigualdad
continuous data a set of values for which there is at least one value between any two given values	**datos continuos** conjunto de valores para el que existe al menos un valor entre dos valores dados
continuous decay rate k in the continuous decay model $p(t) = ae^{kt}$; the k-value shows the amount by which the initial value decreases continuously at any given time. The k-value is negative in continuous decay models.	**tasa de decaimiento continuo** k en el modelo de desintegración continua $p(t) = ae^{kt}$; el valor k muestra la cantidad en la que el valor inicial disminuye continuamente en un momento dado. El valor k es negativo en modelos de desintegración continua.
continuous distribution the graphed set of values, a curve, in a continuous data set	**distribución continua** conjunto de valores representado gráficamente, una curva, en un conjunto de datos continuos
continuous exponential decay model an exponential decay model that uses the number e; given by the equation $p(t) = ae^{kt}$. In this model, a is initial value, k is the continuous decay rate in decimal form, and t is time. The k-value will be negative in decay models.	**modelo de decaimiento exponencial continuo** un modelo de desintegración exponencial que usa el número e; dado por la ecuación $p(t) = ae^{kt}$. En este modelo, a es el valor inicial, k es la tasa de disminución continua en forma decimal y t es el tiempo. El valor k será negativo en los modelos de desintegración.
continuous exponential growth model an exponential growth model that uses the number e; given by the equation $p(t) = ae^{kt}$. In this model, a is initial value, k is the continuous growth rate in decimal form, and t is time. The k-value will be positive in growth models.	**modelo de crecimiento exponencial continuo** un modelo de crecimiento exponencial que usa el número e; dado por la ecuación $p(t) = ae^{kt}$. En este modelo, a es el valor inicial, k es la tasa de crecimiento continuo en forma decimal y t es el tiempo. El valor k será positivo en los modelos de crecimiento.

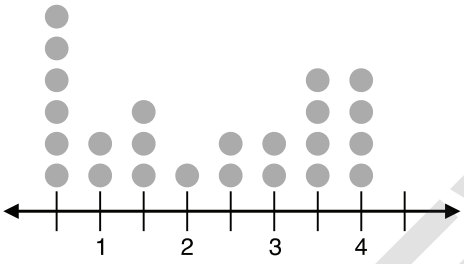
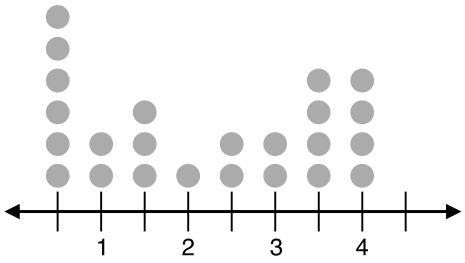
English	Español
continuous function a function that does not have a break in its graph across a specified domain	**función continua** función que no tiene una interrupción en su curva a lo largo de un dominio específico
continuous growth rate k in the continuous growth model $p(t) = ae^{kt}$; the k-value shows the amount by which the initial value increases continuously at any given time. The k-value is positive in continuous growth models.	**tasa de crecimiento continuo** k en el modelo de crecimiento continuo $p(t) = ae^{kt}$; el valor k muestra la cantidad en la que el valor inicial aumenta continuamente en un momento dado. El valor k es positivo en modelos de crecimiento continuo.
continuous random variable a random variable that has an infinite number of possible values	**variable aleatoria continua** es una variable aleatoria que tiene una cantidad infinita de valores posibles
control group the group of participants in a study who are not subjected to the treatment, action, or process being studied in the experiment, in order to form a comparison with participants who are subjected to it	**grupo de control** grupo de participantes en un estudio que no están sujetos al tratamiento, acción o proceso que está en estudio en el experimento con el fin de establecer una comparación con participantes que sí lo están.
convenience sample a sample in which members are chosen to minimize the time, effort, or expense involved in sampling	**muestreo de conveniencia** muestreo en el cual se eligen los miembros para minimizar el tiempo, esfuerzo o gasto involucrado en este proceso
conversion factor a ratio of quantities given in different units that are equivalent. For example, the ratio $\frac{12 \text{ inches}}{1 \text{ foot}}$ is a conversion factor.	**factor de conversión** una relación de cantidades dadas en diferentes unidades que son equivalentes. Por ejemplo, la relación $\frac{12 \text{ inches}}{1 \text{ foot}}$ es un factor de conversión.
coordinate plane a plane determined by a set of two number lines, called the axes, that intersect at right angles	**plano de coordenadas** un plano determinado por un conjunto de dos líneas numéricas, llamadas los ejes, que se cruzan en ángulos rectos
corporation a company or group of people authorized to act as a single entity	**corporación** una empresa o grupo de personas autorizadas para actuar como una sola entidad

English	Español
correlation a relationship between two events, where a change in one event is related to a change in the second event. A correlation between two events does not imply that the first event is responsible for the change in the second event; the correlation only shows how likely it is that a change also took place in the second event.	**correlación** relación entre dos eventos en la que el cambio en un evento se relaciona con un cambio en el segundo evento. Una correlación entre dos eventos no implica que el primero sea responsable del cambio en el segundo; la correlación sólo demuestra cuán probable es que también se produzca un cambio en el segundo evento.
correlation coefficient a quantity that assesses the strength of a linear relationship between two variables, ranging from –1 to 1; a correlation coefficient of –1 indicates a strong negative correlation, a correlation coefficient of 1 indicates a strong positive correlation, and a correlation coefficient of 0 indicates a very weak or no linear correlation	**coeficiente de correlación** cantidad que evalúa la fuerza de una relación lineal entre dos variables, que varía de –1 a 1; un coeficiente de correlación de –1 indica una fuerte correlación negativa, un coeficiente de correlación de 1 indica una fuerte correlación positiva, y un coeficiente de correlación de 0 indica una correlación muy débil o no lineal
corresponding entries entries that are in the same position	**entradas correspondientes** entradas que están en la misma posición
cosine a trigonometric function of an acute angle in a right triangle that is the ratio of the length of the side adjacent to the length of the hypotenuse; the cosine of $\theta = \cos\theta = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$	**coseno** función trigonométrica de un ángulo agudo en un triángulo rectángulo que es la proporción de la longitud de lado adyacente a la longitud de la hipotenusa; el coseno de $\theta = \cos\theta = \frac{\text{longitud del lado adyacente}}{\text{longitud de la hipotenusa}}$
cosine function a trigonometric function of the form $f(x) = a\cos[b(x-c)] + d$, in which a, b, c, and d are constants and x is a variable defined in radians over the domain $(-\infty,\infty)$	**función del coseno** función trigonométrica de la forma $f(x) = a\cos[b(x-c)] + d$, donde a, b, c y d son constantes y x es una variable definida en radianes a lo largo del dominio $(-\infty,\infty)$

English	Español
critical value a measure of the number of standards of error to be added to or subtracted from the mean in order to achieve the desired confidence level; also known as z_c-*value*	**valor crítico** medida de la cantidad de estándares de error que se suma o se resta de la media para lograr el nivel de confianza deseado; también conocido como *valor* z_c
cycle the smallest representation of a cosine or sine function graph as defined over a restricted domain; equal to one repetition of the period of a function	**ciclo** la representación más pequeña de una gráfica de la función coseno o seno definida a través de un dominio restringido; igual a una repetición del período de una función
D	
data numbers in context	**datos** números en contexto
debit card a card issued by the bank that allows you to access your account funds for purchases	**tarjeta débito** una tarjeta emitida por el banco que le permite acceder a los fondos de su cuenta para realizar compras
decay factor $1 - r$ in the exponential decay model $f(t) = a(1 - r)^t$, or b in the exponential function $f(t) = ab^t$ if $0 < b < 1$; the multiple by which a quantity decreases over time. The general form of an exponential function modeling decay is $f(t) = a(1 - r)^t$.	**factor de decaimiento** $1 - r$ en el modelo de decaimiento exponencial $f(t) = a(1 - r)^t$, o b en la función exponencial $f(t) = ab^t$ si $0 < b < 1$; el múltiplo por el que una cantidad disminuye con el tiempo. La forma general de una función exponencial que determina decaimiento es $f(t) = a(1 - r)^t$.
decay rate r in the exponential decay model $f(t) = a(1 - r)^t$	**tasa de decaimiento** r en el modelo de decaimiento exponencial $f(t) = a(1 - r)^t$
degrees of freedom (*df*) the number of data values that are free to vary in the final calculation of a statistic; that is, values that can change or move without violating the constraints on the data	**grados de libertad (*df*)** la cantidad de valores de datos que varían libremente en el cálculo final de una estadística; es decir, los valores que pueden cambiar o moverse sin violar las restricciones en los datos
dependent events events that are not independent. The outcome of one event affects the probability of the outcome of another event.	**eventos dependientes** eventos que no son independientes. El resultado de un evento afecta la probabilidad del resultado de otro.

English	Español
dependent variable labeled on the *y*-axis; the quantity that is based on the input values of the independent variable; the output variable of a function	**variable dependiente** designada en el eje de *y*; cantidad que se basa en los valores de entrada de la variable independiente; variable de salida de una función
deposit adding money to an account	**depositar** agregar dinero a una cuenta
desirable outcome the data sought or hoped for, represented by *p*; also known as *favorable outcome* or *success*	**resultado deseado** datos buscados o esperados, representado por *p*; también conocido como *resultado favorable* o *éxito*
determinant a specific value that is associated with a square matrix and has multiple applications	**determinante** valor específico asociado con las matrices cuadradas que tiene muchas aplicaciones
dilation a transformation in which a figure is either enlarged or reduced by a scale factor in relation to a center point	**dilatación** transformación en la que una figura se amplía o se reduce por un factor de escala en relación con un punto central
dimensions of a matrix the size of a matrix, as determined by the number of rows and columns. The dimensions of a matrix are listed as rows × columns (pronounced "rows by columns").	**dimensiones de una matriz** es el tamaño de una matriz, determinado por la cantidad de filas y columnas. Las dimensiones de una matriz se indican como filas × columnas (se dice "filas por columnas").
direct deposit payments are made directly from one account to another electronically, without the need of a check or other intermediate step	**depósito directo** los pagos se realizan directamente de una cuenta a otra de forma electrónica, sin la necesidad de un cheque u otro paso intermedio
directed line segment a line segment $\overrightarrow{PQ}$ directed from point *P* to point *Q*; note that because the line segment is directed, $\overrightarrow{QP}$ points in the opposite direction, thus the order of letters is important	**segmento de recta dirigido** segmento de recta $\overrightarrow{PQ}$ dirigido del punto *P* al punto *Q*; como el segmento de recta es dirigido, $\overrightarrow{QP}$ apunta en el sentido opuesto, por lo tanto el orden de las letras es importante
direction the way in which a vector points; may be specified by an angle, a slope, or a pair of vector components	**dirección** hacia dónde apunta un vector; puede estar especificada por un ángulo, una pendiente o un par de componentes de vector

English	Español
discontinuous function a function with a graph that is undefined at certain domain values or over certain domain intervals	**función discontinua** función con una gráfica indefinida en determinados valores de dominios o a través de ciertos intervalos de dominios
discrete individually separate and distinct	**discreto** individualmente aparte y distinto
discrete data a set of values with gaps between successive values	**datos discretos** conjunto de valores con interrupciones entre valores sucesivos
discrete function a function in which every element of the domain is individually separate and distinct	**función discreta** función en la cual cada elemento del dominio está individualmente separado y distinguible
discrete random variable a random variable that has a finite or countable number of possible values	**variable aleatoria discreta** es una variable aleatoria que tiene una cantidad finita o numerable de valores posibles
disjoint events events that have no outcomes in common. If A and B are disjoint events, then they cannot both occur. Disjoint events are also called mutually exclusive events.	**eventos disjuntos** eventos que no tienen resultados en común. Si A y B son eventos disjuntos, entonces no pueden producirse ambos. También se denominan eventos mutuamente excluyentes.
displacement vector a vector that represents a distance traveled in the x- and y-directions; the magnitude of a displacement vector is the shortest distance from the initial point to the terminal point	**vector de desplazamiento** vector que representa una distancia recorrida en las direcciones x e y; el módulo de un vector de desplazamiento es la distancia más corta desde el punto inicial hasta el punto final
distance formula a formula that states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	**fórmula de distancia** fórmula que señala la distancia entre puntos (x_1, y_1) y (x_2, y_2) es igual a $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Distributive Property for any quantities a, b, and c, $a(b + c) = a \bullet b + a \bullet c$	**propiedad distributiva** para toda cantidad a, b y c, $a(b + c) = a \bullet b + a \bullet c$

English	Español
divide and conquer algorithm an algorithm that takes the initial problem and divides it into sub-problems. The sub-problems are then divided further and solved recursively. The solutions of the sub-problems are combined to give the overall solution.	**algoritmo divide y vencerás** un algoritmo que toma el problema inicial y lo divide en subproblemas. A continuación, los subproblemas se dividen aún más y se resuelven de forma recursiva. Las soluciones de los subproblemas se combinan para dar la solución global.
dot plot a frequency plot that shows the number of times a response occurred in a data set, where each data value is represented by a dot. Example: 	**diagrama de puntos** diagrama de frecuencia que muestra la cantidad de veces que se produjo una respuesta en un conjunto de datos, en el que cada valor de dato está representado por un punto. Ejemplo: 
dot product the result of matrix multiplication	**producto punto** el resultado de la multiplicación matricial
double-blind study a study in which neither the researcher nor the participants know who has been subjected to the treatment, action, or process being studied, and who is in a control group	**estudio doble-ciego** estudio en el cual ni el investigador ni los participantes saben quién se sometió al tratamiento, acción o proceso que está siendo estudiado y quién está en un grupo de control
E	
e an irrational number with an approximate value of 2.71828; *e* is the base of the natural logarithm ($\ln x$ or $\log_e x$)	***e*** número irracional con un valor aproximado de 2,71828; *e* es la base del logaritmo natural (In x o $\log_e x$)
element an item in a set; also called a member	**elemento** ítem en un conjunto; también se denomina miembro
elementary row operations operations that can be performed on a single row of a matrix	**operaciones elementales de fila** operaciones que se pueden realizar en una sola fila de una matriz

English	Español
empirical probability the number of times an event actually occurs divided by the total number of trials, given by the formula $P(E)=\frac{\text{number of occurrences of the event}}{\text{total number of trials}}$; also called *experimental probability*	**probabilidad empírica** cantidad de veces que se produce un evento dividido por la cantidad total de pruebas, dada por la fórmula $P(E)=\frac{\text{cantidad de ocurrencias del evento}}{\text{cantidad total de pruebas}}$; también se denomina *probabilidad experimental*
Empirical Rule a rule that states percentages of data under the normal curve are as follows: $\mu \pm 1\sigma \approx 68\%$, $\mu \pm 2\sigma \approx 95\%$, and $\mu \pm 3\sigma \approx 99.7\%$; also known as the *68–95–99.7 rule*	**Regla Empírica** regla que establece los siguientes porcentajes de datos bajo la curva normal: $\mu \pm 1\sigma \approx 68\%$, $\mu \pm 2\sigma \approx 95\%$ y $\mu \pm 3\sigma \approx 99{,}7\%$; también se conoce como *la regla 68–95–99,7*
empty set a set that has no elements, denoted by $\varnothing$. The empty set is also called the *null set*.	**conjunto vacío** conjunto que no contiene elementos, indicado con $\varnothing$. También se denomina *conjunto nulo*.
entry each number, variable, or expression in a matrix. Each entry has a specific position within a row and a column.	**entrada** cada número, variable o expresión de una matriz. Cada entrada tiene una posición específica dentro de una fila y una columna.
equal sets sets with all the same elements	**conjuntos iguales** conjuntos con todos los mismos elementos
equation a mathematical sentence that uses an equal sign (=) to show that two quantities are equal	**ecuación** declaración matemática que utiliza el signo igual (=) para demostrar que dos cantidades son equivalentes
Euler's number (*e*) an irrational number with an approximate value of 2.71828	**número de Euler (*e*)** un número irracional con un valor aproximado de 2.71828
event an outcome or set of outcomes of an experiment. An event is a subset of the sample space.	**evento** resultado o conjunto de resultados de un experimento. Un evento es un subconjunto del espacio de muestral.
exemption with regard to taxes, a money amount that the worker can claim in order to pay less tax	**exención** con respecto a los impuestos, una cantidad de dinero que el trabajador puede reclamar para pagar menos impuestos

English

exhaustive algorithm an algorithm that explores all possible solutions to a problem and selects the optimal solution; also known as a *brute force algorithm*

expected value an estimate of value that is determined by finding the product of a total value and a probability of a given event; symbolized by $E(X)$

experiment a process or action that has observable results

experimental probability the number of times an event actually occurs divided by the total number of trials, given by the formula $P(E) = \frac{\text{number of occurrences of the event}}{\text{total number of trials}}$; also called *empirical probability*

explicit formula for a geometric sequence a formula used to find any term in a sequence. The formula is $a_n = a_1 \bullet r^{n-1}$, where n is a positive integer that represents the number of terms in the sequence, r is the common ratio, a_1 is the value of the first term in the sequence, and a_n is the value of the nth term of the sequence.

exponential decay an exponential equation with a base, b, that is between 0 and 1 exclusive (that is, $0 < b < 1$); an example is the formula $y = a(1 - r)^t$, where a is the initial value, $(1 - r)$ is the base (with $0 < r < 1$), t is the variable exponent, and y is the final value

Español

algoritmo exhaustivo un algoritmo que explora todas las posibles soluciones a un problema y selecciona la solución óptima; también conocido como *algoritmo de fuerza bruta*

valor esperado estimación de valor que se determina al encontrar el producto de un valor total y una probabilidad de un evento dado; simbolizado por $E(X)$

experimento proceso o acción con resultados observables

probabilidad experimental cantidad de veces que se produce un evento dividido por la cantidad total de pruebas, dada por la fórmula $P(E) = \frac{\text{cantidad de ocurrencias del evento}}{\text{cantidad total de pruebas}}$; también se denomina *probabilidad empírica*

fórmula explícita para una secuencia geométrica fórmula usada para encontrar algún término en una secuencia. La fórmula es $a_n = a_1 \bullet r^{n-1}$, donde n es un número entero positivo que representa la cantidad de términos en la secuencia, r es la relación común, a_1 es el valor del primer término de la secuencia y a_n es el valor del término n de la secuencia.

decaimiento exponencial una ecuación exponencial con una base, b, que está entre 0 y 1 exclusivo (es decir, $0 < b < 1$); un ejemplo es la fórmula $y = a(1 - r)^t$, donde a es el valor inicial, $(1 - r)$ es la base (con $0 < r < 1$), t es el exponente variable y y es el final valor

English

exponential decay model an exponential function, $f(t) = a(1 - r)^t$, where $f(t)$ is the final output value at the end of t time periods, a is the initial value, r is the percent decrease per time period (expressed as a decimal), and t is the number of time periods

exponential equation an equation of the form $y = ab^x$, where x is the independent variable, y is the dependent variable, and a and b are real numbers

exponential expression an expression that contains a base raised to a power/exponent

exponential function a function of the form $f(x) = ab^{cx}$, in which a, b, and c are constants; a function that has a variable in the exponent, such as $f(x) = 0.4 \bullet 6^{-0.2x}$

exponential growth an exponential equation with a base, b, greater than 1 ($b > 1$); an example is the formula $y = a(1 + r)^t$, where a is the initial value, $(1 + r)$ is the base (with $r > 0$), t is the variable exponent, and y is the final value

exponential growth model an exponential function, $f(t) = a(1 + r)^t$, where $f(t)$ is the final output value at the end of t time periods, a is the initial value, r is the percent increase per time period (expressed as a whole number or decimal), and t is the number of time periods

Español

modelo de decaimiento exponencial función exponencial, $f(t) = a(1 - r)^t$, en la que $f(t)$ es el valor de salida final despues de t períodos de tiempo, a es el valor inicial, r es el porcentaje de disminución por período (expresado como decimal), y t es la cantidad de períodos

ecuación exponencial ecuación de la forma $y = ab^x$, en la que x es la variable independiente, y es la variable dependiente, y a y b son números reales

expresión exponencial expresión que incluye una base elevada a una potencia o exponente

función exponencial función de la fórmula $f(x) = ab^{cx}$ en la cual a, b y c son constantes; una función que tiene una variable en el exponente, tal como $f(x) = 0{,}4 \bullet 6^{-0{,}2x}$

crecimiento exponencial una ecuación exponencial con una base, b, mayor que 1 ($b > 1$); un ejemplo es la fórmula $y = a(1 + r)^t$, donde a es el valor inicial, $(1 + r)$ es la base (con $r > 0$), t es el exponente variable ey y es el valor final

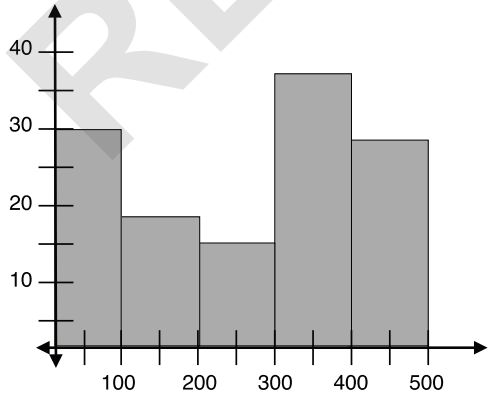
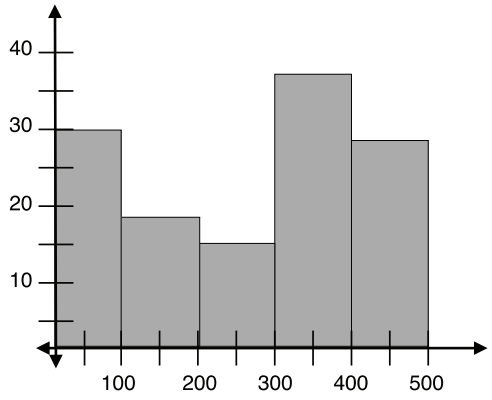
modelo de crecimiento exponencial función exponencial, $f(t) = a(1 - r)^t$, en la que $f(t)$ es el valor de salida final despues de t períodos de tiempo, a es el valor inicial, r es el porcentaje de aumento por período (expresado como entero o decimal), y t es la cantidad de períodos

English	Español
F	
factorial the product of an integer and all preceding positive integers, represented using a ! symbol; $n! = n \bullet (n-1) \bullet (n-2) \bullet \cdots \bullet 1$. For example, 5! = 5 • 4 • 3 • 2 • 1. By definition, 0! = 1.	**factorial** producto de un entero y todos los enteros positivos anteriores, que se representa con el símbolo !; $n! = n \bullet (n-1) \bullet (n-2) \bullet \cdots \bullet 1$. Por ejemplo, 5! = 5 • 4 • 3 • 2 • 1. Por definición, 0! = 1.
failure the occurrence of an event that was not sought out or wanted, represented by *q*; also known as *undesirable outcome* or *unfavorable outcome*	**fracaso** ocurrencia de un evento que no fue buscado ni deseado, representado por *q*; también conocido como *resultado no deseado* o *resultado desfavorable*
fair describes a situation or game in which all of the possible outcomes have an equal chance of occurring	**equitativo** describe una situación o juego en el cual todos los resultados posibles tienen igual probabilidad de producirse
false negative result a determination that an experiment has produced an incorrect negative result	**resultado falso negativo** determinación de que un experimento ha producido un resultado negativo incorrecto
false positive result a determination that an experiment has produced an incorrect positive result	**resultado falso positivo** determinación de que un experimento ha producido un resultado positivo incorrecto
favorable outcome the data sought or hoped for, represented by *p*; also known as *desirable outcome* or *success*	**resultado favorable** datos buscados o esperados, representados por *p*; también conocido como *resultado deseado* o *éxito*
federal income tax portion of your gross income that goes to the federal government for general purposes	**impuestos federales** parte de sus ingresos brutos que va al gobierno federal para fines generales
finite limited in number	**finito** limitado en número
finite geometric series a series that has a limited or definite number of terms; can be written as $\sum_{k=1}^{n} a_1 r^{k-1}$, where *n* is a positive integer that represents the number of terms in the series, a_1 is the first term, *r* is the common ratio, and *k* is the number of the term	**serie geométrica finita** serie que tiene una cantidad limitada o definida de términos; puede ser escrita como $\sum_{k=1}^{n} a_1 r^{k-1}$, donde *n* es un número entero positivo que representa la cantidad de términos en la serie, a_1 es el primer término, *r* es la relación común y *k* es el número del término

English	Español
first quartile the value that identifies the lower 25% of the data; the median of the lower half of the data set; written as Q_1	**primer cuartil** valor que identifica el 25% inferior de los datos; mediana de la mitad inferior del conjunto de datos; se expresa Q_1
flow chart a graphic organizer that shows the order in which to carry out a set of instructions	**diagrama de flujo** un organizador gráfico que muestra el orden en el que llevar a cabo un conjunto de instrucciones
force an influence vector that represents a directed push or pull on an object; the strength of a force vector is its magnitude because it has both magnitude and direction	**fuerza** vector de influencia que representa un empuje o un tirón sobre un objeto; la intensidad de un vector de fuerza es su módulo porque tiene a la vez módulo y dirección
frequency of a periodic function the reciprocal of the period for a periodic function; indicates how often the function repeats	**frecuencia de una función periódica** recíproca del período para una función periódica; indica con que frecuencia se repite la función
function a relation in which each element in the domain is mapped onto exactly one element in the range; that is, for every value of x, there is exactly one value of y	**función** relación en la que cada elemento de un dominio se combina con exactamente un elemento del rango; es decir, para cada valor de x, existe exactamente un valor de y
G	
geometric sequence an exponential function that results in a sequence of numbers separated by a common ratio	**secuencia geométrica** una función exponencial que produce como resultado una secuencia de números separados por una relación común
geometric series the sum of a specified number of terms from a geometric sequence	**serie geométrica** suma de una cantidad específica de términos de una secuencia geométrica
greedy algorithm an algorithm that makes best choices locally in hopes of reaching the global minimum	**algoritmo codicioso** un algoritmo que toma las mejores decisiones a nivel local con la esperanza de alcanzar el mínimo global
gross pay the amount of money you earn before deductions to your paycheck	**sueldo bruto** la cantidad de dinero que gana antes de las deducciones a su cheque de pago

English	Español
gross revenue the amount of money a company makes from selling their product before deducting expenses	**Ingresos brutos** la cantidad de dinero que gana una empresa por vender su producto antes de deducir los gastos
growth factor the multiple by which a quantity increases over time	**factor de crecimiento** múltiplo por el que una cantidad aumenta con el tiempo
growth rate the rate of increase in size per unit of time; r in the exponential growth model $f(t) = a(1 + r)^t$	**tasa de crecimiento** tasa de aumento de tamaño por unidad de tiempo; r en el modelo de crecimiento exponencial $f(t) = a(1 + r)^t$

H

head-to-tail method a way to add two vectors; given two vectors, place the head of one vector at the tail of the other vector, then draw a third vector that connects the tail of the first vector to the head of the second vector. The third vector is the resultant vector, or the sum.	**método poligonal** método para sumar dos vectores; dados dos vectores, coloque el extremo inicial de un vector sobre el extremo final del otro y luego dibuje un tercer vector que conecte el extremo inicial del primer vector con el extremo final del segundo. El tercer vector es el vector resultante, o la suma.
high price the highest price at which a share is sold during the day	**precio alto** el precio más alto al que se vende una acción durante el día
histogram a frequency plot that shows the number of times a response or range of responses occurred in a data set. Example: 	**histograma** una diagrama de frecuencia que muestra la cantidad de veces que se produce una respuesta o rango de respuestas en un conjunto de datos. Ejemplo: 

English	Español
hypothesis a statement that you are trying to prove or disprove	**hipótesis** afirmación que usted intenta probar o desaprobar
hypothesis testing assessing data in order to determine whether the data supports (or fails to support) the hypothesis as it relates to a parameter of the population	**prueba de hipótesis** evaluación de datos para determinar si los datos respaldan (o no respaldan) la hipótesis mientras se relaciona con un parámetro de la población
I	
identity matrix a square matrix that has ones along the main diagonal and zeros everywhere else. For example, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. When a matrix is multiplied by an identity matrix, the original matrix does not change.	**matriz identidad** matriz cuadrada que tiene unos en la diagonal principal y ceros en los demás lugares. Por ejemplo, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Cuando se multiplica una matriz por la matriz identidad, la matriz original no cambia.
independent events events such that the outcome of one event does not affect the probability of the outcome of another event	**eventos independientes** eventos en los que el resultado de un evento no afecta la probabilidad del resultado de otro evento
independent variable generally labeled on the *x*-axis; the quantity that changes based on values chosen	**variable independiente** generalmente designada en el eje *x*; cantidad que cambia según valores seleccionados
inequality a mathematical sentence that shows the relationship between quantities that may or may not be equivalent. An inequality contains one or more of the following symbols: $<$, $>$, $\leq$, $\geq$, or $\neq$.	**desigualdad** enunciado matemático que demuestra la relación entre cantidades que pueden ser o no equivalentes. Una desigualdad contiene uno o más de los siguientes símbolos: $<$, $>$, $\leq$, $\geq$ o $\neq$.
inference a conclusion reached upon the basis of evidence and reasoning	**inferencia** conclusión alcanzada sobre la base de evidencia y razonamiento

English	Español
initial point the point at which a vector begins; the "tail" of a vector	**extremo inicial** punto en el cual comienza un vector; la "cola" de un vector
interest a percentage amount of a loan amount that must be repaid along with the loan amount	**interés** una cantidad porcentual de la cantidad de un préstamo que debe reembolsarse junto con la cantidad del préstamo
interquartile range the difference between the third and first quartiles; 50% of the data is contained within this range	**rango intercuartílico** diferencia entre el tercer y primer cuartil; el 50% de los datos está contenido dentro de este rango
intersection a set whose elements are each in both of two other sets. The intersection of sets A and B, denoted by $A \cap B$, is the set of elements that are in both A and B.	**intersección** conjunto cuyos elementos están todos en otros dos conjuntos. La intersección de los conjuntos A y B, indicada por $A \cap B$, es el conjunto de elementos que se encuentran tanto en A como en B.
interval the set of all real numbers between two given numbers. The two numbers on the ends are the endpoints. The endpoints might or might not be included in the interval depending on whether the interval is open, closed, or half-open/half-closed.	**intervalo** conjunto de todos los números reales entre dos números dados. Los dos números en los finales son los extremos. Los extremos podrían o no estar incluidos en el intervalo, según si el intervalo está abierto, cerrado, o medio abierto o medio cerrado.
inverse matrix a matrix that when multiplied by the original matrix produces the identity matrix; also called the *multiplicative inverse matrix*. The inverse of matrix A is denoted A^{-1}.	**matriz inversa** matriz que, cuando se la multiplica por la matriz original, produce la matriz identidad; también se la llama *matriz inversa multiplicativa*. La inversa de la matriz A se denota A^{-1}.
inverse of a matrix a matrix that when multiplied by the original matrix produces the identity matrix. The inverse of matrix A is denoted A^{-1}.	**inversa de una matriz** una matriz que cuando se multiplica por la matriz original produce la matriz de identidad. La inversa de la matriz A se denomina A^{-1}.
iteration the repetition of a process	**iteración** la repetición de un proceso
iterative process a set of instructions that is repeated; also called a *loop* in computer programming	**proceso iterativo** un conjunto de instrucciones que se repite; también llamado bucle en la programación de computadoras

English	Español
J	
joint frequency the number of times a pair of characteristics appear together	**frecuencia conjunta** cantidad de veces que un par de características aparecen juntas
L	
laws of exponents rules that must be followed when working with exponents	**leyes de los exponentes** normas que deben cumplirse cuando se trabaja con exponentes
level of confidence the probability that a parameter's value can be found in a specified interval; also called *confidence level*	**grado de confianza** probabilidad de que se pueda encontrar el valor de un parámetro en un intervalo específico; también llamado *nivel de confianza*
line chart a graph that illustrates the long-term behavior of stock prices using a series of points on a coordinate plane connected by line segments	**gráfico de líneas** un gráfico que ilustra el comportamiento a largo plazo de los precios de las acciones utilizando una serie de puntos en un plano de coordenadas conectados por segmentos de línea
linear equation a first-degree equation that can be written in the form $ax + by = c$, where a, b, and c are rational numbers; when written as $y = mx + b$, m is the slope of the line, and b is its y-intercept. The graph of a linear equation is a straight line.	**ecuación lineal** ecuación de primer grado que puede expresarse en la forma $ax + by = c$, donde a, b y c son números racionales; cuando se expresa como $y = mx + b$, m es la pendiente de la recta y b es el intercepto de y. La representación gráfica de una ecuación lineal es una línea recta.
linear fit (or linear model) an approximation of data using a linear function	**ajuste lineal (o modelo lineal)** aproximación de datos con el uso de una función lineal
linear function a function that can be written in the form $ax + by = c$, where a, b, and c are constants; can also be written as $f(x) = mx + b$, in which m is the slope and b is the y-intercept. The graph of a linear function is a straight line; its solutions are the infinite set of points on the line.	**función lineal** función que puede expresarse en la forma $ax + by = c$, donde a, b y c son constantes; también puede escribirse como $f(x) = mx + b$, donde m es la pendiente y b es el intercepto de y. El gráfico de una función lineal es una línea recta; sus soluciones son el conjunto infinito de puntos en la línea.

English	Español
logarithmic function the inverse of an exponential function; for the exponential function $g(x) = 5^x$, the inverse logarithmic function is $x = \log_5 g(x)$	**función logarítmica** la inversa de una función exponencial; para la función exponencial $g(x) = 5^x$, la función logarítmica inversa es $x = \log_5 g(x)$
loop a set of instructions that is repeated; in computer programming, refers to a section of code that is repeated a desired number of times	**bucle** un conjunto de instrucciones que se repite; en programación de computadoras, se refiere a una sección de código que se repite un número deseado de veces
low price the lowest price at which a share is sold during the day	**precio bajo** el precio más bajo al que se vende una acción durante el día
M	
magnitude the length of a vector, denoted by $\|\|\mathbf{v}\|\|$; the length of vector $\vec{v} = \langle a, b \rangle$ is $\|\|\mathbf{v}\|\| = \sqrt{a^2 + b^2}$	**módulo (de un vector)** longitud de un vector, denotada por $\|\|\mathbf{v}\|\|$; la longitud del vector $\vec{v} = \langle a, b \rangle$ es $\|\|\mathbf{v}\|\| = \sqrt{a^2 + b^2}$
maintenance fee a fee charged by some bank accounts in order to keep the account open; these fees sometimes have conditions under which they may be applied	**cargo de mantenimiento** un arancel cobrada por algunas cuentas bancarias para mantener la cuenta abierta; estas tarifas a veces tienen condiciones bajo las cuales se pueden aplicar
margin of error the quantity that represents the level of confidence in a calculated parameter, abbreviated MOE. The margin of error can be calculated by multiplying the critical value by the standard deviation, if known, or by the SEM.	**margen de error** cantidad que representa el nivel de confianza en un parámetro calculado, abreviado MOE. El margen de error puede calcularse multiplicando el valor crítico por la desviación estándar, si se conoce, o por el SEM.
marginal frequency the total number of times a characteristic occurs	**frecuencia marginal** cantidad total de veces que aparece una característica
matrix an ordered arrangement of numbers or expressions in rows and columns. The plural of *matrix* is *matrices*.	**matriz** arreglo de números o expresiones ordenado en filas y columnas. El plural de *matriz* es *matrices*.
matrix equation an equation in which a variable stands for a matrix	**ecuación matricial** ecuación en la cual alguna variable representa una matriz

English	Español
maximum the greatest value or highest point of a function	**máximo** el mayor valor o punto más alto de una función
mean a measure of center in a set of numerical data, computed by adding the values in a data set and then dividing the sum by the number of values in the data set; denoted as the Greek lowercase letter *mu*, μ; given by the formula $\mu = \frac{x_1 + x_2 + \ldots + x_n}{n}$, where each *x*-value is a data point and *n* is the total number of data points in the set	**media** medida del centro en un conjunto de datos numéricos, calculada al sumar los valores en un conjunto de datos y luego al dividir la suma por el número de valores en el conjunto de datos; indicada con la letra griega minúscula *mu*, μ; dada por la fórmula $\mu = \frac{x_1 + x_2 + \ldots + x_n}{n}$, donde cada valor de x es un punto de datos y *n* es la cantidad total de puntos de datos en el conjunto
mean absolute deviation the average distance between each data point and the mean; found by summing the absolute values of the difference between each data point and the mean, then dividing this sum by the total number of data points	**desviación media absoluta** distancia promedio entre cada punto de datos y la media; se determina al sumar los valores absolutos de la diferencia entre cada punto de datos y la media y luego dividir esta suma por la cantidad total de puntos de datos
measurement bias bias that occurs when the tool used to measure the data is not accurate, current, or consistent	**sesgo de medición** sesgo que se produce cuando la herramienta utilizada para medir los datos no es exacta, actual o constante
measures of center values that describe expected and repeated data values in a data set; the mean and median are two measures of center	**medidas de centro** valores que describen los valores de datos esperados y repetidos de un conjunto de datos; la media y la mediana son dos medidas de centro
measures of spread a measure that describes the variance of data values, and identifies the diversity of values in a data set	**medidas de dispersión** medidas que describen la varianza de los valores de datos e identifican la diversidad de valores en un conjunto de datos

English	Español
median the middle-most value of a data set; 50% of the data is less than this value, and 50% is greater than it	**mediana** valor máximo-medio de un conjunto de datos; el 50 % de los datos es menor que este valor y el otro 50 % es mayor que él
Medicare tax the portion of your gross income that goes into the Medicare program, equal to 1.45% of gross pay	**impuesto de Medicare** la parte de sus ingresos brutos que se destina al programa de Medicare, equivalente al 1,45% del salario bruto
member an item in a set; also called an element	**miembro** ítem en un conjunto; también se denomina elemento
midline in a cosine function or sine function of the form $f(x) = a + \sin x$ or $g(x) = a + \cos x$, a horizontal line of the form $y = a$ that bisects the vertical distance on a graph between the minimum and maximum function values	**línea media** en una función del coseno o en una función del seno de la forma $f(x) = a + \sin x$ o $g(x) = a + \cos x$, una línea horizontal de la forma $y = a$ que divide en dos la distancia vertical en un gráfico entre los valores de funciones mínimos y máximos
minimum the least value or lowest point of a function	**mínimo** el menor valor o el punto más bajo de una función
minimum balance the least amount of money a bank requires you to have in your bank account; going below this amount may incur a fee	**balance minimo** la menor cantidad de dinero que un banco requiere que tenga en su cuenta bancaria; ir por debajo de esta cantidad puede incurrir en un arancel
minimum wage a federally mandated minimum amount that employers must pay workers per hour	**salario mínimo** una cantidad mínima exigida por el gobierno federal que los empleadores deben pagar a los trabajadores por hora
mu, μ a Greek letter used to represent mean	***mu,*** μ letra griega usada para representar la media
Multiplication Rule the probability of two events, A and B, is $P(A \text{ and } B) = P(A) \bullet P(B \mid A) = P(B) \bullet P(A \mid B)$; for independent events A and B, the rule is $P(A \text{ and } B) = P(A) \bullet P(B)$.	**Regla de multiplicación** probabilidad de que dos eventos, A y B, sea $P(A \text{ y } B) = P(A) \bullet P(B \mid A) = P(B) \bullet P(A \mid B)$; para eventos independientes A y B, la regla es $P(A \text{ y } B) = P(A) \bullet P(B)$.
multiplication rule for independent events The probability of two independent events A and B both occurring is $P(A \text{ and } B) = P(A) \bullet P(B)$.	**regla de multiplicación para eventos independientes** La probabilidad de que dos eventos independientes A y B ocurran ambos es $P(A \text{ y } B) = P(A) \bullet P(B)$.

English	Español
multiplicative identity the element of a set whose multiplication does not change other elements; the multiplicative identity of the real numbers is 1	**identidad de la multiplicación** el elemento de un conjunto cuya multiplicación no cambia a los demás elementos; la identidad de la multiplicación en los números reales es el 1
multiplicative identity matrix a matrix that has ones along the main diagonal and zeros everywhere else; also called the *identity matrix*. For example, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; represented as I. When a matrix is multiplied by an identity matrix, the original matrix does not change.	**matriz identidad para la multiplicación** matriz que tiene unos en la diagonal principal y ceros en los demás lugares; también se la llama *matriz identidad*. Por ejemplo, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, que se representa como I. Cuando se multiplica una matriz por la matriz identidad, la matriz original no cambia.
multiplicative inverse matrix a matrix that when multiplied by the original matrix produces the identity matrix; also called the *inverse matrix*. The inverse of matrix A is denoted A^{-1}.	**matriz inversa multiplicativa** matriz que, cuando se la multiplica por la matriz original, produce la matriz identidad; también se la llama *matriz inversa*. La inversa de la matriz A se denota A^{-1}.
mutually exclusive events events that have no outcomes in common. If A and B are mutually exclusive events, then they cannot both occur. Mutually exclusive events are also called disjoint events.	**eventos mutuamente excluyentes** eventos que no tienen resultados en común. Si A y B son eventos mutuamente excluyentes, entonces no pueden producirse ambos. También se denominan eventos disjuntos.

English	Español
N	
natural logarithm a logarithm whose base is the irrational number *e*; usually written in the form "ln," which means "$\log_e$."	**logaritmo natural** logaritmo cuya base es el número irracional *e*; escrito normalmente en la forma "ln", que significa "$\log_e$".
natural numbers the set of positive integers {1, 2, 3, ...}	**números naturales** conjunto de enteros positivos {1, 2, 3, ...}
negatively skewed a distribution in which there is a "tail" of isolated, spread-out data points to the left of the median. "Tail" describes the visual appearance of the data points in a histogram. Data that is negatively skewed is also called *skewed to the left*.	**sesgado negativamente** distribución en la cual existe una "cola" de puntos de datos aislados y esparcidos a la izquierda de la mediana. La "cola" describe la apariencia visual de los puntos de datos en un histograma. Los datos que están sesgados negativamente también se denominan *sesgados a la izquierda*.
net pay the amount of money you actually take home after taxes	**salario neto** la cantidad de dinero que realmente se lleva a casa después de impuestos
net revenue the amount of money a company makes from selling their product after deducting all expenses	**ingresos netos** la cantidad de dinero que una empresa gana por vender su producto después de deducir todos los gastos
neutral not biased or skewed toward one side or another; regarding surveys, neutral refers to phrasing questions in a way that does not lead the response toward one particular answer or side of an issue	**neutral** no sesgado hacia un lado u otro; respecto de las encuestas, neutral se refiere a la formulación de preguntas de una manera que no conduzca la respuesta hacia una respuesta o lado específico de un tema
nonresponse bias bias that occurs when the respondents to a survey have different characteristics than nonrespondents, causing the population that does not respond to be underrepresented in the survey's results	**sesgo sin respuesta** sesgo que se produce cuando los encuestados de una encuesta tienen características diferentes de los no encuestados, dando pie a que la población que no responde sea subrepresentada en los resultados de la encuesta
normal curve a symmetrical curve representing the normal distribution	**curva normal** curva simétrica que representa la distribución normal

English	Español
normal distribution a set of values that are continuous, are symmetric to a mean, and have higher frequencies in intervals close to the mean than equal-sized intervals away from the mean	**distribución normal** conjunto de valores que son continuos, simétricos a una media y tienen frecuencias más altas en intervalos cercanos a la media que los intervalos de igual tamaño lejos de la media
normal probability distribution a probability distribution whose graph is symmetric and is shaped like a bell; most of the data are near or at the mean	**distribución de probabilidad normal** distribución de probabilidad cuya gráfica es simétrica y con forma de campana; la mayoría de los datos están cerca de la media o en la media
null hypothesis the statement or idea that will be tested, represented by H_0; generally characterized by the concept that there is no relationship between the data sets, or that the treatment has no effect on the data	**hipótesis nula** afirmación o idea que será probada, representada por H_0; caracterizada generalmente por el concepto de que no hay relación entre los conjuntos de datos o que el tratamiento no tiene efecto en los datos
null set a set that has no elements, denoted by $\varnothing$. The null set is also called the *empty set*.	**conjunto nulo** conjunto que no tiene elementos, indicado con $\varnothing$. También se denomina *conjunto vacío*.
O	
observational study a study in which all data, including observations and measurements, are recorded in a way that does not change the subject that is being measured or studied	**estudio observacional** estudio en el cual todos los datos, incluyendo las observaciones y las mediciones, están registrados de tal manera que no cambian el objeto que está siendo medido o estudiado
one-tailed test a *t*-test performed on a set of data to determine if the data could belong in one of the tails of the bell-shaped distribution curve; with this test, the area under only one tail of the distribution is considered	**prueba de una cola o unilateral** una prueba *t* realizada en un conjunto de datos para determinar si estos podrían pertenecer a una de las colas de la curva de distribución con forma de campana; con esta prueba, solo se considera el área debajo de una cola de la distribución
online banking this allows customers to access most banking options via the Internet, at any time	**banca en línea** esto permite a los clientes acceder a la mayoría de las opciones bancarias a través de Internet, en cualquier momento

English	Español
opening price the price per share of a stock at the beginning of a day	**precio de apertura** el precio por acción de una acción al comienzo de un día
opposite matrix the matrix which, when added to matrix A, yields the zero matrix; also called the *additive inverse matrix*. The opposite of matrix A is $-A$.	**matriz opuesta** es la matriz que, cuando se la suma a la matriz A, el resultado es la matriz cero; también se la llama *matriz inversa aditiva*. La opuesta a la matriz A es $-A$.
outcome the observable result of an experiment	**resultado** producto observable de un experimento
outlier a value far above or below other values of a distribution	**valor atípico** valor muy por encima o muy por debajo de otros valores de una distribución
overdraft when the balance of a bank account goes negative	**sobregiro** cuando el saldo de una cuenta bancaria se vuelve negativo
overtime time worked over 40 hours. If a worker works 45 hours, they have worked 40 regular hours and 5 overtime hours. Overtime is paid at a higher rate than regular time. Workers are usually paid 1.5 times their usual rate for overtime.	**horas extraordinarias** tiempo trabajado más de 40 horas. Si un trabajador trabaja 45 horas, ha trabajado 40 horas regulares y 5 horas extraordinarias. Las horas extraordinarias se pagan a una tasa más alta que el tiempo regular. Por lo general, a los trabajadores se les paga 1,5 veces su tarifa habitual por horas extra.
P	
***p*-value** a number between 0 and 1 that determines whether to accept or reject the null hypothesis	**valor *p*** número entre 0 y 1 que determina si se acepta o se rechaza la hipótesis nula
Parallelogram Rule a method for vector addition; when the initial points of two vectors $\vec{u}$ and $\vec{v}$ are aligned, they define a parallelogram whose diagonal represents the vector sum $\vec{u}+\vec{v}$	**regla del paralelogramo** método de suma de vectores; cuando se alinean los extremos iniciales de dos vectores $\vec{u}$ y $\vec{v}$, definen un paralelogramo cuya diagonal representa al vector suma $\vec{u}+\vec{v}$
parameter numerical value(s) representing the data in a set, including proportion, mean, and variance	**parámetro** valores numéricos que representan los datos en un conjunto, incluyendo la proporción, la media y la varianza

English	Español
parent function a function with a simple algebraic rule that represents a family of functions. The graphs of the functions in the family have the same general shape as the parent function.	**función principal** función con una regla algebraica simple que representa una familia de funciones. Los gráficos de las funciones en la familia tienen la misma forma general que la función principal.
partnership a business that is owned by a group of people	**sociedad** una empresa que pertenece a un grupo de personas
payroll the money a company sets aside to pay its workers	**nómina** el dinero que una empresa reserva para pagar a sus trabajadoras
payroll taxes the portion of your gross income that goes into Social Security and Medicare, equal to 7.65% of gross pay	**impuestos sobre la nómina** la parte de su ingreso bruto que se destina al Seguro Social y Medicare, equivalente al 7.65% del salario bruto
payroll withholding the practice of employers keeping a certain amount of a worker's gross pay in order to pay taxes	**retención de nómina** la práctica de que los empleadores se queden con una determinada cantidad del salario bruto de un trabajador para pagar impuestos
period in a cosine or sine function graph, the horizontal distance from a maximum to a maximum or from a minimum to a minimum; one repetition of the period of a function is called a *cycle*	**período** en una curva de la función del seno o coseno, distancia horizontal desde un máximo a un máximo o desde un mínimo a un mínimo; una repetición del período de una función se llama *ciclo*
periodic function a function whose values repeat at regular intervals	**función periódica** función cuyos valores se repiten a intervalos regulares
periodic phenomena real-life situations that repeat at regular intervals and can be represented by a periodic function	**fenómenos periódicos** situaciones de la vida real que se repiten a intervalos regulares y se pueden representar mediante una función periódica
permutation a selection of objects where the order matters and is found either using n^r, if repetitions are allowed, or by using $_nP_r = \frac{n!}{(n-r)!}$, where n is the number of objects to select from and r is the number of objects being selected and ordered.	**permutación** selección de objetos en la que el orden importa y se encuentra con el uso de n^r, si se permiten las repeticiones, o con $_nP_r = \frac{n!}{(n-r)!}$, donde n es la cantidad de objetos de donde seleccionar y r es la cantidad de objetos seleccionados y ordenados.

English	Español
personal check a voucher that tells the account holder's bank to make a payment of a designated amount to a designated payee from the linked account	**cheque personal** un comprobante que le indica al banco del titular de la cuenta que realice un pago de una cantidad designada a un beneficiario designado desde la cuenta vinculada
phase shift on a cosine or sine function graph, the horizontal distance by which the curve of a parent function is shifted by the addition of a constant or other expression in the argument of the function. If $f(x) = \sin(ax + b)$, the phase shift is found by setting the argument of the function equal to 0 and solving for x, resulting in a phase shift of $-\frac{b}{a}$.	**cambio de fase** en una gráfica de la función del seno o del coseno, distancia horizontal por la cual la curva de una función raíz se cambia por la adición de una constante u otra expresión en el argumento de la función. Si $f(x) = \sin(ax + b)$, el cambio de fase se encuentra estableciendo el argumento de la función igual a 0 y resolviendo para x, con el resultado de un cambio de fase de $-\frac{b}{a}$.
piecewise function a function that is defined by two or more expressions on separate portions of the domain	**función por partes** función definida por dos o más expresiones en porciones separadas del dominio
piecework a type of employment in which a worker is paid a fixed rate for each unit produced or action performed	**trabajo a destajo** un tipo de empleo en el que a un trabajador se le paga una tarifa fija por cada unidad producida o acción realizada
placebo a substance that is used as a control in testing new medications; the substance has no medicinal effect on the subject	**placebo** sustancia que se utiliza como control en las pruebas de medicamentos nuevos; la sustancia no tiene efecto medicinal sobre el sujeto
population all of the people, objects, or phenomena of interest in an investigation; the entire data set	**población** todas las personas, los objetos o fenómenos de interés en una investigación; el conjunto completo de datos

English

population average the sum of all quantities in a population, divided by the total number of quantities in the population; typically represented by μ; also known as *population mean*

population mean the sum of all quantities in a population, divided by the total number of quantities in the population; typically represented by μ; also known as *population average*

portfolio a collection of investments owned by an investor

positively skewed a distribution in which there is a "tail" of isolated, spread-out data points to the right of the median. "Tail" describes the visual appearance of the data points in a histogram. Data that is positively skewed is also called *skewed to the right*.

power the quantity that shows the number of times the base is being multiplied by itself in an exponential expression, such as x in the logarithmic function $x = \log_5 g(x)$ and its exponential function, $g(x) = 5^x$

precision the degree to which the accuracy of a measurement is known. In measurement systems, precision also refers to the reproducibility of a result upon repetition.

principal an initial amount on money invested or borrowed

probability a number from 0 to 1 inclusive or a percent from 0% to 100% inclusive that indicates how likely an event is to occur

Español

promedio de la población suma de todas las cantidades de una población, dividida por el número total de cantidades de la población; representada normalmente por μ; también se conoce como *media poblacional*

media poblacional suma de todas las cantidades de una población, dividida por el número total de cantidades de la población; representada normalmente μ; también se conoce como *promedio de la población*

portafolio una colección de inversiones propiedad de un inversor

sesgado positivamente distribución en la cual existe una "cola" de puntos de datos aislados esparcidos hacia la derecha de la mediana. La "cola" describe la apariencia de los puntos de datos en un histograma. Los datos que están positivamente sesgados también se denominan *sesgados a la derecha*.

potencia cantidad que muestra el número de veces que se multiplica la base por sí misma en una expresión exponencial, tal que x en la función logarítmica $x = \log_5 g(x)$ y su función exponencial $g(x) = 5^x$

precisión el grado en que se conoce la exactitud de una medición. En los sistemas de medición, la precisión también se refiere a la reproducibilidad de un resultado en la repetición.

capital una cantidad inicial de dinero invertido o prestado

probabilidad número de 0 a 1 inclusivo o porcentaje de 0% a 100% inclusivo que indica cuán probable es que se produzca un evento

English	Español
probability distribution the values of a random variable with associated probabilities	**distribución de probabilidad** los valores de una variable aleatoria con probabilidades asociadas
probability model a mathematical model for observable facts or occurrences that are assumed to be random; a representation of a random phenomenon	**modelo de probabilidad** modelo matemático para hechos o sucesos observables que se presumen aleatorios; representación de un fenómeno aleatorio
probability of an event *E* denoted $P(E)$, and is given by $P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$ in a uniform probability model	**probabilidad de un evento *E*** se expresa como $P(E)$, y está dado por $P(E) = \frac{\text{número de resultados en } E}{\text{número de resultados en el espacio de muestreo}}$ en un modelo de probabilidad uniforme
properties of equality rules that allow you to balance, manipulate, and solve equations	**propiedades de igualdad** normas que permiten equilibrar, manipular y resolver ecuaciones
properties of inequality rules that allow you to balance, manipulate, and solve inequalities	**propiedades de desigualdad** normas que permiten equilibrar, manipular y resolver desigualdades
Q	
quadratic function a function defined by a second-degree expression of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$ and a, b, and c are constants. The graph of any quadratic function is a parabola.	**función cuadrática** función definida por una expresión de segundo grado de la forma $f(x) = ax^2 + bx + c$, donde $a \neq 0$ y a, b y c son constantes. La representación gráfica de toda función cuadrática es una parábola.
quantify to find, describe, or measure the total amount or number of something	**cuantificar** para encontrar, describir o medir la cantidad total o el número de algo
quantity a value or expression that may be expressed in numbers	**cantidad** valor o expresión que puede expresarse en números

English	Español
R	
radian the measure of the central angle that intercepts an arc equal in length to the radius of the circle; π radians = 180°	**radián** medida del ángulo central que intercepta un arco de longitud igual al radio del círculo; π radianes = 180°
random the designation of a group or sample that has been formed without following any kind of pattern and without bias. Each group member has been selected without having more of a chance than any other group member of being chosen.	**aleatorio** designación de un grupo o muestra que se formó sin seguir ninguna clase de patrón y sin sesgo. Cada miembro del grupo se seleccionó sin tener más probabilidades de ser elegido que cualquier otro miembro del grupo.
random number generator a tool used to select a number without following a pattern, where the probability of generating any number in the set is equal	**generador de números aleatorios** herramienta utilizada para seleccionar un número sin seguir un patrón, donde la probabilidad de generar cualquier número en el conjunto es igual
random sample a subset or portion of a population or set that has been selected without bias, with each item in the population or set having the same chance of being found in the sample	**muestra aleatoria** subconjunto o porción de población o conjunto que ha sido seleccionado sin sesgo, con cada elemento de la población o conjunto con la misma probabilidad de encontrarse en la muestra
random variable a variable whose numerical value changes depending on each outcome in a sample space; the values of a random variable are associated with chance variation	**variable aleatoria** variable cuyo valor numérico cambia según cada resultado en un espacio de muestra; los valores de una variable aleatoria están asociados con una variación al azar
randomization the selection of a group, subgroup, or sample without following a pattern, so that the probability of any item in the set being generated is equal; the process used to ensure that a sample best represents the population	**aleatorización** selección de un grupo, subgrupo o muestra sin seguir un patrón, de manera que la probabilidad de cualquier elemento en el conjunto que está siendo generado sea igual; proceso utilizado para asegurar que la muestra sea la que mejor represente a la población

English	Español
randomized algorithm an algorithm that invokes randomness at some point	**algoritmo aleatorio** un algoritmo que invoca la aleatoriedad en algún momento
rate a ratio that compares different kinds of units	**tasa** proporción en que se comparan distintos tipos de unidades
reassignment assigning a new value to a variable; in computer programing, the new value is often given in terms of the variable itself. For example, $n = n + 1$. Also known as *redefinition*.	**reasignación** asignar un nuevo valor a una variable; en la programación de computadoras, el nuevo valor a menudo se da en términos de la propia variable. Por ejemplo, $n = n + 1$. También conocido como *redefinición*.
recursive algorithm an algorithm that is defined by a recursive formula; each term is determined by some operation on prior terms	**algoritmo recursivo** un algoritmo que se define mediante una fórmula recursiva; cada término está determinado por alguna operación en términos anteriores
recursive formula a formula used to find the next term of a sequence when the previous term or terms are known; the recursive formula for an arithmetic sequence is $a_n = a_{n-1} + d$; the recursive formula for a geometric sequence is $a_n = a_{n-1} \cdot r$	**fórmula recursiva** fórmula que se utiliza para encontrar el término siguiente de una secuencia cuando se conoce el o los términos anteriores; la fórmula recursiva de una secuencia aritmética es $a_n = a_{n-1} + d$; la fórmula recursiva para una secuencia geométrica es $a_n = a_{n-1} \cdot r$
recursive formula for a geometric sequence a formula used to find the next term in a sequence. The formula is $a_n = a_{n-1} \cdot r$, where n is a positive integer that represents the number of terms in the sequence and r is the common ratio.	**fórmula recursiva para una secuencia geométrica** fórmula utilizada para encontrar el término siguiente en una secuencia. La fórmula es $a_n = a_{n-1} \cdot r$, donde n es un número entero positivo que representa la cantidad de términos en la secuencia y r es la relación común.
recursive formula for a geometric sequence a formula used to find the next term in a sequence. The formula is $a_n = a_{n-1} \cdot r$, where n is a positive integer that represents the number of terms in the sequence and r is the common ratio.	**fórmula recursiva para una secuencia geométrica** fórmula utilizada para encontrar el término siguiente en una secuencia. La fórmula es $a_n = a_{n-1} \cdot r$, donde n es un número entero positivo que representa la cantidad de términos en la secuencia y r es la relación común.

English	Español
redefinition assigning a new value to a variable; in computer programing, the new value is often given in terms of the variable itself. For example, $n = n + 1$. Also known as *reassignment.*	**redefinición** asignar un nuevo valor a una variable; en la programación de computadoras, el nuevo valor a menudo se da en términos de la propia variable. Por ejemplo, $n = n + 1$. También conocido como *reasignación*.
reflection matrix a matrix which, when multiplied on the left of a vector, yields a vector that is a reflection of the input vector	**matriz de reflejo** matriz que, cuando se la multiplica a izquierda por un vector, se obtiene un vector que es el reflejo del original
relative frequency (of an event) the number of times an event occurs divided by the number of times an experiment is performed	**frecuencia relativa (de un evento)** cantidad de veces que un evento se produce dividido por la cantidad de veces que se realiza el experimento
reliability the degree to which a study or experiment performed many times would have similar results	**confiabilidad** grado en el cual un estudio o experimento realizado varias veces tendría resultados similares
representative sample a sample in which the characteristics of the people, objects, or items in the sample are similar to the characteristics of the population	**muestra representativa** muestra en la cual las características de las personas, los objetos o elementos en ella son similares a las características de la población
residual the vertical distance between an observed data value and an estimated data value on a line of best fit	**residual** distancia vertical entre un valor de datos observado y un valor de datos estimado sobre una línea de ajuste óptimo
residual plot provides a visual representation of the residuals for a set of data; contains the points (x, residual for x)	**diagrama residual** brinda una representación visual de los residuales para un conjunto de datos; contiene los puntos (x, residual de x)
response bias bias that occurs when responses by those surveyed have been influenced in some manner	**sesgo de respuesta** sesgo que se produce cuando las respuestas de los encuestados fueron influenciadas de alguna manera
resultant vector the result of vector addition; adding vectors $\vec{u}$ and $\vec{v}$ yields the resultant vector $\vec{u}+\vec{v}$	**vector resultante** es el resultado de la sumatoria de vectores; al sumar los vectores $\vec{u}$ y $\vec{v}$, se obtiene el vector resultante $\vec{u}+\vec{v}$

English	Español
retirement when someone stops working at his or her job permanently	**jubilación** cuando alguien deja de trabajar en su trabajo de forma permanente
rotation matrix a matrix which, when multiplied on the left of a vector, yields a vector that is a rotation of the input vector	**matriz de rotación** matriz que, cuando se la multiplica a izquierda por un vector, se obtiene un vector que es una rotación del original
row an arrangement of data in a horizontal line	**fila** arreglo de datos en línea horizontal
row echelon form the form of a matrix in which the first nonzero entry in each row of a matrix has only zeros below it	**escalón de fila forma** la forma de una matriz en la que la primera entrada no nula en cada fila de una matriz tiene sólo ceros por debajo de ella
royalty a payment made to the creator or owner of a patent, text, image, or song for use of the work; may also refer to a payment made by a natural resource company to a property owner for the right to extract and sell the natural resources	**derechos** un pago hecho al creador o propietario de una patente, texto, imagen o canción por el uso de la obra; también puede referirse a un pago realizado por una empresa de recursos naturales a un propietario por el derecho a extraer y vender los recursos naturales
S	
salary a fixed amount that a worker earns regardless of the number of hours worked	**salario** una cantidad fija que gana un trabajador independientemente del número de horas trabajadas
sample a subset of the population	**muestra** subconjunto de la población
sample average the sum of all quantities in a sample divided by the total number of quantities in the sample, typically represented by $\overline{x}$; also known as *sample mean*	**promedio de la muestra** suma de todas las cantidades en una muestra dividida por el número total de cantidades en la muestra, normalmente representada por $\overline{x}$; también se conoce como *media de la muestra*
sample mean the sum of all quantities in a sample divided by the total number of quantities in the sample, typically represented by $\overline{x}$; also known as *sample average*	**media de la muestra** suma de todas las cantidades en una muestra dividida por el número total de cantidades en la muestra, normalmente representada por $\overline{x}$; también se conoce como *promedio de la muestra*

English	Español
sample population a portion of the population; the number of elements or observations in a sample population is represented by *n*	**población de la muestra** porción de la población; la cantidad de elementos u observaciones en una población de muestra se representa por *n*
sample proportion the fraction of favorable results *p* from a sample population *n*; conventionally represented by $\hat{p}$, which is pronounced "p hat." The formula for the sample proportion is $\hat{p} = \frac{p}{n}$, where *p* is the number of favorable outcomes and *n* is the number of elements or observations in the sample population.	**proporción de la muestra** fracción de los resultados favorables *p* de una población de muestra *n*; convencionalmente representada por $\hat{p}$, que se pronuncia "p hat". La fórmula para la proporción de la muestra es $\hat{p} = \frac{p}{n}$, donde *p* es la cantidad de resultados favorables y *n* es la cantidad de elementos u observaciones en la población de la muestra.
sample size the number of members of a population that participate in a survey	**tamaño de la muestra** el número de miembros de una población que participan en una encuesta
sample space the set of all possible outcomes of an experiment	**espacio de muestreo** conjunto de todos los resultados posibles de un experimento
sample survey a survey carried out using a sampling method so that only a portion of the population is surveyed rather than the whole population	**encuesta de muestra** encuesta realizada utilizando un método de muestreo para encuestar solo una porción de la población en lugar de toda la población.
sampling bias errors in estimation caused by flawed (non-representative) sample selection	**sesgo de muestreo** errores de cálculo ocasionados por una selección defectuosa (no representativa) de la muestra
sampling error a measure showing how precisely a sample reflects the population, with smaller sampling errors resulting from large samples and/or when the data clusters closely around the mean; also called *chance variation*	**error de muestreo** medición que demuestra qué tan precisamente refleja una muestra a una población, con pequeños errores de muestreo ocasionados por muestras grandes y/o cuando los datos se agrupan estrechamente alrededor de la media; también se llama *variación aleatoria*

English	Español
savings account a bank account similar to a checking account, except it accrues interest and there may be a cap on the number of withdrawals you can make in a month	**cuenta de ahorros** una cuenta bancaria similar a una cuenta corriente, excepto que devenga intereses y puede haber un límite en la cantidad de retiros que puede hacer en un mes
scalar a quantity, usually a constant; a numerical quantity without an associated direction	**escalar** una cantidad, por lo general una constante; cantidad numérica sin dirección asociada
scale the numbers representing the interval of a variable and the increments into which it is subdivided; usually includes the interval endpoints and the increments of the basic unit of the variable	**escala** números que representan el intervalo de una variable y los incrementos en los cuales esta se subdivide; generalmente incluye los extremos del intervalo y los incrementos de la unidad básica de la variable
scatter plot a graph of data in two variables on a coordinate plane, where each data pair is represented by a point	**diagrama de dispersión** gráfica de datos en dos variables en un plano de coordenadas, en la que cada par de datos está representado por un punto
sequence an ordered list of numbers or elements	**secuencia** lista ordenada de números o elementos
series the sum of the terms of a sequence	**serie** suma de los términos de una secuencia
set a collection or list of items	**conjunto** colección o lista de elementos
share a unit of ownership of a business	**acción** una unidad de propiedad de una empresa
shareholder an owner of shares in a company	**accionista** un propietario de acciones en una empresa
***sigma* (lowercase), σ** a Greek letter used to represent standard deviation	***sigma* (minúscula) o σ** letra griega utilizada para representar la desviación estándar
***sigma* (uppercase), Σ** a Greek letter used to represent the summation of values	***sigma* (mayúscula) o Σ** letra griega utilizada para representar la sumatoria de valores
simple event an event that has only one outcome; sometimes called a single event	**evento simple** evento que sólo tiene un resultado; a veces se denomina evento único

English	Español
simple interest interest calculated only on the principal regardless of the interest earned so far. One simple interest formula is represented $A = P(1 + rt)$, where P is the principal amount, r is the percent rate per period, t represents the number of time periods, and A represents the maturity of a loan or savings account.	**interés simple** interés calculado solo sobre el capital independientemente del interés devengado hasta el momento. Una fórmula de interés simple se representa $A = P(1 + rt)$, donde P es el monto capital, r es la tasa de porcentaje por período, t representa el número de períodos de tiempo y A representa el vencimiento de un préstamo o cuenta de ahorros.
simple moving average (SMA) the average price per share of a stock over a number of days	**media móvil simple** el precio medio por acción de una acción durante varios días
simple random sample a sample in which any combination of a given number of individuals in the population has an equal chance of selection	**muestra aleatoria simple** muestra en la cual cualquier combinación de una cantidad dada de individuos de la población tiene iguales posibilidades de selección
simulation a set of data that models an event that could happen in real life	**simulación** conjunto de datos que imita un evento que podría suceder en la vida real
sine a trigonometric function of an acute angle in a right triangle that is the ratio of the length of the opposite side to the length of the hypotenuse; the sine of $\theta = \sin\theta = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$	**seno** función trigonométrica de un ángulo agudo en un triángulo rectángulo que es la proporción de la longitud del lado opuesto a la longitud de la hipotenusa; sen de $\theta = \text{sen}\,\theta = \frac{\text{longitud del lado opuesto}}{\text{longitud de la hipotenusa}}$
sine curve a curve with a constant amplitude and period, which are given by a sine or cosine function; also called a *sine wave* or *sinusoid*	**curva del seno** curva con amplitud y período constantes que están dados por una función seno o coseno; también se denomina *onda de seno* o *sinusoide*

English

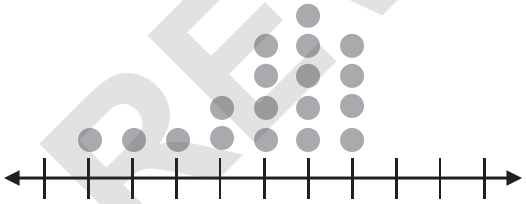
sine function a trigonometric function of the form $f(x) = a \sin [b(x - c)] + d$, in which a, b, c, and d are constants and x is a variable defined in radians over the domain $(-\infty,\infty)$

sine wave a curve with a constant amplitude and period given by a sine or cosine function; also called a *sine curve* or *sinusoid*

sinusoid a curve with a constant amplitude and period given by a sine or cosine function; also called a *sine curve* or *sine wave*

skew to distort or bias, as in data

skewed to the left a distribution in which there is a "tail" of isolated, spread-out data points to the left of the median. "Tail" describes the visual appearance of the data points in a histogram. Data that is skewed to the left is also called *negatively skewed*. Example:



Español

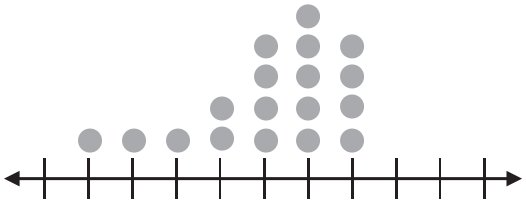
función seno función trigonométrica de la forma $f(x) = a \sin [b(x - c)] + d$, en la cual a, b, c y d son constantes y x es una variable expresada en radianes sobre el dominio $(-\infty,\infty)$

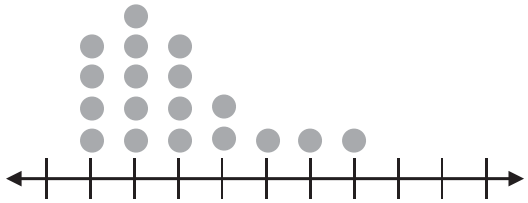
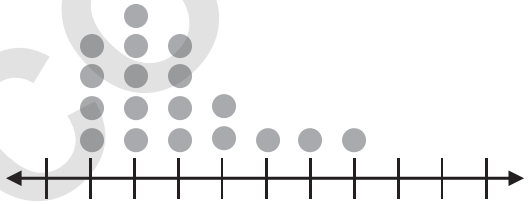
onda senoidal curva con amplitud y período constantes dados por una función seno o coseno; también se denomina *curva del seno* o *sinusoide*

sinusoide curva con amplitud o período constantes dados por una función seno o coseno; también se denomina *curva del seno* u *onda senoidal*

sesgar distorsionar o afectar, como en el caso de los datos

sesgado a la izquierda distribución en la cual existe una "cola" de puntos de datos aislados extendidos hacia la izquierda de la mediana. La "cola" describe la apariencia de los puntos de datos en un histograma. Los datos sesgados a la izquierda también se denominan *negativamente sesgados*. Ejemplo:



English	Español
skewed to the right a distribution in which there is a "tail" of isolated, spread-out data points to the right of the median. "Tail" describes the visual appearance of the data points in a histogram. Data that is skewed to the right is also called *positively skewed*. Example: 	**sesgado a la derecha** distribución en la cual existe una "cola" de puntos de datos aislados extendidos hacia la derecha de la mediana. La "cola" describe la apariencia de los puntos de datos en un histograma. Los datos sesgados a la derecha también se denominan *positivamente sesgados*. Ejemplo: 
slope the measure of the rate of change of one variable with respect to another variable; slope = $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$; the slope in the equation $y = mx + b$ is m	**pendiente** medida de la tasa de cambio de una variable con respecto a otra; pendiente = $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$; la pendiente en la ecuación $y = mx + b$ es m
Social Security tax the portion of your gross income that goes into the Social Security program, equal to 6.2% of gross pay	**impuesto de Seguro Social** la parte de su ingreso bruto que se destina al programa de Seguro Social, equivalente al 6.2% del salario bruto
sole proprietorship a person who owns a business and is personally responsible for its debts	**propietario único** una persona que posee un negocio y es personalmente responsable de sus deudas
solution a value that makes an equation true	**solución** valor que hace verdadera la ecuación
solution set the value or values that make a sentence or statement true; the set of ordered pairs that represent all of the solutions to an equation or a system of equations	**conjunto de soluciones** valor o valores que hacen verdadera una afirmación o declaración; conjunto de pares ordenados que representa todas las soluciones para una ecuación o sistema de ecuaciones

English	Español
speed the magnitude of an object's velocity vector	**velocidad** módulo del vector velocidad de un objeto
spread refers to how data is spread out with respect to the mean; sometimes called *variability*	**dispersión** forma en que los datos se esparcen con respecto a la media; algunas veces se denomina *variabilidad*
square matrix a matrix with the same number of rows and columns	**matriz cuadrada** matriz con la misma cantidad de filas y columnas
standard deviation how much the data in a given set is spread out, represented by s or σ. The standard deviation of a sample can be found using the following formula: $s=\sqrt{\frac{\sum\left(x_i-\overline{x}\right)^2}{n-1}}$. The standard deviation of a population can be found using the following formula: $\sigma=\sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}}$.	**desviación estándar** cuánto se extienden los datos en un conjunto dado, representada por s o σ. Se puede calcular la desviación estándar de una muestra utilizando la siguiente fórmula: $s=\sqrt{\frac{\sum\left(x_i-\overline{x}\right)^2}{n-1}}$. Se puede calcular la desviación estándar de una población utilizando la siguiente fórmula: $\sigma=\sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}}$.
standard error of the mean the variability of the mean of a sample; given by $\text{SEM}=\frac{s}{\sqrt{n}}$, where s represents the standard deviation and n is the number of elements or observations in the sample population	**error estándar de la media** variabilidad de la media de una muestra; dado por $\text{SEM}=\frac{s}{\sqrt{n}}$, donde s representa la desviación estándar y n la cantidad de elementos u observaciones en la población de la muestra

English	Español
standard error of the proportion the variability of the measure of the proportion of a sample, abbreviated SEP. The standard error (*SEP*) of a sample proportion $\hat{p}$ is given by the formula $\text{SEP} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, where $\hat{p}$ is the sample proportion determined by the sample and n is the number of elements or observations in the sample population.	**error estándar de la proporción** variabilidad de la medida de la proporción de una muestra, abreviada SEP. El error estándar (*SEP*) de una proporción de la muestra $\hat{p}$ está dado por la fórmula $\text{SEP} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, donde $\hat{p}$ es la proporción de la muestra determinada por la muestra y n representa la cantidad de elementos u observaciones en la población de la muestra.
standard normal distribution a normal distribution that has a mean of 0 and a standard deviation of 1; data following a standard normal distribution forms a normal curve when graphed	**distribución normal estándar** distribución normal que tiene una media de 0 y una desviación estándar de 1; los datos que siguen una distribución normal estándar forman una curva normal al graficarse
standard units a widely accepted unit of measurement. Standard units are usually defined by law.	**unidades estándar** una unidad de medida ampliamente aceptada. Normalmente, las unidades estándar se definen por ley.
state income tax percentage of your gross income that goes to your state's government for general purposes	**impuestos estatales** porcentaje de su ingreso bruto que va al gobierno de su estado para propósitos generales
statistical significance a measure used to determine whether the outcome of an experiment is a result of the treatment being applied, as opposed to random chance	**relevancia estadística** medida utilizada para determinar si el resultado de un experimento es el resultado del tratamiento aplicado, en oposición al resultado producto del azar

English	Español
statistics a branch of mathematics focusing on how to collect, organize, analyze, and interpret information from data gathered; numbers used to summarize, describe, or represent sets of data	**estadística** rama de la matemática enfocada en la manera de recabar, organizar, analizar e interpretar la información proveniente de los datos reunidos; números utilizados para resumir, describir o representar conjuntos de datos
step function a function that is a series of disconnected constant functions	**función escalonada** función que es una serie de funciones constantes desconectadas
stock the shares in a business; in other words, the stock in a business is divided into individual shares	**acción** las acciones de una empresa; en otras palabras, las acciones de una empresa se dividen en acciones individuales
stock market the buying and selling of shares, usually through public exchanges	**bolsa de valores** la compra y venta de acciones, generalmente a través de bolsas públicas
stockbroker a person who buys or sells an investment for you in exchange for a fee	**corredora de bolsa** una persona que compra o vende una inversión por usted a cambio de un arancel
stratified sample a sample chosen by first dividing a population into subgroups of people or objects that share relevant characteristics, then randomly selecting members of each subgroup for the sample	**muestra estratificada** muestra escogida dividiendo primero una población en subgrupos de personas u objetos que comparten características relevantes, luego seleccionando al azar miembros de cada subgrupo para la muestra
subset a set whose elements are in another set. Set A is a subset of set B, denoted by $A \subset B$, if all the elements of A are also in B.	**subconjunto** conjunto cuyos elementos están en otro conjunto. El conjunto A es un subconjunto del conjunto B, indicado por $A \subset B$, si todos los elementos de A se encuentran también en B.
success the data sought or hoped for, represented by p; also known as *desirable outcome* or *favorable outcome*	**éxito** datos buscados o esperados, representados por p; también conocido como *resultado deseado* o *resultado favorable*

English

sum formula for a finite geometric series $S_n = \frac{a_1(1-r^n)}{1-r}$, where S_n is the sum, a_1 is the first term, r is the common ratio, and n is the number of terms

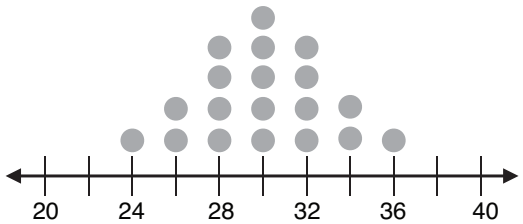
sum formula for an infinite geometric series $S_n = \frac{a_1}{1-r}$, where S_n is the sum, a_1 is the first term, and r is the common ratio

summary statistics information that provides a quick description of a data set as a whole; can include the mean, median, mode, range, expected value, and standard deviation

summation notation a symbolic way to represent a series (the sum of a sequence) using the uppercase Greek letter *sigma*, Σ

survey a study of particular qualities or attributes of items or people of interest to a researcher

symmetric situation in which data is concentrated toward the middle of the range of data; data values are distributed in the same way above and below the middle of the sample. Example:



Español

fórmula de suma para una serie geométrica finita $S_n = \frac{a_1(1-r^n)}{1-r}$ donde S_n es la suma, a_1 es el primer término, r es la relación común y n es la cantidad de términos

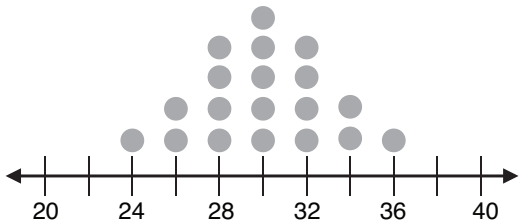
fórmula de suma para una serie geométrica infinita $S_n = \frac{a_1}{1-r}$, donde S_n es la suma, a_1 es el primer término y r es la relación común

resumen estadístico información que describe brevemente un conjunto de datos como un todo; puede incluir la media, la mediana, la moda, el rango, el valor esperado y la desviación estándar

notación sumatoria forma simbólica de representar una serie (la suma de una secuencia) utilizando la letra griega mayúscula *sigma*, Σ

encuesta estudio de las cualidades o atributos particulares de elementos o personas de interés para un investigador

simétrico situación en la que los datos se concentran hacia el medio del rango de datos; los valores de datos se distribuyen de la misma manera por encima y por debajo del medio de la muestra. Ejemplo:



English	Español
symmetric distribution a data distribution in which a line can be drawn so that the left and right sides are mirror images of each other	**distribución simétrica** distribución de datos en la cual se puede trazar una línea de manera que los lados derecho e izquierdo sean imágenes especulares entre sí
system of equations a set of equations with the same unknowns	**sistema de ecuaciones** conjunto de ecuaciones con las mismas incógnitas
system of inequalities a set of two or more inequalities with the same unknowns	**sistema de desigualdades** un conjunto de dos o más desigualdades con las mismas incógnitas
system of measurement a collection of units of measurement, with rules relating the measurements to each other. The metric system, or SI, is an example of a system of measurement.	**sistema de medida** una colección de unidades de medida, con reglas que relacionan las mediciones entre sí. El sistema métrico, o SI, es un ejemplo de un sistema de medición.
systematic sample a sample drawn by selecting people or objects from a list, chart, or grouping at a uniform interval; for example, selecting every fourth person	**muestra sistemática** la muestra se obtiene mediante la selección de personas u objetos a partir de una lista, una tabla o mediante la agrupación a intervalos regulares; por ej., eligiendo una de cada cuatro personas
T	
***t*-test** a procedure to establish the statistical significance of a set of data using the mean, standard deviation, and degrees of freedom for the sample or population	**prueba *t*** procedimiento para establecer la relevancia estadística de un conjunto de datos utilizando la media, la desviación estándar y los grados de libertad para la muestra o población
***t*-value** the result of a *t*-test	**valor *t*** resultado de una prueba *t*
tax bracket a range of different income levels taxed at a specific rate. In the United States, your income is taxed at the lowest rate for some amount, then the next lowest rate for the next amount, and so on.	**tramo impositivo** una gama de diferentes niveles de ingresos gravados a una tasa específica. En los Estados Unidos, sus ingresos se gravan a la tasa más baja para cierta cantidad, luego a la siguiente tasa más baja para la siguiente cantidad, y así sucesivamente.

English

term 1. a number, a variable, or the product of a number and variable(s) 2. an element in a sequence. In the sequence $\{a_1, a_2, a_3, ..., a_n\}$, a_1 is the first term, a_2 is the second term, a_3 is the third term, and a_n is the nth term; a number, a variable, or the product of a number and variable(s)

terminal point the point at which a vector ends; the "head" of a vector

theoretical probability the probability that an outcome will occur as determined through reasoning or calculation, given by the formula $P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$

third quartile value that identifies the upper 25% of the data; the median of the upper half of the data set; 75% of all data is less than this value; written as Q_3

time card a way for employees to keep track of their hours. Many employers now use electronic time cards.

transformation matrix a matrix with special properties that can be used to rotate, reflect, or dilate a vector

translation moving a graph or figure either vertically, horizontally, or both, without changing its shape; a slide

treatment the process or intervention provided to the population being observed

Español

término 1. número, variable, o producto de un número y una o más variables 2. elemento de una secuencia. En la secuencia $\{a_1, a_2, a_3, ..., a_n\}$, a_1 es el primer término, a_2 es el segundo término, a_3 es el tercer término y a_n es el término n; un número, una variable o el producto de un número y variable(s)

extremo final punto en el cual termina un vector; es el extremo "con la punta de flecha"

probabilidad teórica probabilidad de que un resultado se produzca como se determinó mediante razonamiento o cálculo, dado por la fórmula $P(E) = \frac{\text{cantidad de resultados en } E}{\text{cantidad de resultados en el espacio de muestreo}}$

tercer cuartil valor que identifica el 25% superior de los datos; mediana de la mitad superior del conjunto de datos; el 75% de los datos es menor que este valor; se expresa como Q_3

tarjeta de tiempo una forma para que los empleados realicen un seguimiento de sus horas. Muchos empleadores ahora usan tarjetas de tiempo electrónicas.

matriz de transformación matriz con propiedades especiales que se puede usar para rotar un vector, reflejarlo o cambiar su escala

traslación movimiento de un gráfico en sentido vertical, horizontal, o en ambos, sin modificar su forma; deslizamiento

tratamiento proceso o intervención efectuada sobre la población que está siendo observada

English	Español
trend a pattern of behavior, usually observed over time or over multiple iterations	**tendencia** patrón de comportamiento, que se observa por lo general en el tiempo o en múltiples repeticiones
trial each individual event or selection in an experiment or treatment	**ensayo** cada evento o selección individual en un experimento o tratamiento
true negative result a determination that an experiment has produced a correct negative result	**resultado negativo verdadero** determinación de que un experimento ha producido un resultado negativo correcto
true positive result a determination that an experiment has produced a correct positive result	**resultado positivo verdadero** determinación de que un experimento ha producido un resultado positivo correcto
two-tailed test a *t*-test performed on a set of data to determine if the data could belong in either of the tails of the bell-shaped distribution curve; with this test, the area under both tails of the distribution is considered	**prueba de dos colas o prueba bilateral** prueba *t* realizada sobre un conjunto de datos para determinar si esos datos podrían pertenecer a alguna de las colas de una curva de distribución en forma de campana; con esta prueba, se tiene en cuenta el área bajo ambas colas de la distribución
two-way frequency table a frequency table that shows two categories of characteristics, one in rows and the other in columns. Each cell value is a frequency that shows how many times two different characteristics appear together, or how often characteristics are associated with a person, object, or type of item that is being studied.	**tabla de frecuencia de dos vías** tabla de frecuencia que muestra dos categorías de características, una en filas y la otra en columnas. Cada valor de celda es una frecuencia que demuestra cuántas veces dos características diferentes aparecen juntas, o con qué frecuencia las características se asocian con una persona, objeto, o tipo de elemento que se está analizando.

English	Español
U	
undesirable outcome the data not sought or hoped for, represented by *q*; also known as *unfavorable outcome* or *failure*	**resultado no deseado** datos no buscados o esperados, representados por *q*; también conocido como *resultado desfavorable* o *fracaso*
unfavorable outcome the data not sought or hoped for, represented by *q*; also known as *undesirable outcome* or *failure*	**resultado desfavorable** datos no buscados o esperados, representados por *q*; también conocido como *resultado no deseado* o *fracaso*
uniform distribution a set of values that are continuous, are symmetric to a mean, and have equal frequencies corresponding to any two equally sized intervals. In other words, the values are spread out uniformly throughout the distribution.	**distribución uniforme** conjunto de valores que son continuos, simétricos respecto de la media y tienen frecuencias iguales que corresponden a cualquiera de dos intervalos del mismo tamaño. En otras palabras, los valores se extienden uniformemente en la distribución.
uniform probability distribution a probability distribution in which the possible outcomes of a statistical experiment occur with equal probability	**distribución de probabilidad uniforme** distribución de probabilidad en la cual los resultados posibles de un experimento estadístico ocurren con igual probabilidad
uniform probability model a probability model in which all the outcomes of an experiment are assumed to be equally likely	**modelo de probabilidad uniforme** modelo de probabilidad en el que se presume que todos los resultados de un experimento son igualmente probables
union a set whose elements are in at least one of two other sets. The union of sets *A* and *B*, denoted by $A \cup B$, is the set of elements that are in either *A* or *B* or both *A* and *B*.	**unión** conjunto cuyos elementos están al menos en uno de otros dos conjuntos. La unión de los conjuntos *A* y *B*, indicada por $A \cup B$, es el conjunto de elementos que están en *A* o en *B*, o a la vez en *A* y *B*.
unit circle a circle with a radius of 1 unit. The center of the circle is located at the origin of the coordinate plane.	**círculo unitario** círculo con un radio de una unidad. El centro del círculo está ubicado en el origen del plano de coordenadas.

English	Español
unit of measurement a defined quantity of the subject being measured. For example, the current formal definition of a meter is "the length of the path traveled by light in a vacuum during a time interval of $\frac{1}{299{,}792{,}458}$ of a second."	**unidad de medida** una cantidad definida del sujeto que se está midiendo. Por ejemplo, la definición formal actual de un metro es "la longitud del camino recorrido por la luz en un vacío durante un intervalo de tiempo de $\frac{1}{299{,}792{,}458}$ de un segundo".
unit rate a ratio of two measurements, the second of which is 1	**tasa unitaria** una proporción de dos medidas, de las que la segunda es 1
unit vector a vector with magnitude 1. Given any vector $\vec{v}$, the scalar multiple $\frac{\vec{v}}{\|\mathbf{v}\|}$ is the unique unit vector that points in the same direction as $\vec{v}$.	**vector unitario** vector de módulo 1. Dado cualquier vector $\vec{v}$, el múltiplo escalar $\frac{\vec{v}}{\|\mathbf{v}\|}$ es el único vector unitario que apunta en la misma dirección que $\vec{v}$.
universal set a set of all elements that are being considered in a particular situation. In a probability experiment, the universal set is the sample space.	**conjunto universal** conjunto de todos los elementos que se consideran en una situación particular. En un experimento de probabilidad, el conjunto universal es el espacio de muestreo.
V	
validity the degree to which the results obtained from a sample measure what they are intended to measure	**validez** el grado en el cual los resultados obtenidos de una muestra miden lo que se pretende que midan
variability refers to how data is spread out with respect to the mean; sometimes called *spread*	**variabilidad** hace referencia al modo en que se distribuyen los datos respecto de la media; algunas veces se denomina *dispersión*
variable a letter used to represent an unknown value or a value that changes	**variable** una letra utilizada para representar un valor desconocido o un valor que cambia

English	Español
vector a quantity having both direction and magnitude	**vector** cantidad que tiene tanto dirección como módulo
velocity vector a vector that represents the motion of an object	**vector velocidad** vector que representa el movimiento de un objeto
Venn diagram a diagram that shows how two or more sets in a universal set are related	**diagrama de Venn** diagrama que muestra cómo se relacionan dos o más conjuntos en un conjunto universal
volatile market the term for when many or most of the stocks traded on a stock exchange show large amounts of variation in their share price	**mercado volátil** el término para cuando muchas o la mayoría de las acciones negociadas en una bolsa de valores muestran grandes variaciones en el precio de sus acciones
volatility when stock prices undergo large and/or frequent changes	**volatilidad** cuando los precios de las acciones sufren cambios importantes y / o frecuentes
voluntary response bias bias that occurs when the sample is not representative of the population due to the sample having the option of responding to the survey	**sesgo de respuesta voluntaria** sesgo que se produce cuando la muestra no es representativa de la población debido a que en la muestra existe la opción de responder a la encuesta (la respuesta es optativa)
W	
weighted average a type of arithmetic mean in which some elements carry more importance (weight) than others. This number is calculated by multiplying each element by a number that represents the element's relative importance or weight.	**promedio ponderado** tipo de media aritmética en la cual a algunos elementos se les asigna más importancia (peso) que a otros. Para calcular este valor, se multiplica cada elemento por un número que representa la importancia relativa o peso del elemento.
withdrawal taking money out of an account	**retiro** sacar dinero de una cuenta

English	Español
X	
***x*-intercept** the *x*-coordinate of the point where a line or a curve intersects the *x*-axis	**intersección *x*** la coordenada *x* del punto en que una recta o curva corta el eje *x*
Y	
***y*-intercept** the *y*-coordinate of the point where a line or a curve intersects the *y*-axis	**intersección *y*** la coordenada *y* del punto en que una recta o curva corta el eje *y*
Z	
***z*-score** the number of standard deviations that a score lies above or below the mean; given by the formula $z=\frac{x-\mu}{\sigma}$	**puntuación *z*** cantidad de desviaciones estándar por encima o por debajo de la media que presenta la muestra; dada por la fórmula $z=\frac{x-\mu}{\sigma}$
z_c-value a measure of the number of standards of error to be added or subtracted from the mean in order to achieve the desired confidence level; also known as *critical value*	**valor-z_c** medida de la cantidad de estándares de error a sumar o restar de la media para alcanzar el nivel de confianza deseado; conocido también como *valor crítico*
zero matrix a matrix for which all entries are 0; the matrix whose addition does not change other matrices. Also known as the *additive identity matrix*.	**matriz cero** es una matriz cuyas entradas son todas 0; es la matriz cuya sumatoria no cambia a las demás matrices. También se la conoce como *matriz identidad de la suma*.