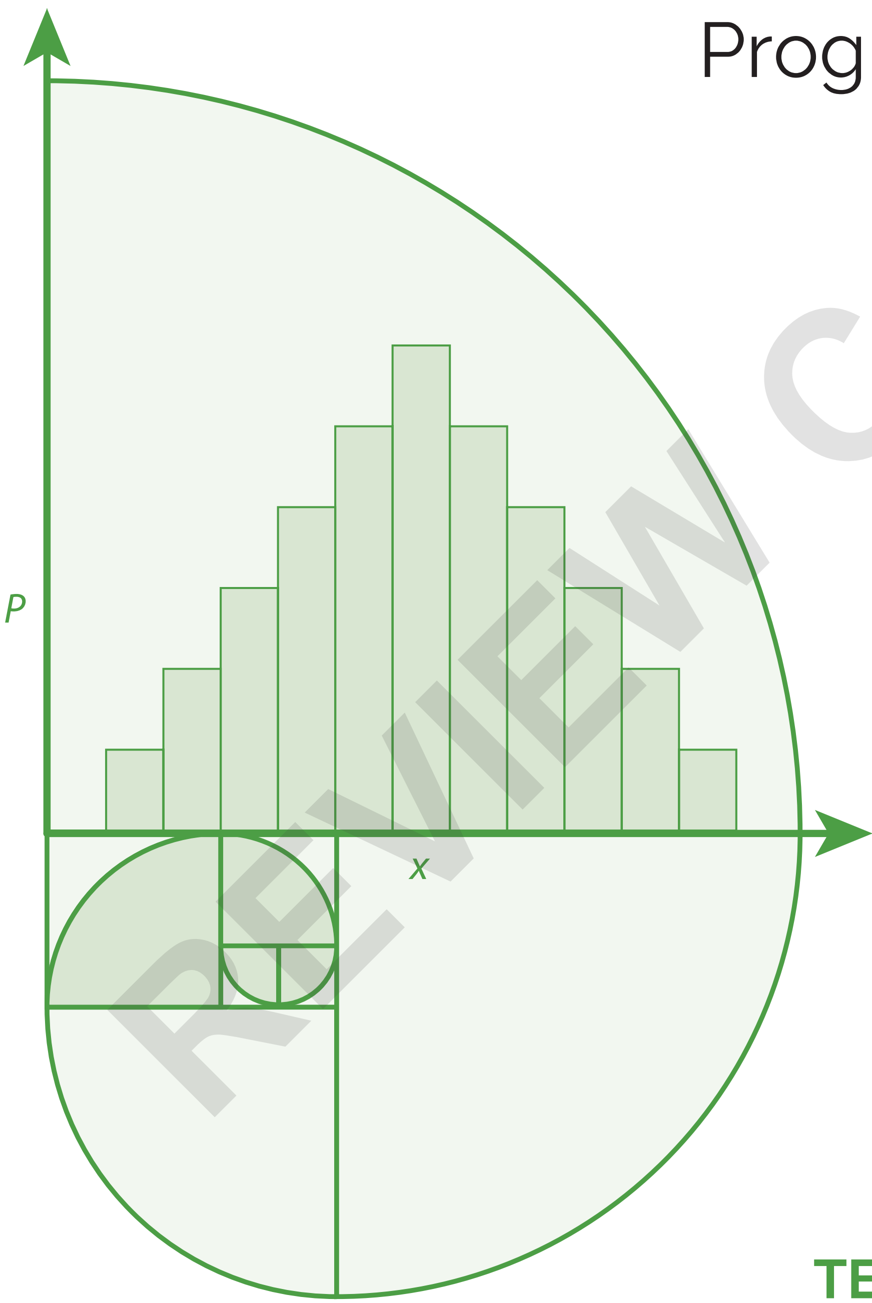
South Carolina CCR Mathematics

Algebra 1

Program Overview



TEACHER RESOURCE

BW WALCH

This program was developed and reviewed by experienced math educators who have both academic and professional backgrounds in mathematics. This ensures: freedom from mathematical errors, grade level appropriateness, freedom from bias, and freedom from unnecessary language complexity.

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Introduction to the Program

Introduction

The *South Carolina CCR Mathematics Standards: Algebra 1* program is a complete set of materials developed around the South Carolina College and Career-Ready (SC CCR) Mathematics Standards, the overview of the for the South Carolina College and Career-Ready Mathematics Standards, and the Mathematics I content map found in Appendix A of the South Carolina College and Career-Ready Mathematics Standards. Topics are built around accessible core curricula, ensuring that the South Carolina College and Career-Ready Mathematics Standards Algebra I Program is useful for striving students and diverse classrooms.

This program realizes the benefits of exploratory and investigative learning and employs a variety of instructional models to meet the learning needs of students with a range of abilities.

The *South Carolina CCR Mathematics Standards: Algebra 1* program includes components that support problem-based learning, instruct and coach as needed, provide practice, and assess students' skills. Instructional tools and strategies are embedded throughout.

The program includes:

- More than 150 hours of lessons addressing the SC CCR Mathematics Standards
- Essential Questions for each instructional topic
- Vocabulary
- Instruction and Guided Practice
- Problem-based Tasks and Coaching questions
- Step-by-step graphing calculator instructions for the TI-Nspire and the TI-83/84
- Station activities to promote collaborative learning and problem-solving skills

Purpose of Materials

The *South Carolina CCR Mathematics Standards: Algebra 1* program has been organized to coordinate with the SC CCR Math Standards Algebra I content map and specifications from the SC CCR Mathematics Standards.

Each topic includes activities that offer opportunities for exploration and investigation. These activities incorporate concept and skill development and guided practice, then move on to the application of new skills and concepts in problem-solving situations. Throughout the lessons and activities, problems are contextualized to enhance rigor and relevance.

This program includes all the topics addressed in the South Carolina Algebra 1 content map. These include:

- Relationships Between Quantities
- Linear and Exponential Relationships
- Reasoning with Equations
- Descriptive Statistics
- Extending the Number System
- Quadratic Functions and Modeling
- Expressions and Equations

The five Mathematical Process Standards are infused throughout:

- **MPS.PS.1:** Make sense of problems and persevere in solving them strategically.
- **MPS.RC.1:** Explain ideas using precise and contextually appropriate mathematical language, tools, and models.
- **MPS.C.1:** Demonstrate a deep and flexible conceptual understanding of mathematical ideas, operations, and relationships while making real-world connections.
- **MPS.AJ.1:** |Use critical thinking skills to reason both abstractly and quantitatively.
- **MPS.SP.1:** Identify and apply regularity in repeated reasoning to make generalizations.

Structure of the Teacher Resource

The *South Carolina Algebra 1* program is completely reproducible. Online materials can be provided in your Learning Management System (such as Canvas or Schoolology) or in BW Walch's proprietary course management platform, the Curriculum Engine. The nested folder organization in the Curriculum Engine allows you to access the materials quickly and easily. The digital format also facilitates printing and copying student pages and/or making assignments online.

The Program Overview is the first section. This section helps you to navigate the materials, offers a collection of research-based Instructional Strategies along with their literacy connections and implementation suggestions, and shows the correlation between the South Carolina CCR for Mathematics and the South Carolina Algebra 1 course.

The remaining materials focus on content, knowledge, and application of the units in the

South Carolina Algebra 1 curriculum: Relationships Between Quantities; Linear and Exponential Relationships; Reasoning with Equations; Descriptive Statistics; Extending the Number System; Quadratic Functions and Modeling; and Expressions and Equations. These units are designed to be flexible so that you can mix and match activities as the needs of your students and your instructional style dictate.

The Station Activities correspond to the content in the units and provide students with the opportunity to apply concepts and skills, while you have a chance to circulate, observe, speak to individuals and small groups, and informally assess and plan.

Each topic begins with a pre-assessment and ends with a progress assessment. These allow you to assess students' progress as you move from topic to topic, enabling you to gauge how well students have understood the material and to differentiate as appropriate.

Glossary

The Glossary contains vocabulary terms and formulas from throughout the program, organized alphabetically. Each listing provides the term and the definition in both English and Spanish. The listings include the lesson number(s) where the terms can be found in the Words to Know.

Correspondence to NCTM *Principles to Actions* Teaching Practices

How Do BW Walch's High School Math Resources Address the NCTM *Principles to Actions* Mathematics Teaching Practices?

BW Walch's programs for South Carolina's College and Career Ready Mathematics Standards for high school courses were designed by experienced educators and curriculum developers, informed by best-practice research, and refined through an iterative process of implementation and feedback. Together with professional development, these materials support and sustain good teaching practices.

NCTM Mathematics Teaching Practices	Relevant Attributes of BW Walch's High School Math Resources
Establish mathematics goals to focus learning.	Each lesson in BW Walch's programs addresses specified standards which can be used as goals to focus learning. Essential Questions offer further focus.
Implement tasks that promote reasoning and problem solving.	Each lesson in BW Walch's programs is built around a Problem-Based Task (PBT), set in a meaningful real-world context and designed to promote reasoning and problem solving. The courses include dozens of PBTs as well as warm-up and practice problems.
Use and connect mathematical representations.	BW Walch's High School Math programs make frequent use of, and connections among and between, equations, tables, and graphs. PBTs often require students to use and connect two or more of these representations, and the representations are modeled through guided practice.
Facilitate meaningful mathematical discourse.	Several features of the programs support mathematical discourse, including warm-up debriefs with connections to the upcoming lesson, implementation guides and optional coaching questions for the PBTs, and discussion guides for Station Activities. Explanations of PBT solutions are another opportunity for discourse. Please note: Mathematical discourse is an important topic for professional development, in conjunction with implementation of these materials.
Pose purposeful questions.	The implementation guides, coaching questions and discussion guides provide samples of purposeful questions. Note that this is another important topic for professional development.
Build procedural fluency from conceptual understanding.	The programs develop conceptual understanding through modeling, guided practice, and application, and then provide additional opportunities to practice and develop fluency.
Support productive struggle in learning mathematics.	The PBTs require "productive struggle;" implementation guides include suggestions for facilitation and monitoring, and coaching questions provide an option for additional support as appropriate, allowing students to proceed through the task and ensuring that the struggle remains productive rather than too frustrating.
Elicit and use evidence of student thinking.	Various discussions and PBTs require students to display their thinking. Implementation guides offer specific prompts and suggestions for eliciting and responding to student thinking. Professional development supports teachers in using that evidence to respond in instructionally appropriate ways.

Unit Structure

All of the instructional units have common features. Each unit begins with a Unit Overview listing all the standards addressed in the topics. The Unit Overview also includes Essential Questions; vocabulary (titled "Words to Know"); and lists of recommended websites and conceptual activities to be used as needed.

Each topic begins with a pre-assessment, lists the specific vocabulary, Essential Questions, and resources for that topic, and ends with a progress assessment to evaluate students' learning.

Each lesson begins with a list of identified prerequisite skills that students need to have mastered in order to be successful with the new material in the upcoming lesson. This is followed by an introduction, key concepts, common errors/misconceptions, scaffolded practice problems, guided practice examples, a problem-based task with coaching questions and sample responses, a closure activity, and practice.

All of the components are described below and on the following pages for your reference.

Standards Correlations

In this section, you'll find a comprehensive list of the SC CCR standard(s) addressed in each lesson.

Pre-Assessment

This can be used to gauge students' prior knowledge and to inform instructional planning.

South Carolina College and Career-Ready Standards for the Topic

All standards that are addressed in the entire topic are listed.

Essential Questions

These are intended to guide students' thinking as they proceed through the topic. By the end of each topic, students should be able to respond to the questions.

Words to Know

Vocabulary terms and formulas are provided as background information for instruction or to review key concepts that are addressed in the topic.

Recommended Resources

This is a list of websites that can be used as additional resources. Some websites are games; others provide additional examples and/or explanations. (*Note*: Links will be monitored and repaired or replaced as necessary.) Each Recommended Resource is also accessible through BW Walch's cloud-based Curriculum Engine Learning Object Repository as a separate learning object that can be assigned to students.

Conceptual Activities

Conceptual understanding serves as the foundation on which to build deeper understanding of mathematics. In an effort to build conceptual understanding of mathematical ideas and to provide more than procedural fluency and application, links to interactive open education and Desmos resources are included. (*Note*: These website links will be monitored and repaired or replaced as necessary.) These and many other open educational resources (OERs) are also accessible through the Learning Object Repository as separate objects that can be assigned to students.

Warm-Up

Each warm-up takes approximately 5 minutes and addresses either prerequisite and critical-thinking skills or previously taught math concepts.

South Carolina College and Career-Ready Standards for the Lesson

When topics are broken down into lessons, the specific standard or standards that are addressed are presented at the beginning of the instructional portion of the lesson.

Warm-Up Debrief

Each debrief provides the answers to the warm-up questions, and offers suggestions for situations in which students might have difficulties. A section titled Connection to the Lesson is also included in the debrief to help answer students' questions about the relevance of the particular warm-up activity to the upcoming instruction. Warm-Ups with debriefs are also provided in PowerPoint presentations.

Identified Prerequisite Skills

This list cites the skills necessary to be successful with the new material.

Introduction

This brief paragraph gives a description of the concepts about to be presented and often contains some Words to Know.

Key Concepts

Provided in bulleted form, this instruction highlights the important ideas and/or processes for meeting the standard.

Graphing Calculator Directions

Step-by-step instructions for using a TI-Nspire and a TI-83/84 are provided whenever graphing calculators are referenced.

Common Errors/Misconceptions

This is a list of the common errors students make when applying Key Concepts. This list suggests what to watch for when students arrive at an incorrect answer or are struggling with solving the problems.

Scaffolded Practice (Printable Practice)

This set of 10 printable practice problems provides introductory level skill practice for the lesson. This practice set can be used during instruction time.

Guided Practice

This section provides step-by-step examples of applying the Key Concepts. The three to five examples are intended to aid during initial instruction, but are also for individuals needing additional instruction and/or for use during review and test preparation.

Enhanced Instructional PowerPoint (Presentation)

Each lesson includes an instructional PowerPoint presentation with the following components: Warm-Up, Key Concepts, and Guided Practice. Selected Guided Practice examples include links to GeoGebra applets. These instructional PowerPoints are downloadable and editable.

Problem-Based Task

This activity can serve as the centerpiece of a problem-based lesson, or it can be used to walk students through the application of the standard, prior to traditional instruction or at the end of instruction. The task makes use of critical-thinking skills.

Optional Problem-Based Task Coaching Questions with Sample Responses

These questions scaffold the task and guide students to solving the problem(s) presented in the task. They should be used at the discretion of the teacher for students requiring additional support. The Coaching Questions are followed by answers and suggested appropriate responses to the coaching questions. In some cases answers may vary, but a sample answer is given for each question.

Recommended Closure Activity

Students are given the opportunity to synthesize and reflect on the lesson through a journal entry or discussion of one or more of the Essential Questions.

Problem-Based Task Implementation Guide

This instructional overview, found with selected Problem-Based Tasks in each unit, highlights connections between the task and the lesson's key concepts and Mathematical Process Standards. The Implementation Guide also offers suggestions for facilitating and monitoring, and provides alternative solutions.

Printable Practice (Sets A and B) and Interactive Practice (Set A)

Each lesson includes two sets of practice problems to support students' achievement of the learning objectives. They can be used in any combination of teacher-led instruction, cooperative learning, or independent application of knowledge. Each Practice A is also available as an interactive Learnosity activity with Technology-Enhanced Items.

Progress Assessment

Each lesson ends with 10 multiple-choice questions, as well as one extended-response question that incorporates critical thinking and writing components. This can be used to document the extent to which students grasp the concepts and skills addressed during instruction.

Unit Assessment

Each unit ends with 12 multiple-choice questions and three extended-response questions that incorporate critical thinking and writing components. This can be used to document the extent to which students grasped the concepts and skills of each unit.

Answer Key

Answers for all of the Warm-Ups and practice problems are provided at the end of each unit.

Station Activities

Most units include a collection of station-based activities to provide students with opportunities to practice, reinforce, and apply mathematical skills and concepts. The debriefing discussions after each set of activities provide an important opportunity to help students reflect on their experiences and synthesize their thinking.

Conceptual Tasks

These engaging tasks provide opportunities for students to deepen their understanding and develop their conceptual knowledge of math concepts. These tasks provide multiple entry points and are accessible for ALL learners.

Standards Correlations

Each lesson in this program was written specifically to address the South Carolina College and Career-Ready Standards for Mathematics. Each topic lists the standards covered in all the lessons, and each lesson lists the standards addressed in that particular lesson. In this section, you'll find a comprehensive list mapping the lessons to the SC CCR.

Unit 1: Relationships Between Quantities			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Interpreting Structure in Expressions**		
	1.1	Identifying Terms, Factors, and Coefficients	A1.PAFR.1.1
	1.2	Interpreting Linear and Exponential Expressions	A1.PAFR.1.1 A1.PAFR.1.3
Topic B	**Creating Equations and Inequalities in One Variable**		
	1.3	Creating Linear Equations in One Variable	A1.MGSR.1.1
	1.4	Creating Linear Inequalities in One Variable	A1.PAFR.2.4 A1.PAFR.1.3
	1.5	Creating Exponential Equations	A1.PAFR.1.3
Topic C	**Creating and Graphing Equations in Two Variables**		
	1.6	Creating and Graphing Linear Equations in Two Variables	A1.MGSR.1.1 A1.PAFR.1.3
	1.7	Creating and Graphing Exponential Equations	A1.MGSR.1.1
Topic D	**Representing Constraints**		
	1.8	Representing Constraints	A1.MGSR.1.1 A1.PAFR.2.3
Topic E	**Rearranging Formulas**		
	1.9	Rearranging Formulas	A1.PAFR.1.2

Unit 2: Linear and Exponential Relationships			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Graphs As Solution Sets and Function Notation**		
	2.1	Graphing the Set of All Solutions	A1.PAFR.2.8
	2.2	Intersecting Graphs	A1.PAFR.2.8
	2.3	Domain and Range	A1.PAFR.2.8 A1.PAFR.3.2
	2.4	Function Notation and Evaluating Functions	A1.PAFR.3.1 A1.PAFR.3.2
Topic B	**Solving Linear Inequalities in Two Variables and Systems of Inequalities**		
	2.5	Solving Linear Inequalities in Two Variables	A1.PAFR.2.4
	2.6	Solving Systems of Linear Inequalities	A1.PAFR.2.4 A1.PAFR.2.7
Topic C	**Sequences As Functions**		
	2.7	Sequences As Functions	A1.PAFR.2.6
Topic D	**Interpreting Graphs of Functions**		
	2.8	Identifying Key Features of Linear and Exponential Graphs	A1.PAFR.2.3 A1.PAFR.2.10 A1.PAFR.4.2
	2.9	Average Rate of Change	A1.PAFR.4.2
	2.10	Recognizing Average Rate of Change	A1.PAFR.2.10 A1.PAFR.4.2
Topic E	**Analyzing Linear and Exponential Functions**		
	2.11	Graphing Linear Functions	A1.PAFR.2.1 A1.PAFR.3.4
	2.12	Graphing Exponential Functions	A1.PAFR.2.1 A1.PAFR.3.4 A1.PAFR.2.10
Topic F	**Comparing Functions**		
	2.13	Comparing Linear Functions	A1.PAFR.2.1 A1.PAFR.4.3
	2.14	Comparing Exponential Functions	A1.PAFR.2.1 A1.PAFR.2.10 A1.PAFR.4.3
	2.15	Comparing Linear to Exponential Functions	A1.PAFR.2.1 A1.PAFR.4.3

Topic	Lesson number	Title	Standard(s)
Topic G	**Building Functions**		
	2.16	Building Functions from Context	A1.PAFR.2.3
	2.17	Constructing Functions from Graphs and Tables	A1.PAFR.2.6
Topic H	**Operating on Functions and Transformations**		
	2.18	Operating on Functions	A1.PAFR.4.1
	2.19	Transformations of Linear and Exponential Functions	A1.PAFR.3.4 A1.PAFR.4.1
Topic I	**Arithmetic and Geometric Sequences**		
	2.20	Arithmetic Sequences	A1.PAFR.2.5 A1.PAFR.2.6
	2.21	Geometric Sequences	A1.PAFR.2.5 A1.PAFR.2.6
Topic J	**Interpreting Parameters**		
	2.22	Interpreting Parameters	A1.PAFR.2.3 A1.PAFR.2.10

Unit 3: Reasoning with Equations

Topic	Lesson number	Title	Standard(s)
Topic A	**Solving Equations and Inequalities**		
	3.1	Properties of Equality	A1.PAFR.1.3
	3.2	Solving Linear Equations	A1.PAFR.1.3
	3.3	Solving Linear Inequalities	A1.PAFR.2.4
	3.4	Solving Exponential Equations	A1.PAFR.1.3
Topic B	**Solving Systems of Equations**		
	3.5	Solving Systems of Linear Equations by Substitution and Elimination	A1.PAFR.2.7 A1.PAFR.2.9
	3.6	Solving Systems of Linear Equations by Graphing	A1.PAFR.2.7 A1.PAFR.2.9

Unit 4: Descriptive Statistics			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Working with Two Categorical and Quantitative Variables**		
	4.1	Summarizing Data Using Two-Way Frequency Tables	A1.DPSR.1.1 A1.DPSR.1.2 A1.DPSR.2.1
	4.2	Solving Problems Given Functions Fitted to Data	A1.DPSR.1.3 A1.DPSR.2.1 A1.DPSR.2.3
	4.3	Analyzing Data Sets	A1.DPSR.1.1 A1.DPSR.1.2 A1.DPSR.1.3 A1.DPSR.2.1 A1.DPSR.2.3
	4.4	Fitting Linear Functions to Data	A1.DPSR.1.2 A1.DPSR.1.3 A1.DPSR.2.3
Topic B	**Interpreting Linear Models**		
	4.5	Interpreting Slope and y-intercept	A1.DPSR.1.3 A1.DPSR.2.2
	4.6	Calculating and Interpreting the Correlation Coefficient	A1.DPSR.1.4

Unit 5: Extending the Number System			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Working with the Number System**		
	5.1	Working with Rational Exponents	A1.NR.1.1
	5.2	Rational and Irrational Numbers and Their Properties	A1.NR.1.1 A1.NR.2.1
Topic B	**Operating with Polynomials**		
	5.3	Adding and Subtracting Polynomials	A1.NR.2.1 A1.PAFR.1.4
	5.4	Multiplying Polynomials	A1.NR.2.1 A1.PAFR.1.4

Unit 6: Quadratic Functions and Modeling			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Analyzing Quadratic Functions**		
	6.1	Graphing Quadratic Functions	A1.PAFR.2.1 A1.PAFR.2.3
	6.2	Interpreting Various Forms of Quadratic Functions	A1.PAFR.2.1
Topic B	**Interpreting Quadratic Functions**		
	6.3	Interpreting Key Features of Quadratic Functions	A1.PAFR.2.1 A1.PAFR.2.3 A1.PAFR.3.4
	6.4	Identifying the Domain of a Quadratic Function	A1.PAFR.2.1 A1.PAFR.2.3
	6.5	Identifying the Average Rate of Change	A1.PAFR.2.10 A1.PAFR.4.2
Topic C	**Building Functions**		
	6.6	Building Functions from Context	A1.PAFR.2.3 A1.PAFR.3.3 A1.PAFR.4.2
	6.7	Operating on Functions	A1.PAFR.2.3 A1.PAFR.3.3
Topic D	**Graphing Other Functions**		
	6.8	Square Root and Absolute Value Functions	A1.PAFR.2.1 A1.PAFR.3.4
Topic E	**Transforming Functions**		
	6.9	Replacing $f(x)$ with $f(x) + k$ and $f(x + k)$	A1.PAFR.4.1
	6.10	Replacing $f(x)$ with $k \cdot f(x)$ and $f(k \cdot x)$	A1.PAFR.4.1

Unit 7: Expressions and Equations			
Topic	**Lesson number**	**Title**	**Standard(s)**
Topic A	**Interpreting Structure in Expressions**		
	7.1	Identifying Terms, Factors, and Coefficients	A1.PAFR.2.1
	7.2	Interpreting Complicated Expressions	A1.PAFR.2.1
Topic B	**Creating and Solving Quadratic Equations in One Variable**		
	7.3	Taking the Square Root of Both Sides	A1.PAFR.1.3 A1.PAFR.2.2
	7.4	Factoring Expressions by the Greatest Common Factor	A1.PAFR.1.3 A1.PAFR.2.2
	7.5	Factoring Expressions with $a = 1$	A1.PAFR.1.3 A1.PAFR.2.2
	7.6	Factoring Expressions with $a > 1$	A1.PAFR.1.3 A1.PAFR.2.2
	7.7	Solving Quadratic Equations by Factoring	A1.PAFR.1.3 A1.PAFR.2.2
	7.8	Completing the Square	A1.PAFR.1.3 A1.PAFR.2.2
	7.9	Applying the Quadratic Formula	A1.PAFR.1.3 A1.PAFR.2.2
Topic C	**Creating Quadratic Equations in Two or More Variables**		
	7.10	Creating and Graphing Equations Using Standard Form	A1.PAFR.1.1 A1.PAFR.2.3
	7.11	Creating and Graphing Equations Using the x-intercepts	A1.PAFR.1.1 A1.PAFR.2.3
	7.12	Creating and Graphing Equations Using Vertex Form	A1.PAFR.1.1 A1.PAFR.2.3
	7.13	Rearranging Formulas	A1.PAFR.1.1
Topic D	**Writing Exponential Expressions in Equivalent Forms**		
	7.14	Writing Exponential Expressions in Equivalent Forms	A1.PAFR.2.1

Conceptual Activities

Use these interactive open education and/or Desmos resources to build conceptual understanding of mathematical ideas. (*Note*: Activity links will be monitored and repaired or replaced as necessary.)

Unit 1

- Desmos. "Function Carnival."

  https://www.walch.com/ca/01006

  This activity focuses attention on graphs as expressing relationships between variables. It lays the informal groundwork for the more formal definitions and properties of functions.

- Desmos. "Marbleslides: Lines."

  https://www.walch.com/ca/01008

  Restrict, reposition, and rotate lines at will using slope-intercept form, and describe transformations using words and/or symbols.

- Desmos. "Put the Point on the Line."

  https://www.walch.com/ca/01009

  The focus of this activity is slope. Participants are asked to estimate, calculate, and notice proportionality as they place points on an imaginary line.

- Desmos, Inc. "Polygraph: Linear Inequalities."

  https://www.walch.com/ca/10000

  In this activity, students will engage in vocabulary-rich conversations about linear inequalities. Key vocabulary terms that may appear in student questions include *shading*, *above*, *below*, *boundary*, *solid*, *dotted*, *horizontal*, *vertical*, *slanted*, *axis*, and *quadrant*.

Unit 2

- Desmos. "Avi and Benita's Repair Shop."

  https://www.walch.com/ca/01014

  Compare linear and exponential growth in the context of daily payments. One plan increases by $100 each day, while another grows by doubling the previous day's payment. This activity is appropriate for students who have studied linear functions but may not have an experience with exponential growth.

- Desmos. "Card Sort: Functions."

  https://www.walch.com/ca/01005

  Sort graphs, equations, and contexts according to whether each one represents a function.

- Desmos. "Card Sort: Linear Functions."

  https://www.walch.com/ca/01010

  Notice and use properties of linear functions to make groups of three. Different properties will lead to different groupings by different participants.

- Desmos. "Function Carnival, Part 2."

  https://www.walch.com/ca/01007

  This activity follows up on "Function Carnival" by using the contexts in that activity to develop an understanding of function notation.

- Desmos. "Game, Set, Flat."

  https://www.walch.com/ca/01015

  Develop understanding of the exponential relationship that describes a bouncing tennis ball. Learn to examine successive terms in a sequence to determine if it represents an exponential relationship or not, and how to construct the exponential equation itself.

- Desmos. "Marbleslides: Exponentials."

  https://www.walch.com/ca/01016

  Restrict, reposition, and otherwise transform exponential curves at will by modifying the basic form $y = b^x$, and use precision in describing these transformations using words and/or symbols.

- Desmos. "Match My Line."

  https://www.walch.com/ca/01013

  Work through a series of scaffolded linear graphing challenges to develop proficiency with direct variation, slope-intercept, point-slope, and other linear function forms.

- Desmos. "Polygraph: Exponentials."

  https://www.walch.com/ca/01019

  This Custom Polygraph is designed to spark vocabulary-rich conversations about exponentials, including how they differ from linear functions. Key vocabulary terms that may appear in student questions include *increasing*, *decreasing*, *intercept*, *rate*, *asymptote*, and *curve*.

- Desmos. "What Comes Next?"

  https://www.walch.com/ca/01020

  Predict "what comes next" for linear and exponential functions based first on graphs and then on tables of values, then explore connections between graphs, tables, and equations of linear and exponential functions.

Unit 3

- Desmos. "Card Sort: Linear Systems."

  https://www.walch.com/ca/01000

  In this activity, students practice what they've learned about solving systems of linear equations. The activity begins with a review of the graphical meaning of a solution to a system. Later, students consider which algebraic method is most efficient for solving a given system. Finally, students practice solving equations using substitution and elimination. Prior to beginning this activity, students should have experience solving systems of linear equations graphically and algebraically.

- Desmos. "The Intersection."

  https://www.walch.com/ca/01011

  Predict the point of intersection for a system of two linear equations: first without a grid, then with one. With the grid in play, participants are able to use the slope of the lines (formally or informally) to improve the accuracy of their predictions.

- Desmos. "Solutions to Systems of Linear Equations."

  https://www.walch.com/ca/01001

  This activity will help students understand what it means for a point to be a solution to a system of equations—both graphically and algebraically.

- Desmos. "Systems of Two Linear Equations."

  https://www.walch.com/ca/01002

  This resource gives a progression of written explanations, equations, and graphs to explain what the algebraic or graphical solution to a system of equations represents.

Unit 4

- Desmos. "LEGO Prices."

  https://www.walch.com/ca/01012

  Use the concept of linear regression to predict the cost of a LEGO set with x pieces. (This activity does NOT use the calculator, just the concept. Participants draw the line on the graph, and Desmos calculates the equation.)
- Desmos. "Predicting Movie Ticket Prices."

  https://www.walch.com/ca/01018

  Build a model to describe the relationship between average movie ticket prices and time, then use that model to make predictions about past and future ticket prices. Participants also interpret the parameters of their equation in context.
- Desmos. "Polygraph: Histograms."

  https://www.walch.com/ca/01024

  This activity is designed to spark vocabulary-rich conversations about histograms. Key vocabulary terms that may appear in student questions include *shape*, *center*, *spread*, *roughly symmetric*, *skew right*, *skew left*, *mean*, *median*, *range*, *peak*, *unimodal*, and *bimodal*.

Unit 5

- GeoGebra. "Operations on Polynomials."

  https://www.geogebra.org/m/QEbGNJcE

  Simplify the polynomial and write in standard form (highest to lowest degree). Click the check box to check your answer.
- GeoGebra. "Classifying Rational Numbers."

  https://www.geogebra.org/m/bzJN8J7f

  This applet by Celia Jimenez asks students to classify each of a set of numbers as a whole number, integer, or rational number.
- GeoGebra. "Rational Exponents."

  https://www.geogebra.org/m/bzJN8J7f

  This applet provides practice with simplifying fractional exponents.

Unit 6

- Desmos. "Card Sort: Parabolas."

  http://www.walch.com/ca/01022

  Find the shape of a parabola by using its form to reveal its characteristics. The activity begins with a review of both the characteristics and forms of a parabola, then moves on to determine characteristics of the graph of a parabola given in standard form, vertex form, or intercept form.

- Desmos. "Free-Range Functions."

  http://www.walch.com/ca/01028

  This activity challenges students to strengthen their ideas about the range of quadratic functions.

- Desmos. "Polygraph: Absolute Value."

  http://www.walch.com/ca/01050

  This activity is designed to spark vocabulary-rich conversations about transformations of the absolute value parent function. Key vocabulary terms that may appear in student questions include *translation*, *shift*, *slide*, *dilation*, *stretch*, *horizontal*, *vertical*, and *reflect*.

- Desmos. "Polygraph: Square Root Functions."

  http://www.walch.com/ca/01032

  This activity is designed to spark vocabulary-rich conversations about transforming square root functions. Key vocabulary terms that may appear in student questions include *translation*, *reflection*, *intercept*, and *quadrant*.

Unit 7

- Desmos. "Build a Bigger Field."

  https://www.walch.com/ca/01021

  Use quadratic models to optimize the area of a field for a given perimeter.

- Desmos. "Match My Parabola."

  https://www.walch.com/ca/01017

  A series of graphing challenges builds understanding of quadratic functions in various forms and graphing transformations of quadratic functions.

- Desmos. "Penny Circle."

  https://www.walch.com/ca/01023

  Gather data, build a model, and use that model to answer the question, "How many pennies fit in a large circle?"

Station Activities Guide

Introduction

Each unit includes a collection of station-based activities to provide students with opportunities to practice and apply the mathematical skills and concepts they are learning. You may use these activities in addition to the instructional topics, or, especially if the pre-test or other formative assessment results suggest it, instead of direct instruction in areas where students have the basic concepts but need practice. The debriefing discussions after each set of activities provide an important opportunity to help students reflect on their experiences and synthesize their thinking. Debriefing also provides an additional opportunity for ongoing, informal assessment to guide instructional planning.

Implementation Guide

The following guidelines will help you prepare for and use the activity sets in this section.

Setting Up the Stations

Each activity set consists of four or five stations. Set up each station at a desk, or at several desks pushed together, with enough chairs for a small group of students. Place a card with the number of the station on the desk. Each station should also contain the materials specified in the teacher's notes, and a stack of student activity sheets (one copy per student). Place the required materials (as listed) at each station.

When a group of students arrives at a station, each student should take one of the activity sheets to record the group's work. Although students should work together to develop one set of answers for the entire group, each student should record the answers on his or her own activity sheet. This helps keep students engaged in the activity and gives each student a record of the activity for future reference.

Forming Groups of Students

All activity sets consist of four or five stations. You might divide the class into four or five groups by having students count off from 1 to 4 or 5. If you have a large class and want to have students working in small groups, you might set up two identical sets of stations, labeled A and B. In this way, the class can be divided into eight groups, with each group of students rotating through the "A" stations or "B" stations.

Assigning Roles to Students

Students often work most productively in groups when each student has an assigned role. You may want to assign roles to students when they are assigned to groups and change the roles occasionally. Some possible roles are as follows:

- Reader—reads the steps of the activity aloud
- Facilitator—makes sure that each student in the group has a chance to speak and pose questions; also makes sure that each student agrees on each answer before it is written down
- Materials Manager—handles the materials at the station and makes sure the materials are put back in place at the end of the activity
- Timekeeper—tracks the group's progress to ensure that the activity is completed in the allotted time
- Spokesperson—speaks for the group during the debriefing session after the activities

Timing the Activities

The activities in this section are designed to take approximately 10 minutes per station. Therefore, you might plan on having groups change stations every 10 minutes, with a two-minute interval for moving from one station to the next. It is helpful to give students a "5-minute warning" before it is time to change stations.

Since each activity set consists of four or five stations, the above time frame means that it will take about 50 to 60 minutes for groups to work through all stations.

Guidelines for Students

Before starting the first activity set, you may want to review the following "ground rules" with students. You might also post the rules in the classroom.

- All students in a group should agree on each answer before it is written down. If there is a disagreement within the group, discuss it with one another.
- You can ask your teacher a question only if everyone in the group has the same question.
- If you finish early, work together to write problems of your own that are similar to the ones on the activity sheet.
- Leave the station exactly as you found it. All materials should be in the same place and in the same condition as when you arrived.

Debriefing the Activities

After each group has rotated through every station, bring students together for a brief class discussion. At this time, you might have the groups' spokespersons pose any questions they had about the activities. Before responding, ask if students in other groups encountered the same difficulty or if they have a response to the question. The class discussion is also a good time to reinforce the essential ideas of the activities. The questions that are provided in the teacher's notes for each activity set can serve as a guide to initiating this type of discussion.

You may want to collect the student activity sheets before beginning the class discussion. However, it can be beneficial to collect the sheets afterward so that students can refer to them during the discussion. This also gives students a chance to revisit and refine their work based on the debriefing session. If you run out of time to hold class discussions, you might want to have students journal about their experiences and follow up with a class discussion the next day.

Digital Enhancements Guide

Introduction

With this program, you have access to the following digital components, described here with guidelines and suggestions for implementation.

Digital Instruction PowerPoints (Presentations)

These optional versions of the Warm-Ups, Warm-Up Debriefs, Introductions, Key Concepts, and Guided Practices for each lesson run on PowerPoint. (*Please note*: Computers may render PowerPoint images differently. For best viewing and display, use a PowerPoint Viewer and adjust your settings to optimize images and text.)

Each PowerPoint begins with the lesson's Warm-Up and is followed by the Warm-Up Debrief, which reveals the answers to the Warm-Up questions.

In the notes section of the last Warm-Up slide, you will find the "Connections to the Lesson," which describes concepts students will glean or skills they will need in the upcoming lesson. The "Connections" help transition from the Warm-Up to instruction.

GeoGebra Applets (Interactive Practice Problems)

One or two interactive GeoGebra applets are provided for most lessons. The applets model the mathematics in the Guided Practice examples for these lessons. Links to these applets are also embedded within the Instructional PowerPoints. With an Internet connection, simply click on the "Play" button slide that follows selected examples.

Once you've accessed the GeoGebra applet, please adjust your view to maximize the image. Each applet illustrates the specific problem addressed in the Guided Practice example. The applets allow you to walk through the solution by visually demonstrating the steps, such as defining points and drawing lines. Variable components of the applets (usually fill-in boxes or sliders) allow you to substitute different values in order to explore the mathematics. For example, "What happens to the line when we increase the amount of time?" or "What if we cut the number of students in half?" This experimentation and discussion supports development of conceptual understanding.

GeoGebra for PC/MAC

GeoGebra is not required for using the applets, but can be downloaded for free for further exploration at the following link:

https://www.geogebra.org/download

Curriculum Engine Learning Object Repository

BW Walch's Curriculum Engine comes loaded with thousands of curated learning objects that can be used to build formative and summative assessments, as well as practice worksheets, instructional components, and an item bank. District leaders and teachers can search for items by standard and create assessments or worksheets in minutes using the three-step assessment builder.

For more information about the Curriculum Engine, or for additional support, please contact us at (207) 828-8800 or success@bwwalch.com.

PROGRAM OVERVIEW

SC CCR Mathematical Process Standards Implementation Guide

Introduction

The five Mathematical Process Standards describe features of lesson design, teaching pedagogy, and student actions that will lead to a true conceptual understanding of the mathematics standards. The Walch lessons, practice problems, and Problem-Based Tasks lend themselves to teaching through this framework. When the Walch resources are combined with high-level questioning and engaging teacher decisions in the classroom, it will lead to high-level math instruction and student achievement.

Here is a brief description of the MPSs and how they can be applied in the classroom:

MPS.PS.1: Make sense of problems and persevere in solving them strategically.

Students understand there are multiple entry points that can identify and explain a problem. Using prior knowledge, a variety of methods, and continual self-reflection, students can check for reasonable solutions. Students can monitor progress and confidently change course if necessary to plan a solution pathway. Teacher prompts that can enhance this standard include:

- What do you already know that might help you solve the problem?
- What is the problem asking you to solve?
- What are some different strategies you could use to solve this problem?
- How can you explain your strategy to someone else?
- Compare your answer with a classmate's answer. Who is correct? Why?
- How can you check your solution for reasonableness and accuracy?
- Using the context of the problem, is your solution reasonable?

MPS.RC.1: Explain ideas using precise and contextually appropriate mathematical language, tools, and models.

Students can consider the available and relevant tools that are helpful to explore, model, and deepen their understanding of concepts. They can use precise mathematical language to model, explain, and justify valid solutions. Students can engage in constructive dialogue individually and collaboratively through writing, speaking, and listening. Teacher prompts that can enhance this standard include:

- Can you graph this equation in the calculator to see a relationship?
- What formula or strategy might help you determine the answer to this question?

- How can you represent the situation using handheld tools (rulers, protractors, etc.) to determine an answer?
- Can you represent this situation with a visual model?
- How will it help you solve the problem?
- What information is needed to solve this problem?
- Is there another way to solve this problem?

MPS.C.1: Demonstrate a deep and flexible conceptual understanding of mathematical ideas, operations, and relationships while making real-world connections.

Students can make connections between different areas of mathematics, other content areas, and real-world context. They can identify applicable quantities, interpret mathematical models, and describe their relationships in the context of relevant situations. Teacher prompts that can enhance this standard include:

- How can this concept be applied in a real-world context?
- Is your answer reasonable based on your initial estimate?
- While working to solve this problem, what misconceptions might someone have with this?
- What do the characteristics of the graph tell us about the situation?
- What do each of the variables and numbers in the equation/formula represent?
- How are these situations the same and different based on their representations?
- How does this concept relate to concepts we have learned previously?
- What connections can you make between this concept and other areas of mathematics?
- What units of measure help describe your numerical answer?

MPS.AJ.1: Use critical thinking skills to reason both abstractly and quantitatively.

Students can construct arguments using multiple representations (objects, symbols, drawings, and actions). They can recognize and explain bias and errors in an argument. Mathematical students can listen to and read the arguments of others to critique whether they make sense and ask questions for clarification. Students can use reasoning to make and explore the truth of conjectures. Teacher prompts that can enhance this standard include:

- How can you represent this problem with diagrams and models?
- What evidence supports your argument? Can you justify your reasoning?
- Will your strategy work for any number?

- How does your solution compare with your classmates?
- For which categories of numbers (negative integers, all real numbers, etc.) will your strategy work?
- How did you determine your answer?
- Why did you choose that strategy?

MPS.SP.1: Identify and apply regularity in repeated reasoning to make generalizations.

Students can make and test conjectures, express regularities as generalizations about relationships, and then use the generalizations to solve problems. They can recognize complex mathematical objects and situations as being composed of multiple parts. Teacher prompts that can enhance this standard include:

- Can you identify any patterns or regularities in the data or problem?
- How can you express your observations about the problems as a rule?
- What relationship do you notice in the graph/table/numbers?
- Why did you choose to use this process to solve this word problem/equation?
- How can you apply this process in other situations?
- How can you generalize your reasoning to apply to similar problems?
- How might this generalization help you solve similar problems?

Source

- South Carolina Department of Education. (n.d.)." 2023 South Carolina College- and Career-Ready Mathematics Standards." Accessed June 4, 2024. https://www.walch.com/SCCCR/00001

Instructional Strategies

Ensuring Access for All Students

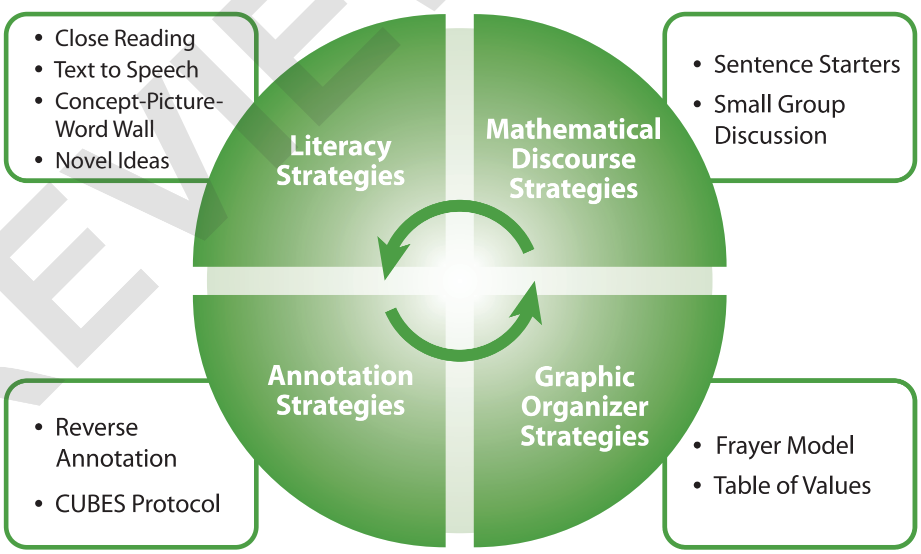
Introduction

The increased focus on literacy in math instruction can help some students navigate mathematical contexts, but for struggling readers, it can further complicate calculations. English language learners struggle to master difficult mathematical concepts while simultaneously processing a new language. Students with learning and behavioral disabilities struggle with the math concepts in their own contexts. This is where teachers and the strategies they select for their classrooms become essential.

The strategies presented here can help all students succeed in math, literacy, school, and, ultimately, in life. These instructional strategies provide teachers with a wide range of instructional support to aid English as a Second Language (ESL) students, students with disabilities (SWD), and struggling readers. These strategies provide support for the Mathematics Standards and the Mathematical Process Standards (MPS), English Language Development (ELD) Standards, English Language Arts Standards, and WIDA English Language Development Standards.

Within each lesson throughout this course, you will find suggested instructional strategies. These instructional strategies are research-based strategies and best practices that work well for all students.

The instructional strategies detailed here fall into four main categories: Literacy, Mathematical Discourse, Annotation, and Graphic Organizers. These strategies provide teachers with research-based strategies to address the needs of all students.



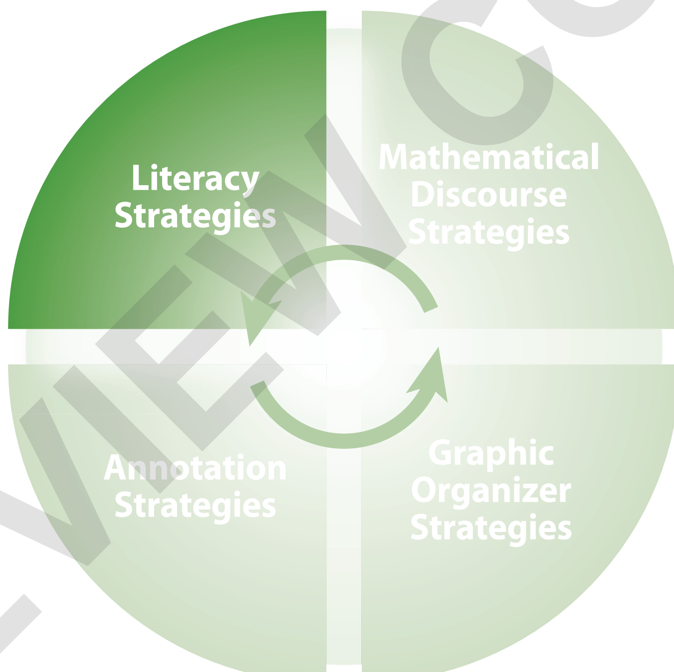
Mathematical Modeling

Source

- WIDA: https://www.walch.com/rr/09052

Understanding the Language of Mathematics: Literacy

Mathematics has its own language consisting of words, notations, formulas, and visuals. In education, the language of mathematics is often regarded solely in the context of word problems and articles. This neglects the vocabulary and other mathematical representations students must be able to interpret. The strategies presented here help students navigate the language of mathematics so that they can understand text and feel confident speaking in and listening to mathematical discussions. For students with disabilities, the stress on repetition and different representations in this approach is essential to their ability to grasp the math concepts. For ESL students, repetition and different representations can strip out some of the English language barriers to understanding the language of mathematics, as well as provide multiple means of accessing the content. Literacy strategies include Close Reading, Text-to-Speech, Concept-Picture-Word Walls, and Novel Ideas.



Literacy Strategies

Close Reading with Guiding Questions

What is Close Reading with Guiding Questions?

Close Reading with Guiding Questions is a process that allows students to preview mathematical reading and problems by answering questions related to the text in advance and reviewing their responses during and/or after reading. Multiple reading protocols can be used in conjunction with guiding questions to enhance their effectiveness.

How do you implement Close Reading with Guiding Questions in the classroom?

When utilizing a textbook, task, or article in a math class, literacy struggles are often a strong barrier to entry into the mathematical ideas. Asking students to answer accessible questions before and/or as they read can lead them to the key information.

Prior to implementation, the teacher should determine the most important information students need to obtain from a text, whether it is a math problem to solve, a task to complete, or an informational lesson or article to read. Then, the teacher should come up with some questions to guide students before they read. These questions can:

- assess and relate prior knowledge
- define key vocabulary words
- discuss non-mathematical concepts in the text

The teacher should also prepare some questions to guide students as they read. These questions can:

- point out key concepts within the text
- relate the text and concepts to future learning
- assist students in identifying key facts in the text
- highlight the importance of text features (graphics, headings, etc.) in the text

To ensure the questions are accessible for students and to encourage reflection and debate after reading, many of these questions should be designed as either "True/False" or "Always True/Sometimes True/Never True." Students can represent their reasoning for their answer in writing, numbers, or graphic/pictorial representations. Students should complete the guiding questions and reading individually, with discussion to follow.

After students complete the reading, they should be given some time to individually evaluate their initial answers. Then, in partners or in groups, they can discuss their answers and come to final conclusions that will help them find the important information initially identified by the teacher. After deciphering the text through close reading, students will be able to complete the given activity.

When would I use Close Reading with Guiding Questions in the classroom?

Close Reading with Guiding Questions can be used for any activity in which literacy could be a barrier to learning or demonstrating mastery of mathematical concepts. The number of questions and length of the discussions can be altered based on the length, importance, and difficulty of the text and concept. As students become more accustomed to mathematical literacy, the text complexity can be increased, but the adherence to close reading strategies must be maintained to ensure students can access the mathematical concepts. The length of time spent on the literacy aspect can be shortened as students become more skilled, but the questioning and discussions must occur to ensure students are properly interpreting the text in the mathematical context.

How can I use Close Reading with Guiding Questions with students needing additional support?

For struggling readers, including ESLs, Close Reading with Guiding Questions can help make an intimidating lesson, word problem, or task much more accessible. Questions focusing more on Tier 2 and Tier 3 vocabulary, text features, and real-world concepts can help struggling readers relate to the text and learn how to decipher the text in context. Discussions around the questions will help students grasp the math concepts.

Allowing struggling readers to explain their answers using words, numbers, or graphics/pictures ensures that they can express their opinion and rationale despite a potential lack of vocabulary. Through these representations and the ensuing discussion, students will begin to learn the necessary vocabulary to be successful.

What other standards does Close Reading with Guiding Questions address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E1.AOR.2
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1
- ELA.E1.AOR.8.1

Sources

- Anne Adams, Jerine Pegg, and Melissa Case. "Anticipation Guides: Reading for Mathematics Understanding."

  https://www.walch.com/rr/09053

- Diane Staehr Fenner and Sydney Snyder. "Creating Text Dependent Questions for ELLs: Examples for 6th to 8th Grade."

  https://www.walch.com/rr/09054

Literacy Strategies

Text-to-Speech Technology

What is Text-to-Speech Technology?

Text-to-Speech Technology is an adaptive technology that reads text aloud from a text source for students. It is usually accessed through an application or program on a computer, smartphone, or tablet. Some new programs utilize Mathematical Markup Language (MathML) to read mathematical notation in a common, understandable manner for students. Many programs also highlight the words and notation on the screen as the audio plays, which helps students relate the written representation to the words they hear. The use of Text-to-Speech Technology allows students who struggle with literacy to hear the words and notation and access the text in a different way.

How do you implement Text-to-Speech Technology?

A classroom community focused on everyone's learning and a growth mindset is the first step in implementing Text-to-Speech Technology. One of the main barriers to implementation is encouraging students to use the program. Once they do, they will realize how the audio can help them understand the difficult mathematical texts and interpret the math content within them. After students realize the benefits of Text-to-Speech Technology, it can become part of the regular routine for group and independent work.

The use of headphones can be very important for effective use of Text-to-Speech Technology. Students can use the technology to listen to lessons and texts at their own pace. Extra noise from other students working or other students listening at different paces can confuse students attempting to use Text-to-Speech Technology, and headphones can help mitigate these distractions. Many teachers are nervous about the potential disruption headphones can cause in class. However, well-managed use of headphones can help students successfully utilize the technology to learn.

When would I use Text-to-Speech Technology in the classroom?

Text-to-Speech Technology can be used at any time throughout the year, and if the program speaks in MathML, it can be used with any lesson. Without MathML, effective use could be limited to word problems without unusual notation. For example, if x^2 is read as "x-two" instead of "x-squared" or "x to the second power," that could confuse students more.

During a lesson or small group discussion, Text-to-Speech Technology could detract from students' ability to listen, question, and process information. However, during warm-ups, independent work, or assessments, Text-to-Speech Technology can help students process the information and access the activity. It can become a routine for students to automatically listen to the question, problem, or directions first, and then attempt the activity.

How can I use Text-to-Speech Technology with students needing additional support?

Text-to-Speech Technology is an important adaptation and accommodation for struggling readers. Students who have read-aloud accommodations sometimes don't receive them because they are either embarrassed to accept them or because of staffing restrictions. These students can use Text-to-Speech Technology to supplement their math instruction by having text automatically read to them in a manner in which they can process it.

Additionally, for ESL students, hearing the English mathematical language, especially referring to mathematical representations and notation, can help put English words to the ideas they see. Some Text-to-Speech Technology can translate written and mathematical text into other languages, so students can hear the text in their natural language and see the English highlighted on the screen as they hear it. In this way, students are learning English vocabulary as well as learning the mathematical content in a language they can understand.

What other standards does Text-to-Speech Technology address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E1.AOR.2
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1
- ELA.E1.AOR.8.1

Source

- Steve Noble. "Using Mathematics eText in the Classroom: What the Research Tells Us." https://www.walch.com/rr/09055

Literacy Strategies

Concept-Picture-Word Wall

more> than
Concept/Word Cards

What is a Concept-Picture-Word Wall?

A Concept-Picture-Word Wall is a classroom display, often a bulletin board or a set of posters, that exposes students to important vocabulary words they will use in math class.

Posting vocabulary words in class helps reinforce the words students will see in textbooks, videos, websites, and test questions on math concepts. These Tier 3 vocabulary words are often not used in everyday language, and the exposure to the words visually through Concept-Picture-Word Walls can help students connect them to the math content.

How do you implement Concept-Picture-Word Walls in the classroom?

Just seeing the vocabulary on a Concept-Picture-Word Wall by itself will help students; more importantly, referring to the words as the teacher uses them in class helps students connect the visual to the application. A simple gesture to the wall makes a very explicit reference to the word as it is used and allows students to connect the unfamiliar word to its meaning in context. Additionally, students can be taught to refer to the wall as they use the words in class, and they can be asked to make sure they say at least 3 words from the wall during each class period in small-group discourse or as answers to whole-class questions. The comfort gained from using these Tier 3 words will help students to use appropriate math vocabulary while solving problems and will help students connect concepts more explicitly.

Postings on the Concept-Picture-Word Wall can be arranged strategically to connect concepts, units of study, or groups of words where appropriate. Having three sections of the Concept-Picture-Word Wall—for example, an "In the Future" section, a "Live in the Present" section, and a "Remember the Past" section—can help students see and remember the vocabulary throughout the entire course. Even without regular use of some words, just seeing the words before a unit can help instill a familiarity with the vocabulary. Leaving the words on the Concept-Picture-Word Wall after a unit is taught can help students connect "old" concepts to the current lesson and ensure that students still have access to the vocabulary.

When would I use Concept-Picture-Word Walls in the classroom?

Concept-Picture-Word Walls can be used for the entire year. The actual words might have to change, or at least be moved to different areas of the Concept-Picture-Word wall. The more exposure students have to the words, the more familiar and comfortable they will become. The constant exposure to the math context is beneficial for students throughout the entire course, especially for words with multiple meanings (bias, tangent, etc.) that could exist as Tier 2 words in everyday conversation but are Tier 3 words in the math classroom.

How can I use Concept-Picture-Word Walls with students needing additional support?

For all students learning mathematics, knowing and using the math vocabulary is often a major barrier. This is a problem especially for ESL students, who are learning the English language along with math content. If teachers try to simplify the words too much for students, it does them a disservice as they seek out information from other teachers, textbooks, and online sources that use the proper vocabulary. Most tests, especially state tests, will expect students to have knowledge of the Tier 3, math-specific vocabulary. The more students see these words, the more familiarity they will have when they apply them.

Concept-Picture-Word Walls can also be written in multiple languages. Especially for students who are on-grade-level in their native language, a multi-lingual Concept-Picture-Word Wall can help students connect the content they already know in another language to the English vocabulary necessary for success on English-language math activities and tests.

This website can help you get started on an English-Spanish Concept-Picture-Word Wall: https://www.walch.com/rr/09056

What other standards do Concept-Picture-Word Walls address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E1.AOR.2
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1
- ELA.E1.AOR.8.1

Source

- Janis M. Harmon, Karen D. Wood, Wanda B. Hedrick, Jean Vintinner, and Terri Willeford. "Interactive Word Walls: More Than Just Reading the Writing on the Walls."

  https://www.walch.com/rr/09057

Literacy Strategies

Novel Ideas

What is Novel Ideas?

Novel Ideas is a classroom activity that explores students' understanding of important Tier 2 vocabulary words they will use in math class. Instead of asking students to look up vocabulary words in the dictionary, Novel Ideas allows students to have conversations with their peers about vocabulary words in class. This reinforces the mathematical vocabulary students will see in textbooks, videos, websites, and test questions. These Tier 2 vocabulary words are often used in everyday language, but have specific meaning in mathematics. Exposure to the words through Novel Ideas can help students connect them to the math content.

How do you implement Novel Ideas in the classroom?

While building a rich representation of math content words and connecting the words to other words and concepts has inherent merit, it is more important to consider that pre-teaching the words before they are used in class helps students connect to the application. The understanding gained from discussing these Tier 2 words will help students apply them in a mathematical context to solve problems and connect concepts.

Here is a step-by-step process for implementing Novel Ideas:

1. Students separate into groups of four.
2. Students copy the teacher generated prompt/sentence starters and number their papers 1–8.
3. One student offers an idea, another echoes it, and all write it down.
4. After three minutes, students draw a line under the last item in the list.
5. All students stand, and the teacher calls one student from a group to read the group's list.
6. The student starts by reading the prompt/sentence starters, "We think a _______ called _______ may be about ... ," and then adds whatever ideas the team has agreed on.
7. The rest of the class must pay attention because after the first group has presented all their ideas, the teacher asks them to sit down and calls on a student from another team to add that team's "novel ideas only." Ideas that have already been presented cannot be repeated.
8. As teams complete their turns and sit down, each seated student should record novel ideas from other groups below the line that marks the end of his or her team's ideas.

When would I use Novel Ideas in the classroom?

Novel Ideas can be used for the entire year. The more students are exposed to mathematical vocabulary, the more familiar and comfortable they become, leading to increased usage of these math terms in their conversation and writing. Using math vocabulary in context is beneficial for students throughout the entire course, especially for words with multiple meanings (bias, tangent, etc.) that could exist as Tier 2 words in everyday conversation but are Tier 3 words in the math classroom.

How can I use Novel Ideas with students needing additional support?

Most tests, especially state tests, will expect students to have knowledge of the Tier 3, math-specific vocabulary. The more students use these words in conversation, the more familiarity they will have when they apply them. Understanding Tier 2 words also helps students avoid misconceptions in mathematics. Twice a week before the start of a lesson, allow students to use sentence starters in small groups that include all students. Prepare the sentence starter "When I hear the word _______, I think about _______" to share out with whole class. This will allow students who know the vocabulary words to share their knowledge, and will allow other students to hear the meaning of the vocabulary words. This strategy is particularly helpful for ESL students.

What other standards does Novel Ideas address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E1.AOR.2
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1
- ELA.E1.AOR.8.1

Sources

- Colorín Colorado. "Selecting Vocabulary Words to Teach English Language Learners."

  https://www.walch.com/rr/09058

- Elsa Billings and Peggy Mueller, WestEd. "Quality Student Interactions: Why Are They Crucial to Language Learning and How Can We Support Them?"

  https://www.walch.com/rr/09059

Novel Ideas Sentence Starters

Slope

- When I hear the word climb, I think about ...
- When I hear the word steep, I think about ...

Volume

- When I hear the word filling, I think about ...

Equations

- When I hear the word balance, I think about ...
- When I hear the word equal, I think about ...

Graphing

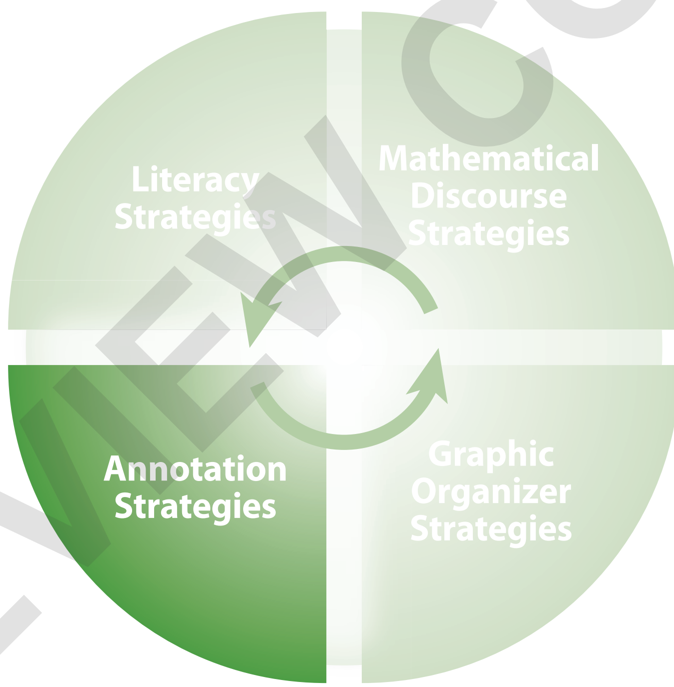
- When I hear the word grid, I think about ...
- When I hear the word graph, I think about ...

Scatter Plots

- When I hear the word scattered, I think about ...

Understanding Mathematical Content: Annotation

Understanding mathematical content is an extremely important skill, both in the math classroom and in life. When students read word problems, articles, charts, graphs, equations, tables, or other forms of mathematical text, they must be able to decode and extract meaning from the text. Annotation can help. The strategies presented here help students identify and focus on key characteristics and facts from various forms of text while ignoring the non-essential information. For students with disabilities, many of whom struggle with the distractions inherent in many high-school level texts, making notes and drawing pictures to explain a problem can help them focus. ESL students will be pointed to certain Tier 3 vocabulary words and determine which Tier 2 vocabulary words they must learn to be proficient in math class and in the English language. Annotation strategies include Reverse Annotation and CUBES protocol.



Annotation Strategies

Reverse Annotation Protocol

What is Reverse Annotation?

Reverse Annotation is a strategy that asks students to identify and write down key information from math problems. This is especially helpful for problems given on a computer or tablet, where students can't annotate directly on the problem. A template is given at the end of this section.

How do you implement Reverse Annotation in the classroom?

Many annotation strategies ask students to write, underline, or mark directly on the text of a problem. While those forms of annotation are also beneficial, they are not always possible with technology. Whether the problem is given on paper or using technology, having students write the answers to these questions will ensure that they are thinking strategically and specifically about the strategies and information needed to solve the problem.

The three questions at the top of the Reverse Annotation template are the key to understanding mathematical problems. For every problem given in class, ask students:

1. What is the problem asking us to solve?
2. What key words tell us the mathematical steps we need to perform?
3. What information in the problem can help us figure it out?

After answering the initial questions, students should make a guess, or estimate, of what they think the answer will be. This helps grow their number sense, and provides an initial, reasonable solution to guide their work. Students can then use the strategies they selected to solve the problem and evaluate their solution using the questions at the bottom of the template.

When students first begin to use Reverse Annotation, the teacher should walk them through the steps individually to ensure they can accurately identify the question, key words, and important information. Teachers can also lead students through the estimation process, making a game out of which student has the closest estimate.

Work through each step individually for several "easy" problems first, so that difficult math doesn't interfere with the process. Increase the problem difficulty incrementally as students begin to master the process. This may seem like a long process at first, but the ultimate result is worth the time investment.

When would I use Reverse Annotation in the classroom?

Reverse Annotation can be used to solve any math problem, and is especially helpful for word problems. When Reverse Annotation is initially implemented, the steps should be discussed in detail. As students become accustomed to Reverse Annotation and begin thinking about problems in this manner automatically, the individual steps become less important and can be scaffolded out to improve efficiency. Students should reach the point where they immediately ask themselves the three initial questions when they first see a problem. However, the teacher should ensure that students are truly evaluating all the key information before routine discussions of the individual steps are removed.

How can I use Reverse Annotation with students needing additional support?

Annotation strategies can help students identify key information, even when certain vocabulary words are not known. As teachers introduce the content-specific Tier 3 vocabulary to their classes, annotation strategies such as reverse annotation can help students use these words to apply appropriate strategies while problem solving. Answering the three initial questions can help students organize the key facts and vocabulary, and the identification of key information can simplify the problem. This strategy is especially beneficial for ESL students.

Using reverse annotation with graphic organizers benefits ESL students by removing a lot of the confusing wording and allowing them to focus on the important pieces of a problem. When using Reverse Annotation, all students, including ESL students, will begin to think about problem solving in a way that encourages them to use the appropriate information to find a solution.

What other standards does the Reverse Annotation Protocol address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.RC.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E2.C.7.1
- ELA.E4.C.9.1
- ELA.E1.C.2.1
- ELA.E1.AOR.7.1

Source

- Alliance for Excellent Education. "Six Key Strategies for Teachers of English Language Learners."
  https://www.walch.com/rr/09060

Reverse Annotation Template

Name: ____________________________ Problem/Assignment: ____________________________

Analyze the Problem

What is the problem asking us to solve?	
What key words will tell us the mathematical steps we need to perform?	
What information in the problem can help us figure it out?	

Initial estimate of solution:

Work Space

Remember to box in your solution!

Name: ____________________________ Problem/Assignment: ____________________________

Check It Over

How close was your estimate?	
Does your answer make sense? Is it reasonable? How do you know?	
Did you perform the calculations correctly?	
What does your answer mean in context?	

Annotation Strategies

CUBES Protocol

What is the annotation strategy CUBES?

CUBES is an annotation strategy in which students use different written designs to highlight the key aspects of word problems. It can help them choose the correct mathematical strategy to solve the problem accurately.

CUBES

How do you implement CUBES in the classroom?

The steps for CUBES are:

1. **C**: **C**ircle all the key numbers.
2. **U**: **U**nderline the question.
3. **B**: **B**ox in the key words that will determine the operation(s) necessary and write the mathematical symbol for the operation(s).
4. **E**: **E**valuate the information given to determine the strategy needed. **E**liminate any unnecessary information.
5. **S**: **S**olve the problem, **s**how your work, and check your answer.

As students learn to use CUBES, walk them through the steps individually to ensure they can accurately identify the key numbers, question, key words, unnecessary information, and strategy. Work through each step individually for several "easy" problems first, so that difficult math doesn't interfere with the process. Increase the problem difficulty incrementally as students begin to master the process. This may seem like a long process at first, but the ultimate result is worth the time investment.

A graphic organizer can help students master the process, especially when problems are given on a computer or tablet where students can't always annotate directly on the problem. Students can write down the key numbers and circle them, write down the question and underline it, and so on. This will encourage students to truly think about the different pieces of the problem they are identifying, and how these pieces will guide the strategy and affect the solution.

When would I use CUBES in the classroom?

CUBES can be used to solve any math problem, and is especially helpful for word problems. When CUBES is initially implemented, the steps should be discussed in detail. As students become accustomed to using CUBES and begin thinking about problems in this manner automatically, the individual steps become less important and can be scaffolded out to improve efficiency. However, the teacher should ensure that students are truly evaluating all the key information before routine discussions of the individual steps are removed.

How can I use CUBES with students needing additional support?

Design features can help students identify key words and features, even when certain vocabulary words are not known. As teachers introduce the content-specific Tier 3 vocabulary to their classes, annotation strategies such as CUBES can help students use these words to apply appropriate strategies while problem solving. Using circles, underlines, and boxes can help students organize the key facts and vocabulary, and the elimination of unnecessary information can simplify the problem. This strategy is especially beneficial for ESL students.

Combining CUBES with graphic organizers also benefits ESL students by removing a lot of the confusing wording and allowing them to focus on the important facts of a problem. When using CUBES with a graphic organizer, all students, including ESL students, will begin to think about problem solving in a way that helps encourage them to use the appropriate information to find a solution.

What other standards does the CUBES Protocol address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.RC.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

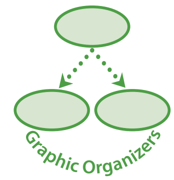
- ELA.E1.C.1.1
- ELA.E2.C.7.1
- ELA.E4.C.9.1
- ELA.E1.C.2.1
- ELA.E1.AOR.7.1

Source

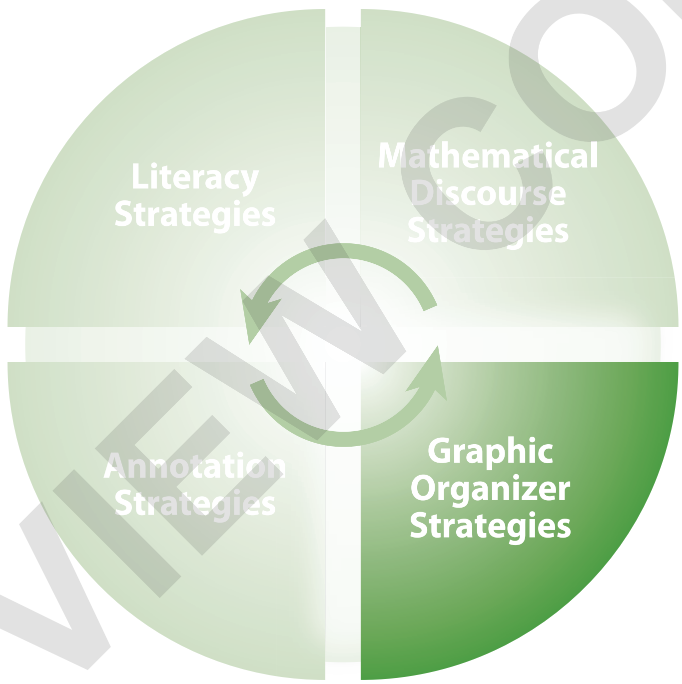
- Margaret Tibbett. “Comparing the effectiveness of two verbal problem solving strategies: Solve It! and CUBES.”

  https://www.walch.com/rr/09061

Organizing Mathematical Content: Graphic Organizers



Organizing mathematical content is a crucial skill for problem solving, exploring other possible methods for finding solutions, and managing math content. All students need strategies for organizing content to build conceptual understanding. For students with disabilities, visual representations and graphic organizers can help them clarify their thoughts and focus on the math. ESL students also benefit from visual representations and graphic organizers. Organizing mathematical knowledge with visuals can help ESL students navigate math content while learning the language. Graphic organizers include Frayer Models and Tables of Values.



Graphic Organizers

Frayer Models

What is a Frayer Model?

A Frayer Model is a graphic organizer that can help students understand new vocabulary words and concepts by exploring their characteristics. A Frayer model lists the definition of a word or concept, describes some key facts, and gives examples and non-examples. Examples and non-examples can come from a mathematical or real-world context.

How do you implement Frayer Models in the classroom?

Students can learn to create Frayer Models the first week of school, and the process can be used throughout the year each time students experience a new word or concept.

While it is important for teachers to give students precise mathematical definitions with appropriate content vocabulary, it is maybe more important for students to understand the application of mathematical words and concepts in their own context. As students learn new information, small group discussions and think-pair-share activities are great ways for students to formulate their own definitions, review the characteristics and facts they have learned, and discuss examples and non-examples.

Discussions of the examples and non-examples can help lead to the mathematical definition. For example, if students use a Frayer Model to define a quadratic function, they would notice that all examples have a highest exponent of 2, and all non-examples would not have a highest exponent of 2. All examples would have parabolic graphs, and all non-examples would have other graphs. Through these comparisons, students will understand the definition of quadratics using different representations, and they will be able to apply it in different contexts.

When would I use Frayer Models in the classroom?

Frayer Models can be used at different points during instruction. They are appropriate as introductions to new concepts, summaries to ensure understanding of new concepts, or as note-organizers throughout the lesson for students to fill in as they learn new concepts. At first, students might need help figuring out how to list and differentiate between the definition, facts and characteristics, examples, and non-examples. As students adapt to the process, they will be able to categorize information on their own or in small groups. As they compare newer Frayer Models to previous models, they will also be able to see how concepts build upon each other.

How can I use Frayer Models with students needing additional support?

Frayer Models can be a point of reference for students as they progress throughout the year. As students determine their own definitions for math-specific words and concepts, and use the examples and non-examples to determine the key facts, they will be able to put them in their own context and apply them to solve complicated problems. As math concepts build upon each other both within a unit and throughout the year, the use of Frayer Models to remind students of their initial definitions of words or concepts can help solidify their understanding. Using Frayer Models as part of a Word Wall or Concept Wall, or having a consistent notebook process to reference past Frayer models, can help consistently reinforce learning.

What other standards do Frayer Models address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E2.C.1.1
- ELA.E1.C.8.1
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1

Source

- Deborah K. Reed. "Building Vocabulary and Conceptual Knowledge Using the Frayer Model." https://www.walch.com/rr/09062

Frayer Model

<table>
<tr><td>Definition</td><td>Characteristics</td></tr>
<tr><td colspan="2">WORD</td></tr>
<tr><td>Examples from Life</td><td>Non-Examples</td></tr>
</table>

Graphic Organizers

Tables of Values

What is a Table of Values?

A Table of Values is an organized way to list numbers that represent different categories of values. These values can be represented as ordered pairs, graphs, word problems, or lists. Tables can help students see and compare values in a different way.

How do you implement Tables of Values in the classroom?

Tables can be used throughout the year to support various mathematical standards. Some standards mention tables specifically, and in others, tables can be an effective support to help students organize and understand the meaning and application of values.

Tables can be set up with numerical values in rows or columns. The key to understanding the values lies in the headings. The headings must be specific enough to show students the meaning and/or application of the numerical values, but not so wordy that they interfere with the clarity of the numbers in the table. For example:

***x* (year)**	***y* (population in millions)**
1960	219
1970	230
1980	258
1990	312
2000	342

Mean (statistical average)	50	45
Median (middle value)	52	43
Quartile 1 (median of the lower 50%)	40	38
Quartile 3 (median of the upper 50%)	72	80
Range (difference of max and min values)	80	61
Interquartile Range (difference of quartiles)	32	42
Standard Deviation (measure of spread of data)	7.24	10.23

When would I use Tables of Values in the classroom?

Various mathematical topics can be represented by tables. For example:

- An (x, y) table of values to represent coordinates on a graph or independent and dependent variables for a given context
- A table to represent coefficients and/or constants in an equation
- A table to show different statistical measures when comparing sets of data
- A table to compare output values for the same input given different functions

Each time numbers or values are being listed, compared, or graphed, a table can help students differentiate between the values. Tables are easy to create, and students can be encouraged to create them as another representation to clarify and compare numbers for nearly any topic.

How can I use Tables of Values with students needing additional support?

Tables of Values can help students focus on numerical values and their meaning in context without distraction. They clarify what each number represents, what numbers can be compared, and what ordered pairs can be graphed to give a visual representation. Additionally, headings can be used to either highlight the relevant facts from a context or to describe mathematical vocabulary.

In general, graphic organizers benefit students by removing much of the confusing wording and focusing on the important facts and numbers of a problem.

What other standards do Tables of Values address?

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.RC.1

WIDA English Language Development Standards:

- ELD Standard 3

SC English Language Arts standards:

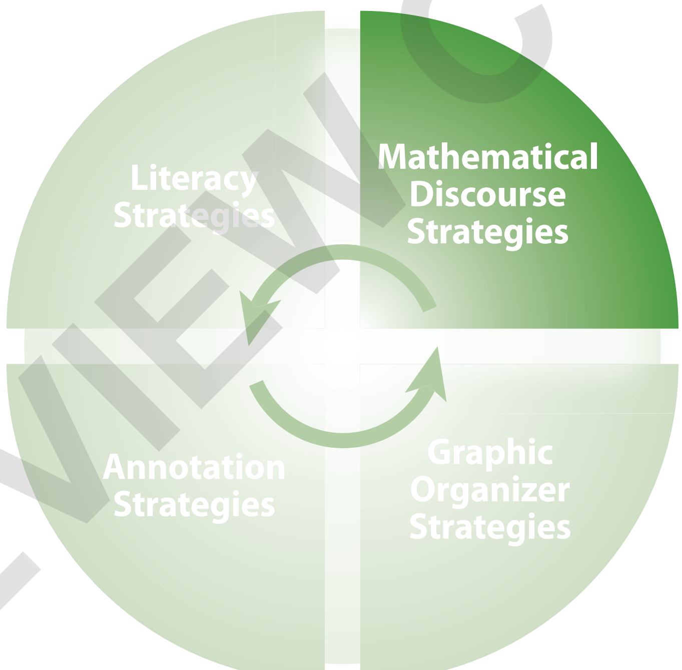
- ELA.E1.C.1.1
- ELA.E2.C.1.1
- ELA.E1.C.8.1
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1

Source

- Alliance for Excellent Education. "Six Key Strategies for Teachers of English Language Learners." https://www.walch.com/rr/09060

Communicating Mathematical Content: Mathematical Discourse

Reading, writing, speaking, and listening are all important ways to learn and express information, but the last two ways are often slighted in the math classroom. The mathematical discourse strategies presented here promote speaking and listening in a math-focused literacy context. Working these strategies into the daily routine of a classroom can help students become comfortable speaking and listening in a mathematical context, which will help them become comfortable with the mathematical content. Routines and structures are essential to support students with disabilities, as they often benefit from following a routine. This can lead to developing capability in their mathematical skills. These strategies also remove the barrier to entry for many ESL students, as structure and routine can help them focus on the math content rather than English language deficiencies. Mathematical Discourse strategies include Sentence Starters and Small Group Discussion.



Mathematical Discourse Strategies

Sentence Starters

What is a Sentence Starter?

A Sentence Starter is a common phrase or mathematical sentence frame that can help students begin and sustain academic conversations around mathematical content. It helps guide students through the discussion and bring out pertinent ideas that can lead to greater understanding.

How do you implement Sentence Starters in the classroom?

Many people view math class as a place to calculate solutions to math problems. However, to ensure the conceptual understanding and proper application of a math concept, students need to be able to explain the concepts and reasoning behind a solution to a problem. As many students are not accustomed to having academic conversations about math, sentence starters can help begin and continue these conversations in a productive manner.

There are two main types of sentence starters for mathematical discussions: discourse starters and math starters. For example, a poster with these or other sentence starters can be displayed from the beginning of the year, and the expectation can be set that any answer to a question or comment in a discussion should be framed using one of these starters. As students become accustomed to framing mathematical conversations in this way, they can expand on the given sentence starters and create some of their own. They will begin to realize how these statements ensure that their conversations revolve around math, enhance understanding of the concept, and force them not only to state, but also to explain their thinking. They will gain confidence from the ability to engage, as the first step has already been taken for them.

When would I use Sentence Starters in the classroom?

Sentence Starters can be used throughout the entire school year with any concept. However, they are most important to use at the beginning of the school year to build a mathematical community in the classroom centered on a comfort with mathematical discourse. Especially at the beginning of the year, students should be encouraged to use these sentence starters for every math statement. Appropriate settings include during small group discussion, while responding to whole class questions, and when writing explanations for problem solutions.

Modifications can be introduced so that students must use certain mathematical vocabulary within the sentences, or must use certain sentence starters at different points in conversations or for different conversation types and situations. However the starters are implemented, it is important for students to realize that these are intended to enhance and focus their conversations, not limit them.

How can I use Sentence Starters with students needing additional support?

Often, students are reluctant to talk about math concepts because they either lack confidence in their knowledge, are afraid to be "wrong," or don't know how to start or continue the conversation. Sentence starters can help students overcome this reluctance. The non-threatening, easy-to-interpret sentence starters remove the barrier to entry for students who don't know how to engage, and the respectful, mathematical focus promoted by sentence starters can help build confidence and provide a structure so that students will not fear being wrong.

For ESL students specifically, sentence starters can provide the English language support to help students engage with and discuss the math. The support of sentence structure removes language barriers to entry for students who don't fully understand English sentence structure.

Discourse Starters	Math Starters
I agree/disagree with ... because ...	My answer was ... because ...
I understand/don't understand ...	The next step is ... because ...
First/Next/Finally I ... because ...	I used (insert formula/equation/concept) because ...
I noticed that ...	
I wonder ...	My answer is right/reasonable because ...

What other standards do Sentence Starters address?

WIDA English Language Development Standards

- ELD Standard 3

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.RC.1

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E2.C.1.1
- ELA.E1.C.8.1
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1

Source

- AVID. "Sentence Starters."
  https://www.walch.com/rr/09064

Mathematical Discourse Strategies

Small Group Discussion

What is Small Group Discussion?

Small Group Discussion is a structured way for students to verbalize their mathematical thinking in a comfortable setting to solve a problem, build conceptual understanding, or summarize a concept.

How do you implement Small Group Discussion?

Small Group Discussion in math class depends on a trusting relationship between the teacher and the students. From there, students can build trusting relationships among themselves. Once this trust has been built, students will feel free to explore mathematical topics in groups, take risks, and engage in a productive struggle toward understanding or a solution.

Once these relationships have been established, certain structures should be established for Small Group Discussion to be effective. Discussion norms can be set by the class to ensure discussions are respectful and productive, and discussions should have predetermined time limits. The group composition is also important and should be based on instructional measures. For different activities, homogeneous groups, heterogeneous groups, or groups based on specific data by standard could be appropriate. Students should always be aware that the groups were chosen to maximize their learning.

Another structure that can be effective for Small Group Discussion is assigning group roles. These roles can include group leader, note taker, timekeeper, resource manager, culture keeper, or other roles determined to be appropriate for the classroom context. During the discussion, assigning each student a letter within the group (A, B, C, D, etc.) can help structure the discussion. Different roles can specify certain time limits for talk, which sentence starters to use, or other structured aspects of the discussion.

When implementing a Small Group Discussion, the question or task should inspire students to think in different ways about a concept. Through the structured format of the discussion, students will compare their ideas and arrive at an answer or explanation of the concept. Within the trusting framework of the class and group, students can focus on the common goal of the discussion and develop their thinking around the math concept. These rich discussions will enhance their understanding.

When would I use Small Group Discussion in the classroom?

Small Group Discussion can be used for nearly any topic, and it can be used at a variety of times in the classroom. The questions and tasks may need to change depending on when it is used. Opening activities for lessons can be Small Group Discussions where students explore properties of new math concepts or review/build upon their prior learning. Turn and talks throughout the lesson can be structured as Small Group Discussions if a consistent framework is in place. At the end of class, a Small Group Discussion can be used to come to a common understanding about an essential question from the lesson.

Depending on when the Small Group Discussion is used in class, and what the goal of the discussion is, the discussion reporting may vary. For a warm-up, each group might be asked to share their thinking. For a guided practice, recording answers on chart paper and a gallery walk could be appropriate. For a closing activity, individual written responses to a question could be appropriate.

How can I use Small Group Discussion with students needing additional support?

As discussed in other Mathematical Discourse strategies, struggling students are reluctant to talk about math concepts because they lack confidence in their knowledge and don't always have the needed vocabulary in their toolbox. Structured discussions with effective grouping can help students through these barriers. After a trusting and respectful classroom environment has been established, struggling students often feel more comfortable sharing their ideas with just a few classmates rather than the whole class. Additionally, adding structure can help students engage by providing the expectation that they participate in the process.

The intentional grouping of students can also help them succeed using Small Group Discussion. At times, heterogeneous groups could be appropriate so that stronger students can help struggling students, and at other times, homogeneous groups could be appropriate so the teacher can work with an entire group of struggling students. ESL students can be grouped with other students with the same dominant language to help remove the language barrier from the conversation.

What other standards does Small Group Discussion address?

WIDA English Language Development Standards:

- ELD Standard 3

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.RC.1

SC English Language Arts standards:

- ELA.E1.C.1.1
- ELA.E2.C.1.1
- ELA.E1.C.8.1
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1

Source

- Jessie C. Store. "Developing Mathematical Practices: Small Group Discussions." https://www.walch.com/rr/09065

Modeling Strategies

Mathematical Modeling

What is Mathematical Modeling?

Mathematical modeling is generally understood as the process of applying mathematics to a real-world problem with a view of understanding the connection. According to the CCSSM, mathematical modeling is the ability to apply concepts learned in class to real-world applications and to use the model to analyze a situation, draw conclusions, and make predictions.

How do you implement Mathematical Modeling in the classroom?

Modeling can be implemented by demonstrating how to make or generate mathematical representations or models, how to validate them, and how to use them to solve real-world problems. There are many ways to show understanding in a math classroom, such as using words, drawings or sketches, physical models, computer programs, or math formulas.

The following is a list of questions and answers suggested in order to create a mathematical modeling classroom environment:

- **Why?** What are we looking for? Identify the need for the model.
- **Find?** What do we want to know? List the data we are seeking.
- **Given?** What do we know? Identify the available relevant data.
- **Assume?** What can we assume? Identify the circumstances that apply.
- **How?** How should we look at this model? Identify the parameters.
- **Predict?** What will our model predict? Identify the equations that will be used, the calculations that will be made, and the answers that will result.
- **Valid?** Are the predictions valid? Identify tests that can be made to validate the model; i.e., is it consistent with its principles and assumptions?
- **Verified?** Are the predictions good? Identify tests that can be made to verify the model; i.e., is it useful in terms of the initial reason it was done? *(inspired by Carson and Cobelli, 2001)*

Teachers should expect these questions to recur often during the modeling process, and should regard this list as a fairly general approach to ways of thinking about mathematical modeling.

In a classroom where mathematical modeling is the expectation, teachers will need to establish that students are responsible for coming up with methods for solving the problems presented and that the teacher will only assist and facilitate.

When would I use Mathematical Modeling in the classroom?

It should come as no surprise that many students find mathematics boring. The most common question posed to any mathematics teacher is "When will I ever need to use this?" Often teachers fail to find problems in which students are interested or to even take student interest into account when planning a lesson. Problems that spark students' interest and curiosity will increase their attention and desire to learn. These types of real-world problems provide students an opportunity to think and respond as a mathematician. Students should be exposed to rigorous learning tasks that allow opportunities for mathematical modeling in the classroom.

How can I use Mathematical Modeling with struggling students?

When struggling readers, which includes ELLs and students with learning disabilities, are exposed to rigorous math learning tasks, there must be a level of scaffolding that includes coaching and guided questions that help to make a word problem or learning task much more accessible. Teachers should come up with questions to guide the students before and during the engagement of the task. Teachers should also:

- assess prior knowledge;
- define Tier 2 and 3 vocabulary words;
- discuss non-mathematical concepts in the task; and
- assist students in identifying key concepts and facts within the tasks.

Allowing struggling readers to explain their answers using words, numbers, or graphics/pictures ensures that they can express their opinion and rationale despite a potential lack of vocabulary. Through these representations and the ensuing discussion, students will begin to learn the necessary math concepts to be successful.

What other standards does Mathematical Modeling address?

WIDA English Language Development Standards:

- ELD Standard 3

Mathematical Process Standards:

- MPS.PS.1
- MPS.AJ.1
- MPS.C.1
- MPS.RC.1

English Language Development for Mathematics:

- ELD–A.9–12: Explain (Interpretive)
- ELD–MA.9–12: Explain (Expressive)

SC English Language Arts standards:

- ELA.E1.C.2.1
- ELA.E2.C.7.1
- ELA.E4.C.2.1
- ELA.E1.AOR.7.1
- ELA.E1.AOR.8.1
- ELA.E1.C.1.1
- ELA.E1.AOR.2

Sources

- Carson, Ewart, and Cobelli, Claudio. *Modeling Methodology for Physiology and Medicine*. Academic Press, San Diego, CA, 2001.
- Dym, Clive L. *Principles of Mathematical Modeling*. Amsterdam: Elsevier Academic Press, 2008.
- MathisFun.com. "Mathematical Models." https://www.walch.com/rr/09066
- Oswalt, Selena. "Mathematical Modeling in the High School Classroom." LSU Digital Commons. Accessed February 25, 2021. https://www.walch.com/rr/09067

PROGRAM OVERVIEW

Mathematical Modeling Implementation Guide

Introduction

BW Walch resources support the framework of the Mathematical Process Standards (MPS) and the NCTM Principles of Teaching Practices. Implementing strategies and support from both practices lead to true conceptual understanding of the math standards. One of which includes mathematical modeling, the process of designing and revising representations to solve a problem.

Mathematical modeling is essential to building a deep conceptual understanding of math concepts for students. Teaching students to model boosts engagement, builds student confidence in math concepts, helps them to make sense of problems, and allows them to make connections to the world around them for better understanding. Students then make decisions about the information, create models, interpret the results, and form conclusions.

A Mathematical Modeling Framework

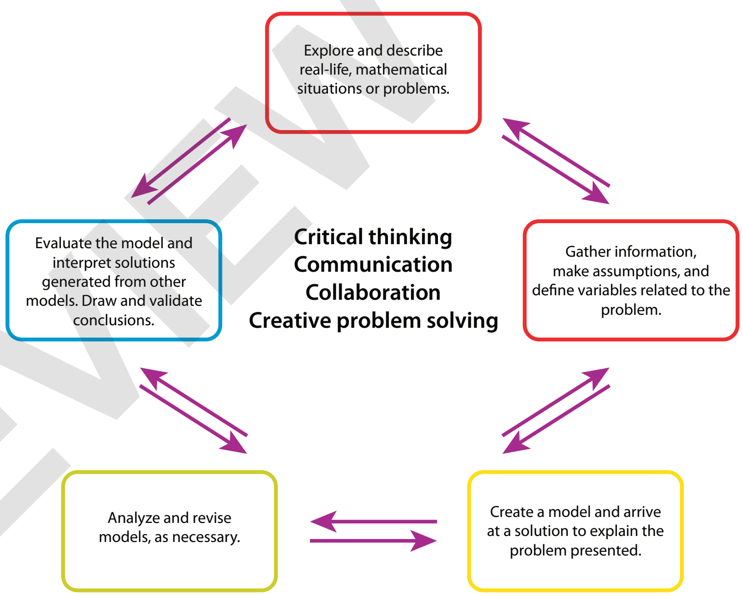


Image adapted from: Suh, Matson, Seshaiyer, 2017

The following is a brief description of how this framework can be applied in the classroom.

Critical Thinking

Students will explore and describe real-life mathematical situations or problems. We want students to discover new ways of thinking and ideas in mathematics. Students do this by developing questions to ask, gathering information, and coming up with solutions. Fostering critical thinking in the classroom not only makes students better at math, but also prepares them for the real world. Below are some ideas and probing questions teachers may use to implement critical thinking.

- Allow for pair-share and small group discussions.
- Encourage students to think and form their own conclusions.
- Encourage the revision of their own thinking and the thinking of others.
- Ask students to think out loud as they work.
- Create a classroom environment that embraces and values student ideas.

Ask students:

- What is the problem asking you to solve?
- Can you think of other strategies you could use to solve this problem?
- What conclusions can you make from this particular problem?
- Will this strategy work in all problems like this? Why or why not? How can we test that?
- Explain how you got to your answer.
- Explain your reasoning.
- How would you respond to a different answer to the same problem?

Communication

When students gather information, make assumptions, and define variables related to the problem, communication allows for them to show their understanding of the math content. Encourage discourse by allowing students to explain their thinking and challenge each other. This encourages students to justify their reasoning. If students communicate their thinking in various ways (including written and oral responses) while doing math, it will improve their understanding of math concepts.

Teachers can do the following to foster communication in the classroom:

- Ask open-ended questions.
- Encourage oral and visual (written and pictorial) communication through journal writing.
- Provide students with detailed feedback.

Ask students:

- Can you explain your thinking?
- How did you get your answer?
- What strategies did you use?
- What information was necessary for you to solve this problem?

Collaboration

Collaboration is an essential component of student success. It allows students to rely on each other during their problem solving. During collaboration, students work in groups, share ideas, ask questions, and discuss math concepts and additional solution strategies while supporting and defending their thinking. Collaboration is most beneficial to students with the use of effective grouping strategies such as assigning students to heterogeneous groups or random grouping.

The following procedures and probing questions can help you implement collaboration in your classroom.

- Establish a classroom culture where all ideas are valued.
- Establish expectations and routines of collaborative learning.
- Discuss "math talk" passages with students.
- Allow students to teach each other.
- Incorporate an accountability piece for students.
- Arrange student seating to support collaboration (group seating).
- Create heterogeneous student groups with varying skill levels.
- Randomize student groups.
- Keep group sizes between 3 and 6 students.
- Assign group roles.

Ask students:

- Come up with as many strategies to solve the problem as you can.
- Explain how you made your calculations.
- Why did you choose that strategy? Why did that strategy work?
- Describe in your own words how your peer-solved the problem.
- Can you make any connections between your strategies?
- Were there any methods that were better than others when solving this problem? Why or why not?
- What did you learn from your group?
- Defend your reasoning behind that solution.

Creative Problem Solving

Creative problem solving is the ability for students to perform math tasks that allow for challenges that increase their conceptual understanding. While performing these tasks, we want students to use mathematical modeling. We want students to evaluate their models and to interpret solutions from other models.

In creative problem solving, students solve problems using different approaches and models, draw on prior knowledge, and justify their thinking. This results in students becoming better problem solvers and increases their understanding of math concepts. Problem solving should be integrated into their math learning and should not be separated.

Here are some tips for implementing creative problem solving.

- Encourage students to challenge different approaches and strategies from their peers as well as the teacher.
- Encourage discourse.
- Allow appropriate wait time for student responses.
- Refrain from telling students how to solve the problem. Instead, allow students to engage and come to their own solutions.
- Allow students to struggle productively.

Ask students:

- How is the information in the problem important to determining the solution?
- How did you go about solving this?
- Can you explain why you chose that model and strategy?
- Are there other ways to model this particular problem? Can you model the problem another way?
- Why did you make that calculation?
- Justify your solution.
- What generalizations can you make about the math concepts based on this particular problem?

Recommended Resource

- Georgia Department of Education. "Scaffolding Instruction for English Learners: A Georgia Mathematics Instructional Resource Guide."

  https://www.walch.com/rr/09047

  The purpose of this document is to provide mathematics teachers and leaders with evidence-based, pragmatic scaffolds and supports for English Learners (ELs). This guide is a useful tool to help teachers provide high-quality instruction aligned to Georgia's K-12 Mathematics Standards.

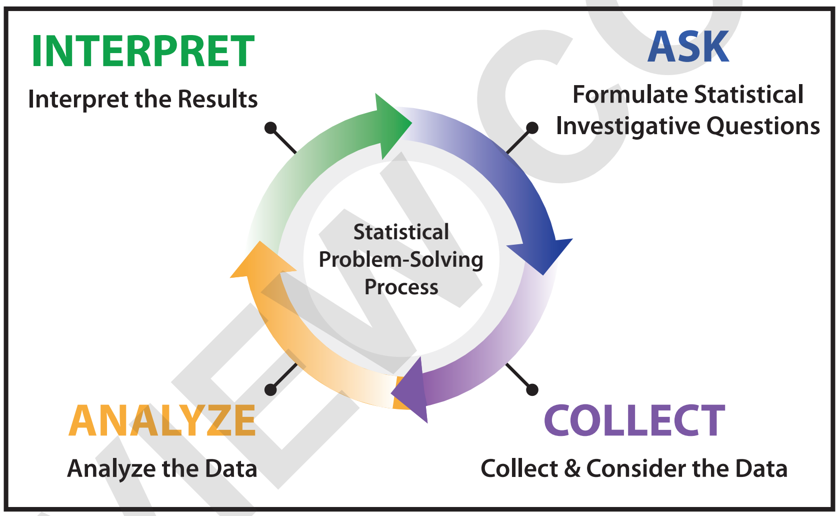
Source

National Council of Teachers of Mathematics. "Problem Solving." Accessed January 11, 2023. https://www.walch.com/rr/09048.

Statistical Reasoning Implementation Guide

Introduction

Statistical reasoning allows students to make sense of ideas, information, and the changing world through questioning and exploration. It provides the foundation necessary for students to fully understand the concept. Statistical reasoning is a continuous cycle consisting of students asking questions, collecting, analyzing, and interpreting data. In order to guide students in this sense-making process, BW Walch resources support this four-step statistical problem-solving strategy to help students develop their understanding in statistical reasoning.



Source: Georgia Department of Education

Here is a brief description of how this framework can be applied in the classroom.

Formulate Statistical Investigative Questions

Students will form and ask investigative questions that allow for various answers. These questions will clarify the problem and lead to questions that can be answered with the data. Best practices and teacher prompts that can foster this framework include:

- Using a student-centered approach.
- Having students prepare ahead of time with an assigned reading to familiarize themselves with words and techniques.

Ask students:

- What do you think?
- What do you notice? What do you wonder?
- What criteria need to be met in order for the question to be statistical?
- How did you determine your question?
- What changes would you make to the question?

Collect and Consider the Data

Students will collect data by creating a plan in order to collect real and relevant data. Making sure the data is relevant to students will increase engagement and lead to more math talk and discussion. Strategies include:

- Refraining from presenting students with procedures.
- Allowing students to use real data sets and to generate their own data.
- Encouraging students to discuss the questions and possible ideas.

Ask students:

- What do you notice about the data?
- In what other ways can the data be collected?
- What are some other methods you can use to collect the data? How do these different methods affect your data collection?
- How can you represent your data? Can you represent it with a visual?
- Are there representations better fit for particular findings? Justify your answer.

Analyze the Data

Students will analyze the data by selecting methods that are appropriate. Exploration of various methods will allow for students to make connections and draw conclusions based on the data. This will deepen their understanding of statistical reasoning. Strategies include:

- Allowing students to use technology tools to explore and analyze their findings.
- Refraining from giving students all the information. Allow students to form their own analysis of the data.
- Creating a classroom environment in which student ideas are valued.

Ask students:

- What conclusions can you draw from the data?
- Do you notice any trends in the data? How can you tell?
- What is the relationship between the data points?
- What evidence may help you distinguish between results?
- Do you agree or disagree? Justify your thinking.
- How can we test that conclusion?
- What do you do about outliers in your data? What do they tell you?
- If extreme values are removed, what happens to the data representation?
- Compare your data with a classmate's. What do you notice?

Interpret the Results

Students will interpret and discuss the results by relating all findings to the original question. Students will discuss these findings and justify their reasoning. Best practices and teacher prompts include:

- Encouraging discourse. Encourage students to present their ideas, answer classmates' questions, and support their responses.
- Focusing on key ideas instead of procedures and calculated answers.
- Making sure students have answered their "I wonder" questions.

Ask students:

- What do the results tell you about the original question?
- Have your "I wonder" questions been answered?
- What conclusions can you make from the results?
- Compare your interpretations to those of your classmates. What connections can you make?
- What do your interpretations represent in a real-world context?

Source

Garfield, Joan and Ben-Zvi, Dani. "Helping Students Develop Statistical Reasoning: Implementing a Statistical Reasoning Learning Environment." Accessed Jan. 11, 2023. https://www.walch.com/rr/09049

Graphic Organizers

Overview

Graphic organizers can be a versatile tool in your classroom. Organizers offer an easy, straightforward way to visually present a wide range of material. Research suggests that graphic organizers support learning in the classroom for all levels of learners. Gifted students, students on grade level, and students with learning difficulties all benefit from the use of graphic organizers. They reduce the cognitive demand on students by helping them access information quickly and easily. Using graphic organizers, learners can understand content more clearly and can take concise notes. Ultimately, learners find it easier to retain and apply what they've learned.

Graphic organizers help foster higher-level thinking skills. They help students identify main ideas and details in their reading. They make it easier for students to see patterns such as cause and effect, comparing and contrasting, and chronological order. Organizers also help students master critical-thinking skills by asking them to recall, evaluate, synthesize, analyze, and apply what they've learned. Research suggests that graphic organizers contribute to better test scores because they help students understand relationships between key ideas, and enable them to be more focused as they study.

Types of Graphic Organizers

There are four main purposes for using graphic organizers in mathematics and a variety of tools within each category:

Purpose 1: Organizing, Categorizing, and Classifying	**Purpose 2: Problem Solving**	**Purpose 3: Understanding Mathematical Information**	**Purpose 4: Communicating Mathematical Information**
Tables Flowcharts Webs Venn Diagrams	Number Lines Geometric Drawings Factor Trees Attribute Tables Cause and Effect Maps Coordinate Plane Probability Trees	Frayer Model Semantic Map/ Concept Map Compare-and-Contrast Diagram	Line Graphs Bar Charts

Tables

A table is simply a grid with rows and columns. Tables are useful because information stored in a table is easy to find—much easier than the same information embedded in text.

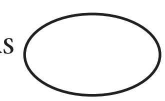
Usually, a table has a row (horizontal) for each item being listed. The columns (vertical) provide places for details about the listed items—the things they have in common. The places where the rows and columns meet are called cells. In each cell, we write information that fits both the topic of the row (the thing being listed) and the topic of the column (the aspect being examined). To create a table, we make rows and columns to fit the number of items and attributes.

Flowcharts

Flowcharts are graphic organizers that show the steps in a process. Flowcharts can be very simple—just a series of boxes with one step in each box. However, there is also a more formal type of flowchart. These flowcharts use special symbols to show different things, such as starting and stopping points, or points where decisions must be made. These symbols make flowcharts especially useful for showing complicated processes.

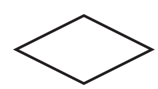
Each step in a flowchart is written in a box. The boxes are connected by arrows to show the sequence of steps. The boxes aren't all rectangular; different shapes are used to indicate different actions. The shapes and symbols are a kind of visual shorthand. Whenever a certain symbol is used, it always has the same meaning.

- Circles and ovals show starting and stopping points. They often contain the words start or stop. The "start" circle or oval has no arrows in and one arrow out. The "stop" circle or oval has one arrow in and no arrows out.

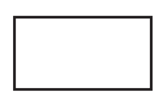


- Arrows show the direction in which the process is moving.

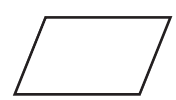
- Diamonds show points where a decision must be made or a question must be answered. The question can usually be answered either "yes" or "no."



- Rectangles and squares show steps where a process or an operation takes place.



- Parallelograms show input or output, such as writing or printing a result or solution.



Webs

Webs are graphic organizers that help take notes, identify important ideas, and show relationships between and among pieces of information. In a web, the main idea is written in the center circle. Details are recorded in other circles with lines to connect related topics. Circles or lines can be added or deleted as necessary.

Number Lines

In its simplest form, a number line is any line that uses equally spaced marks to show numbers. Number lines are used to visualize equalities and inequalities, positive and negative numbers, and measurements of all kinds. They can “map” math problems, especially ones that involve negative numbers or distances.

Geometric Drawings

A geometric drawing is a representation on paper (or some other surface) of a geometric figure. The geometric drawings we make can never be as perfect as the geometric figures they represent, but as long as they are reasonably accurate, they can help us visualize the figures. In fact, it’s often impossible to solve a geometry problem without making a drawing.

Factor Trees

There are several ways to find factors. One that helps to visually keep track of all the factors is called a factor tree. This is a diagram with a tree-like shape. It uses “branches” to show the factors of a number.

All whole numbers other than 1 can be written as the product of factors. A prime number is a number that has only two factors, itself and 1. An example of a prime number is 13. Its only factors are 13 and 1. A composite number is a number that has more than two factors. An example of a composite number is 6. Its factors include 6, 3, 2, and 1. Prime factors are factors that are also prime numbers. The greatest common factor (GCF) of two numbers is the largest number that is a factor of both numbers.

Coordinate Plane

This is the plane determined by a horizontal number line, called the x-axis, and a vertical number line, called the y-axis, intersecting at a point called the origin. A coordinate plane can be used to illustrate locations and relationships using ordered pairs of numbers.

Venn Diagrams

A set is a list of objects in no particular order. Items in a set can be numbers, but they can also be letters or words. Venn diagrams are a visual way of showing how sets of things can include one another, overlap, or be distinct from one another.

Venn diagrams are often used to compare and contrast things. But they are also a useful tool to sort and classify information. You can use Venn diagrams to take notes on material that shows relationships between things or ideas. You can also use them to solve certain types of word problems. When a word problem names two or three different categories and asks you how many items fall into each category, a Venn diagram can be a useful problem-solving tool.

A Venn diagram begins with a rectangle representing the universal set. Then each set in the problem is represented by a circle. Circles can be separate, overlapping, or one within another. When two circles overlap, it means that the two sets intersect. Some members of one set are also members of the other set.

Venn Diagrams AND **Compare-and-Contrast Diagrams**

The Venn diagram is an organizing device for planning comparisons and contrasts. A completed Venn diagram helps students categorize and organize similarities and differences, and provides a blueprint for a comparison-and-contrast exercise. The compare-and-contrast diagram provides a structure to identify or list similarities and differences between two objects.

Attribute Tables

To solve logic problems, you need a way to keep track of the subjects and which attributes they have or don't have. An attribute table can help. This is a table with a row for each subject in the problem, and a column for each attribute. The rows and columns meet to form cells. Because the attributes in logic problems are usually exclusive, you can use Xs or check marks (✔) to show which attribute belongs to which subject.

Cause and Effect Maps

Cause and effect maps help you work through information to make sense of it. Write each cause in the oval. Write all its effects in the boxes. Add or delete ovals and boxes as needed.

Frayer Model

The Frayer Model is a word categorization activity that helps learners to develop their understanding of concepts. Using this model, students provide a definition, list characteristics, and provide examples and non-examples of the concept.

Semantic Map

A semantic word map allows students to conceptually explore their knowledge of a new term or concept by mapping it with other related words, concepts, or phrases that are similar in meaning. Semantic maps portray the schematic relations that compose a concept. It assumes that there are multiple relations between a concept and the knowledge that is associated with the concept.

Line Graphs

Line graphs are often used to show how things change over time. They clearly show trends in data and can let you make predictions about future trends, too. Line graphs use two number lines, one horizontal and one vertical. The horizontal number line is called the x-axis. The vertical line is called the y-axis. The x-axis often shows the passage of time. The y-axis often shows a quantity of some kind, such as height, speed, cost, and so forth.

Bar Charts

Bar charts are useful when you want to compare things or to show how one thing changes over time. They are a good way to show overall trends. Bar charts use horizontal or vertical bars to represent data. Longer bars represent higher values. Different colors can be used to show different variables. When you look at a bar chart, it's easy to see which element has the greatest value—the one with the longest bar.

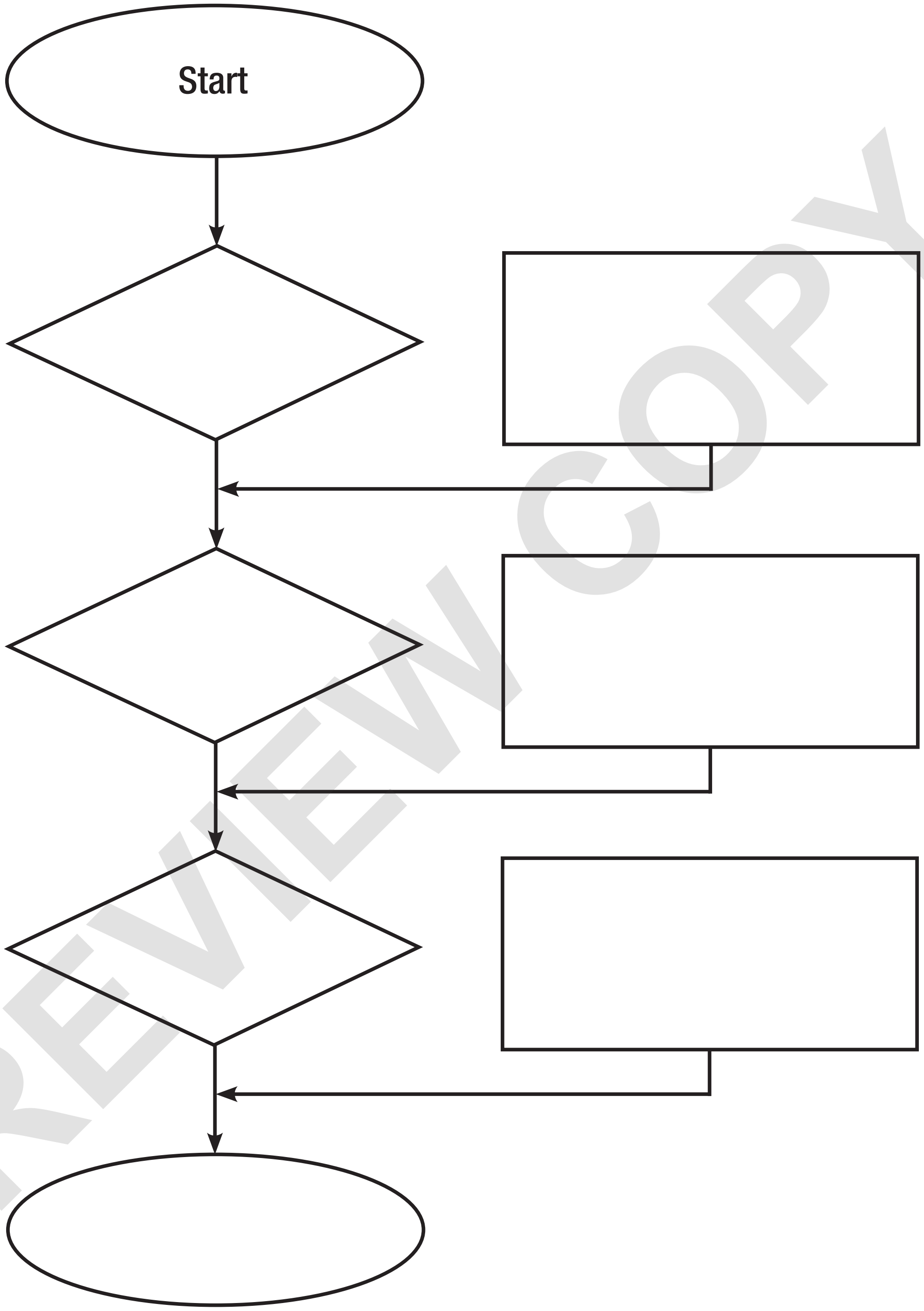
Bar charts have an x-axis (horizontal) and a y-axis (vertical). If the graph is being used to show how something changes over time, the x-axis has numbers for the time period. If the graph is being used to compare things, the x-axis shows which things are being compared. The y-axis has numbers that show how much of each thing there is.

Probability Trees

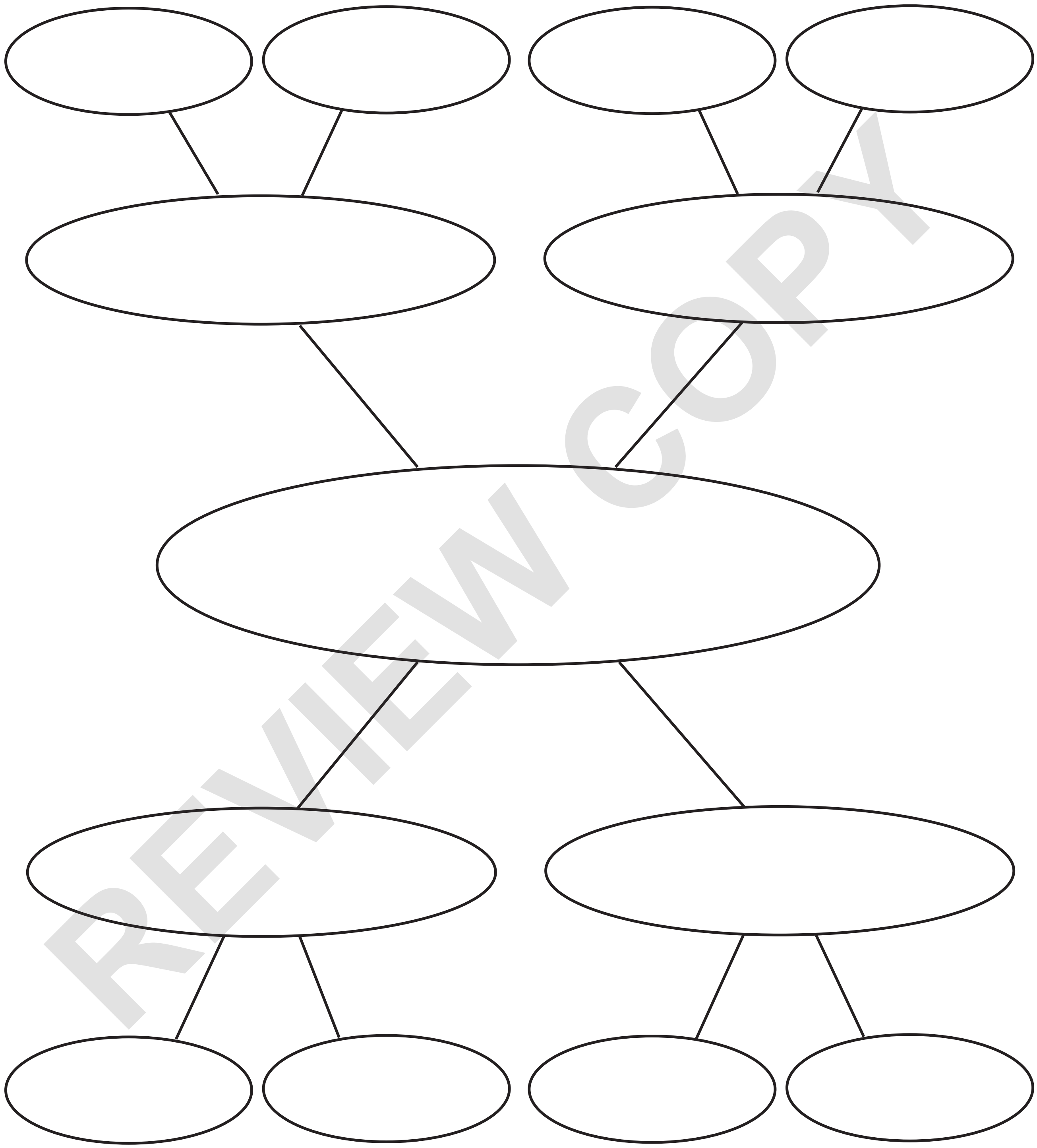
When we have probability problems with many possible outcomes, or events that depend on one another, probability trees can help. Probability trees show all the possible outcomes of an event. Whenever a problem calls for figuring out how many possible outcomes there are, and the probability that any one of them will happen, a probability tree can be useful.

Table

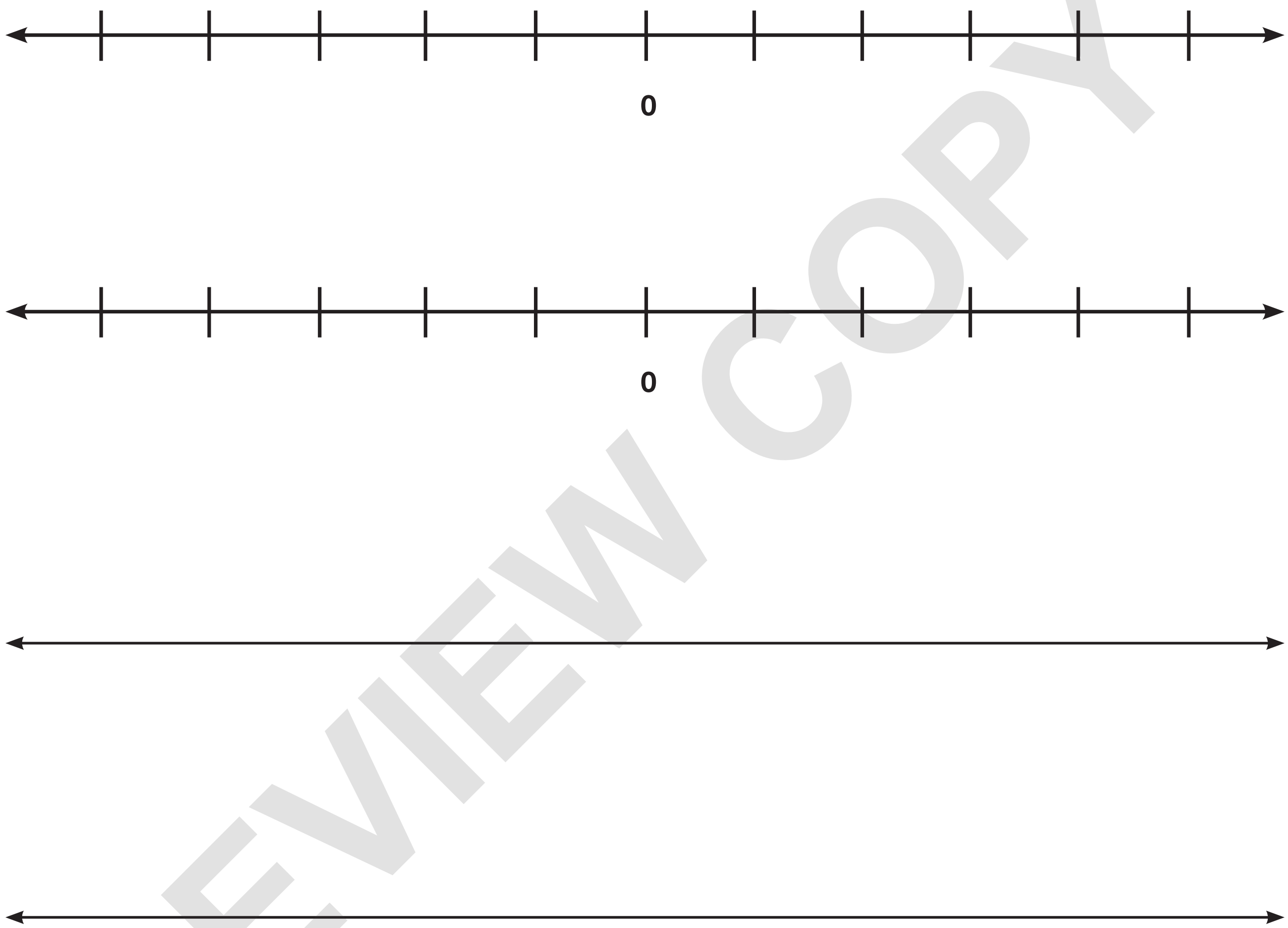
Flowchart



Web

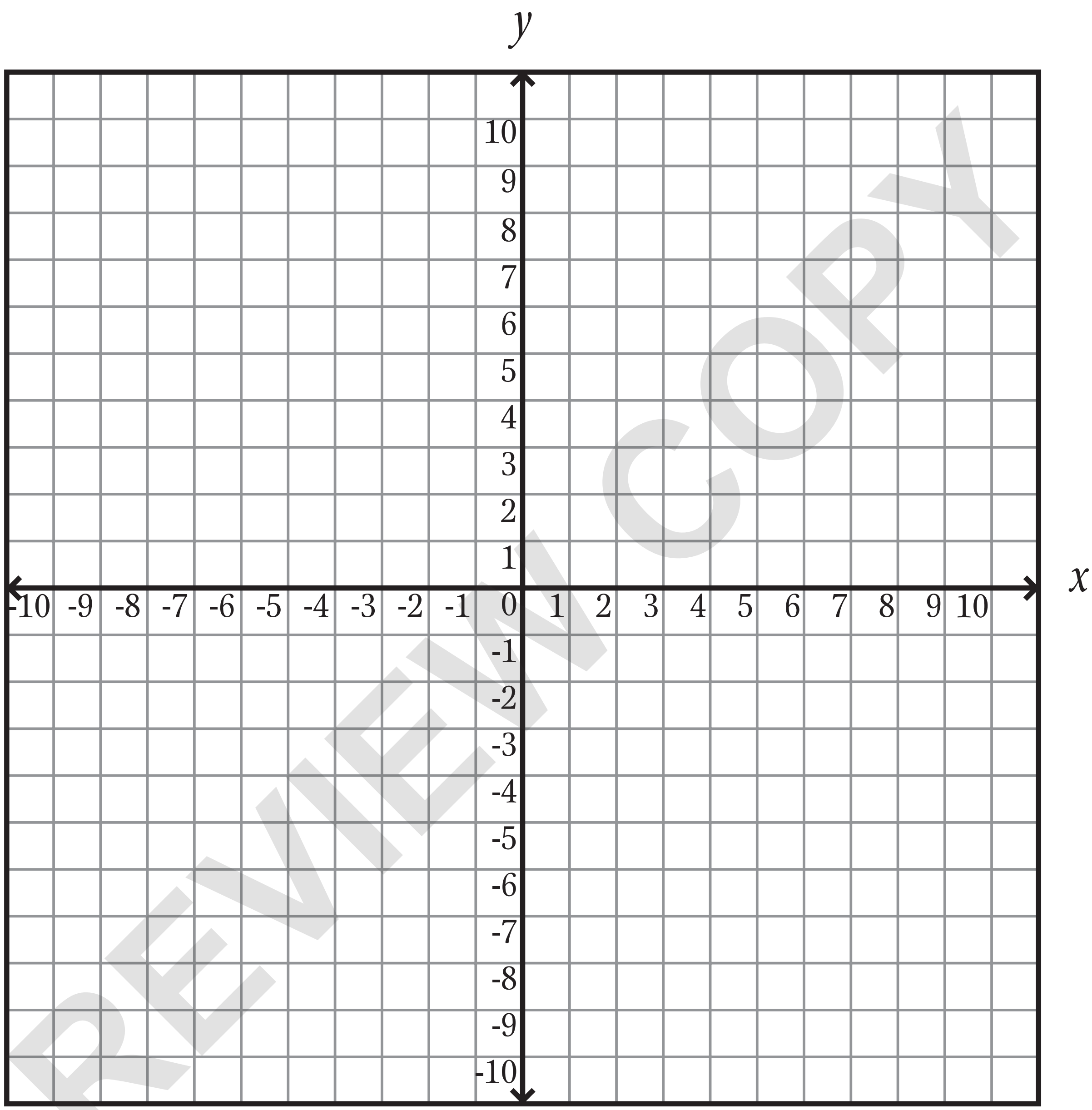


Number Line

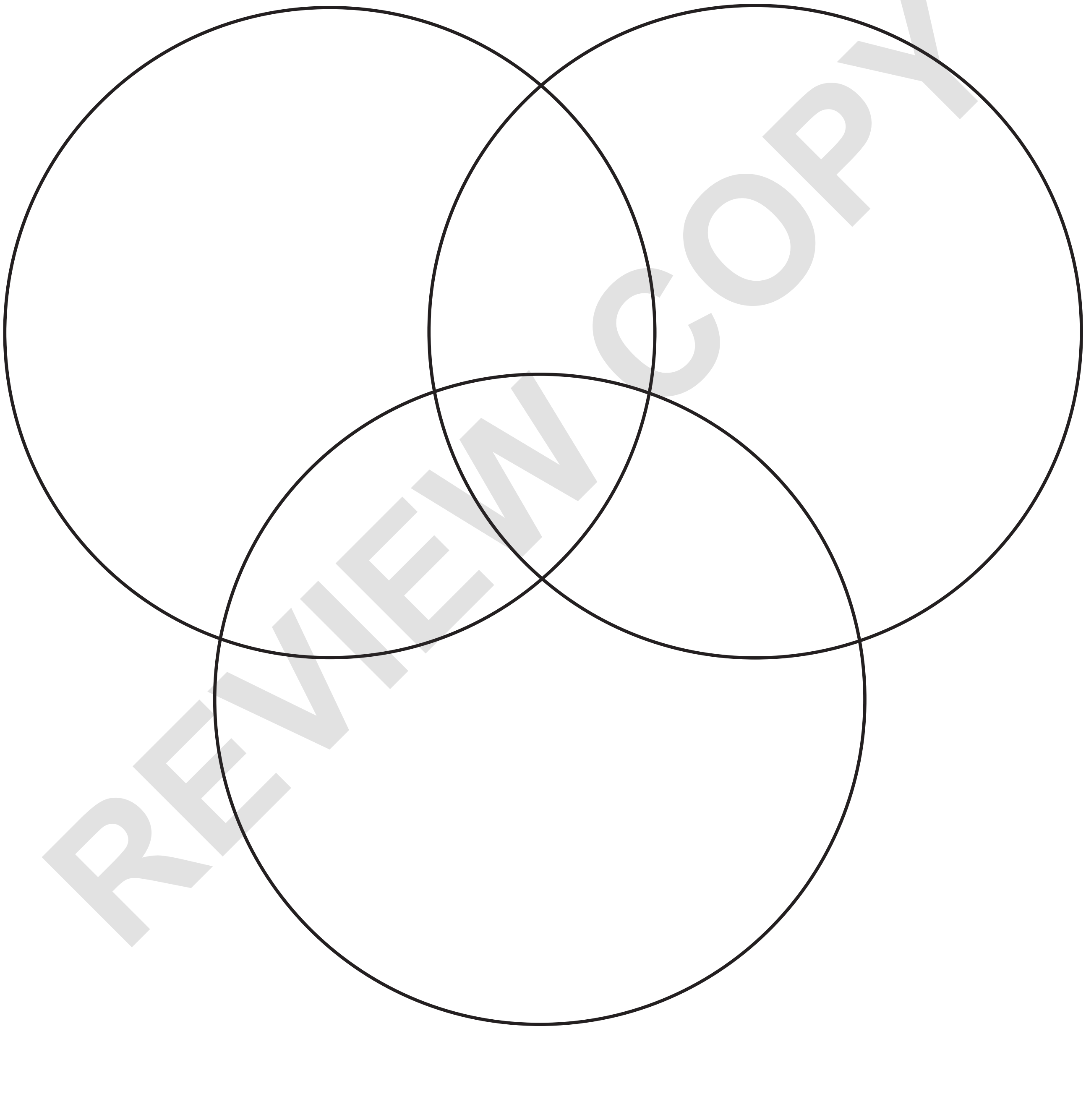


Geometric Drawing

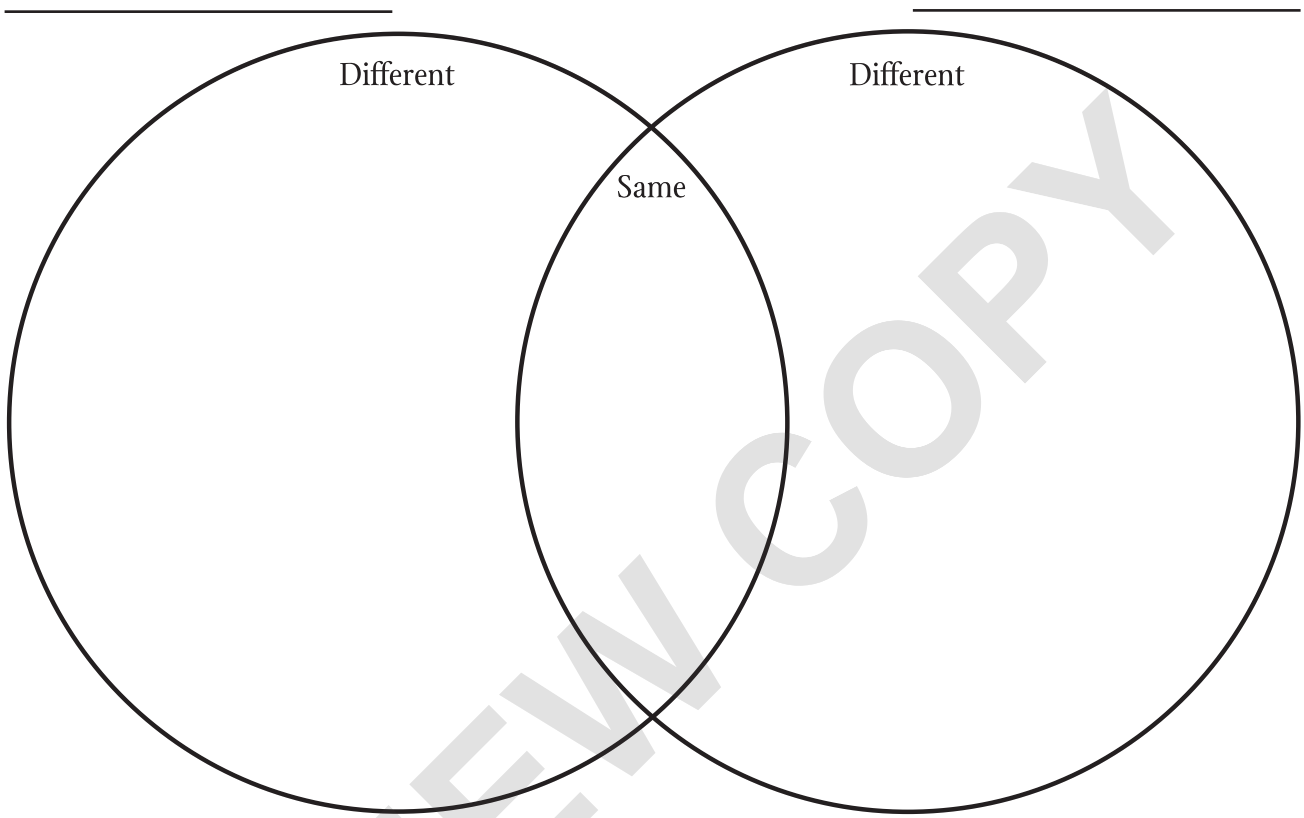
Coordinate Plane



Venn Diagram



Venn Diagram



Compare-and-Contrast Diagram

Item 1 ______________________________ Item 2 ______________________________

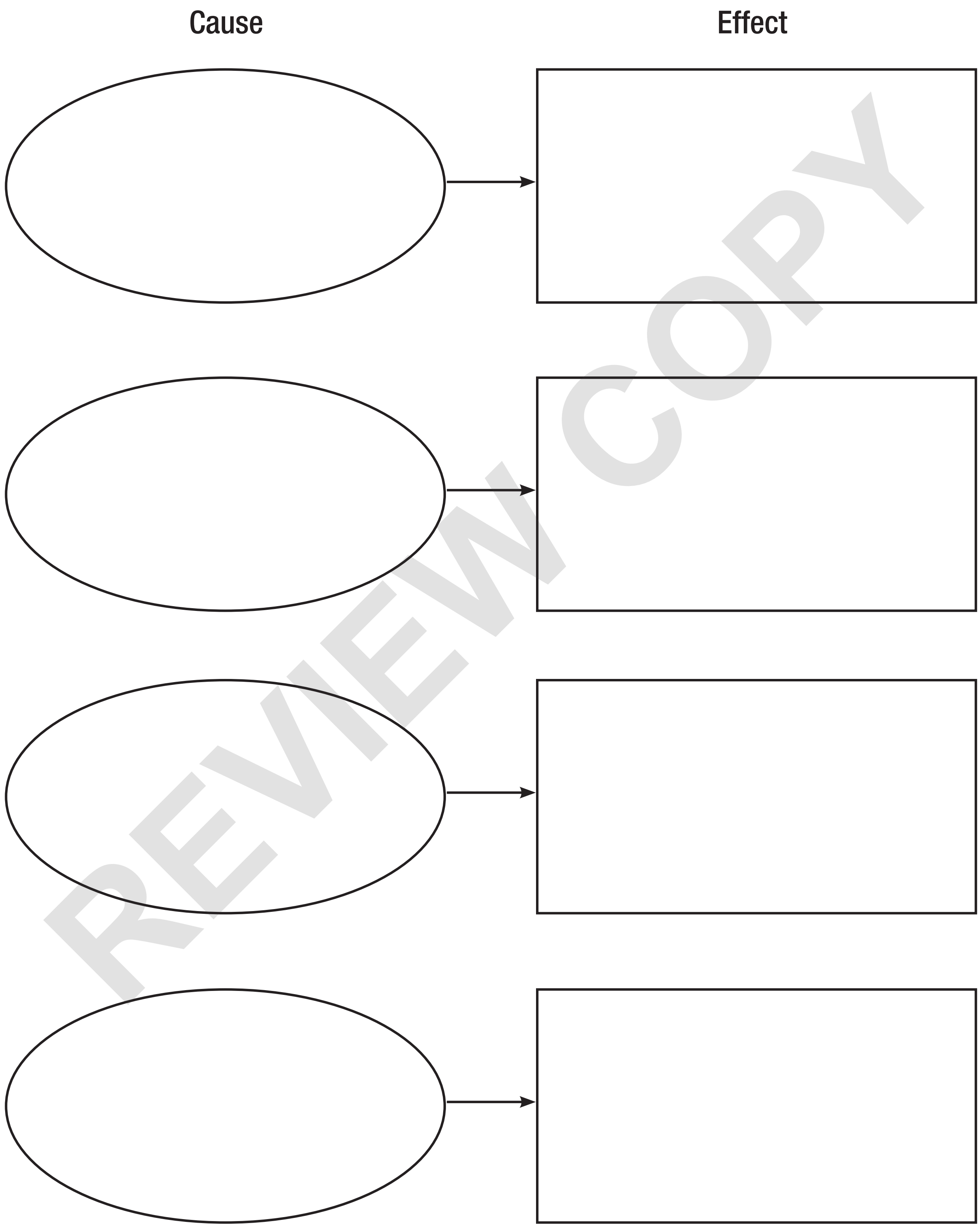
How Alike?

How Different?

With Regard To

Attribute Table

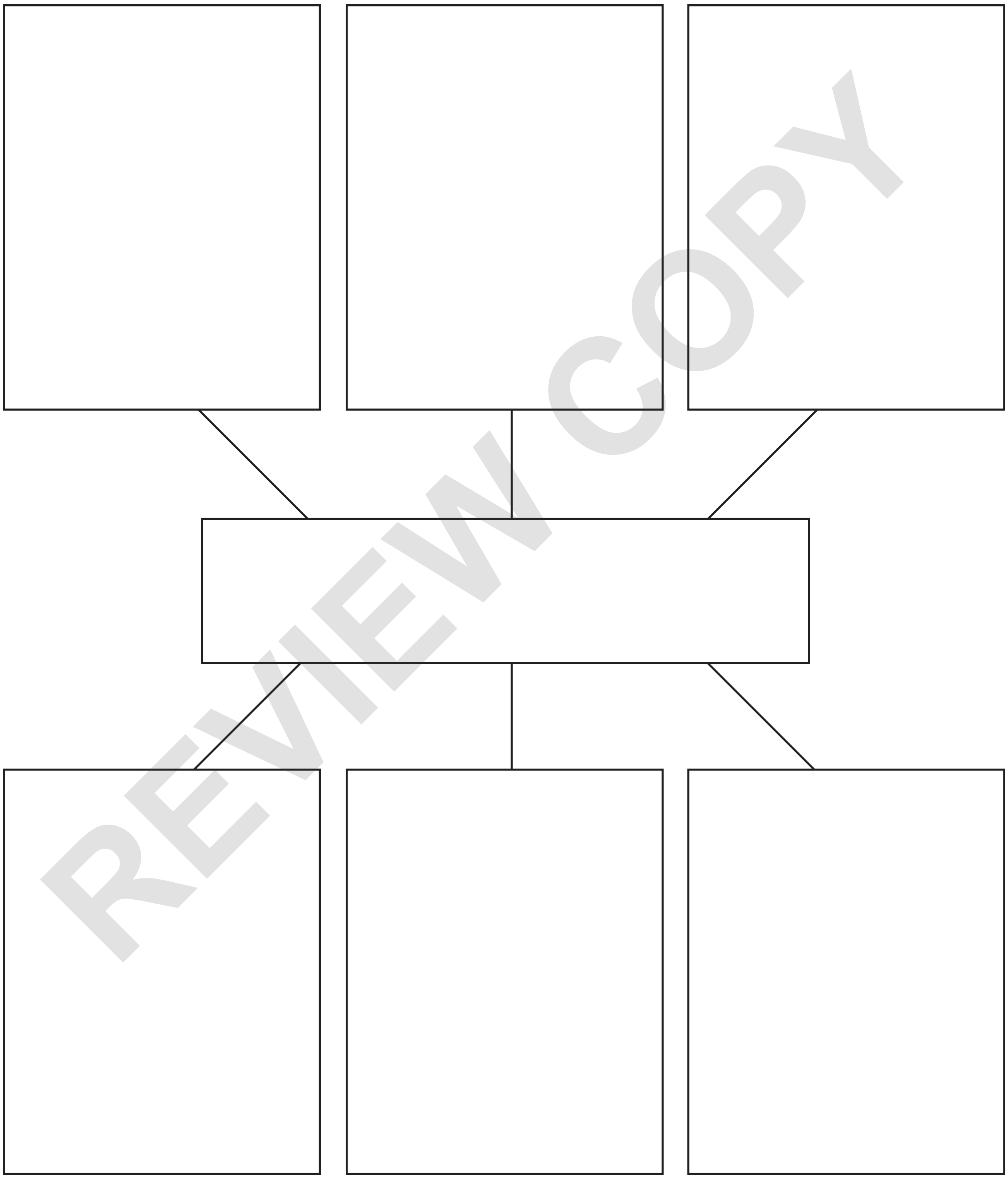
Cause and Effect Map



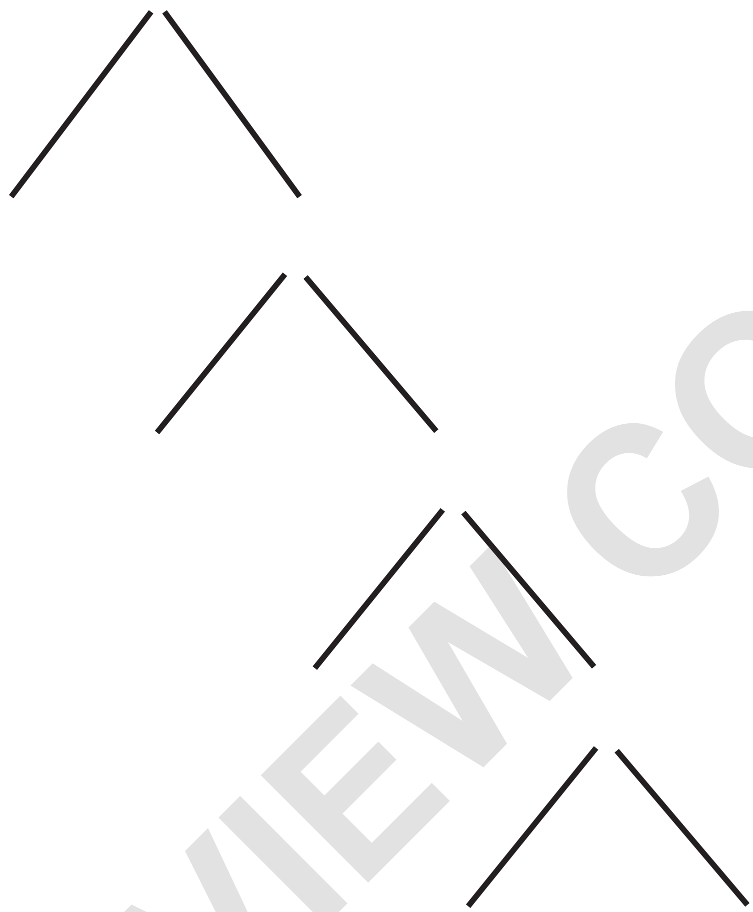
Frayer Model

Definition	Characteristics
WORD	
Examples from Life	Non-Examples

Semantic Map/Concept Map

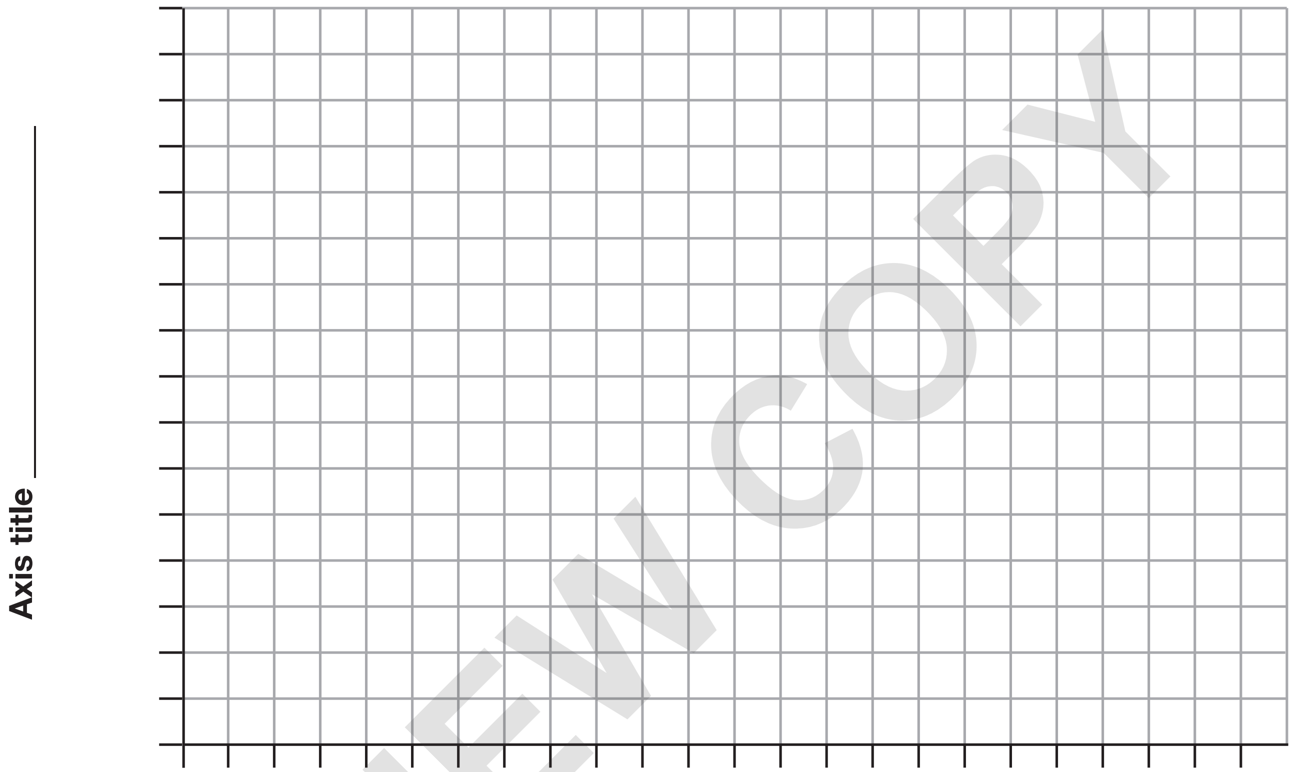


Factor Tree



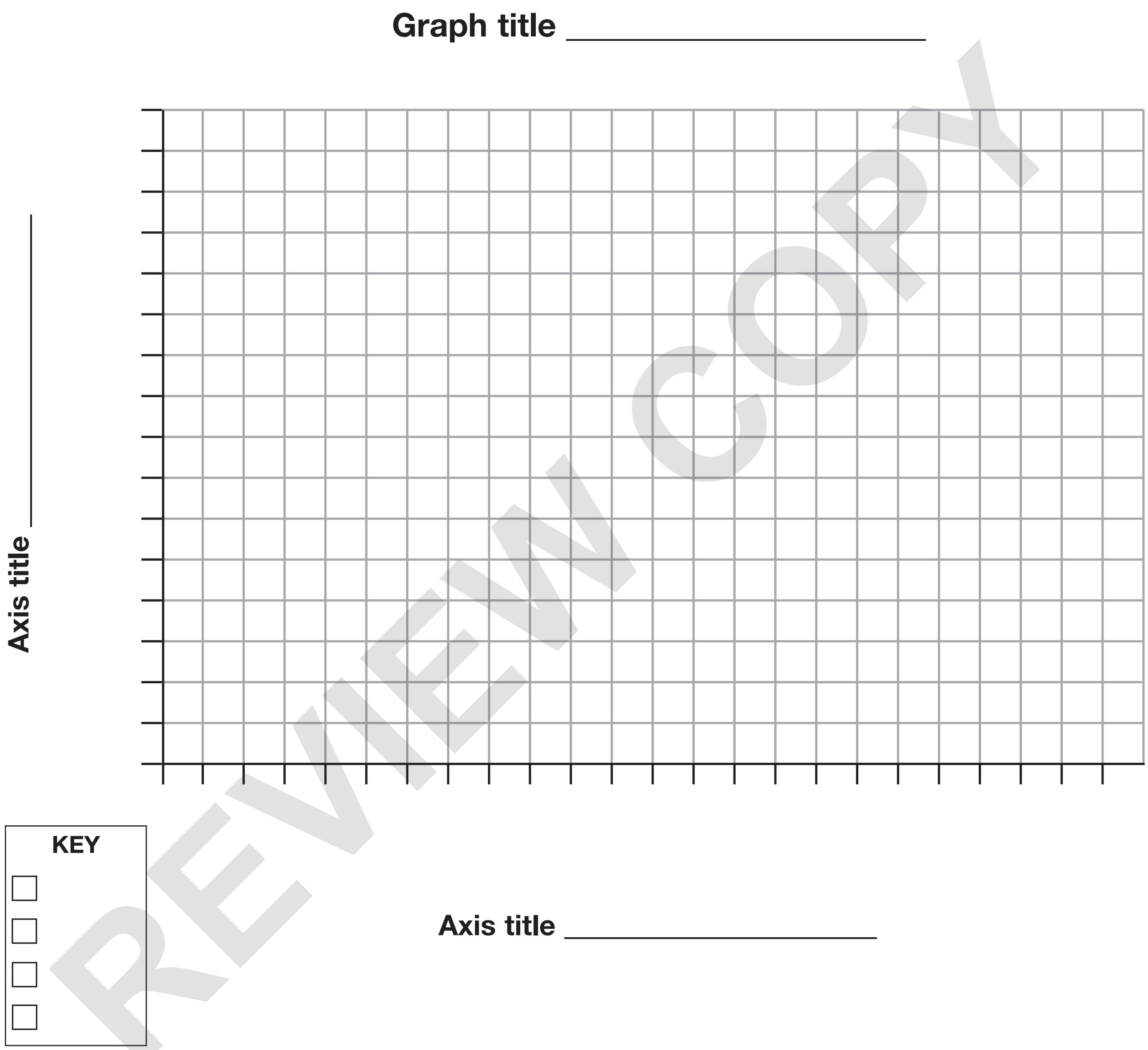
Line Graph

Graph title ______________________

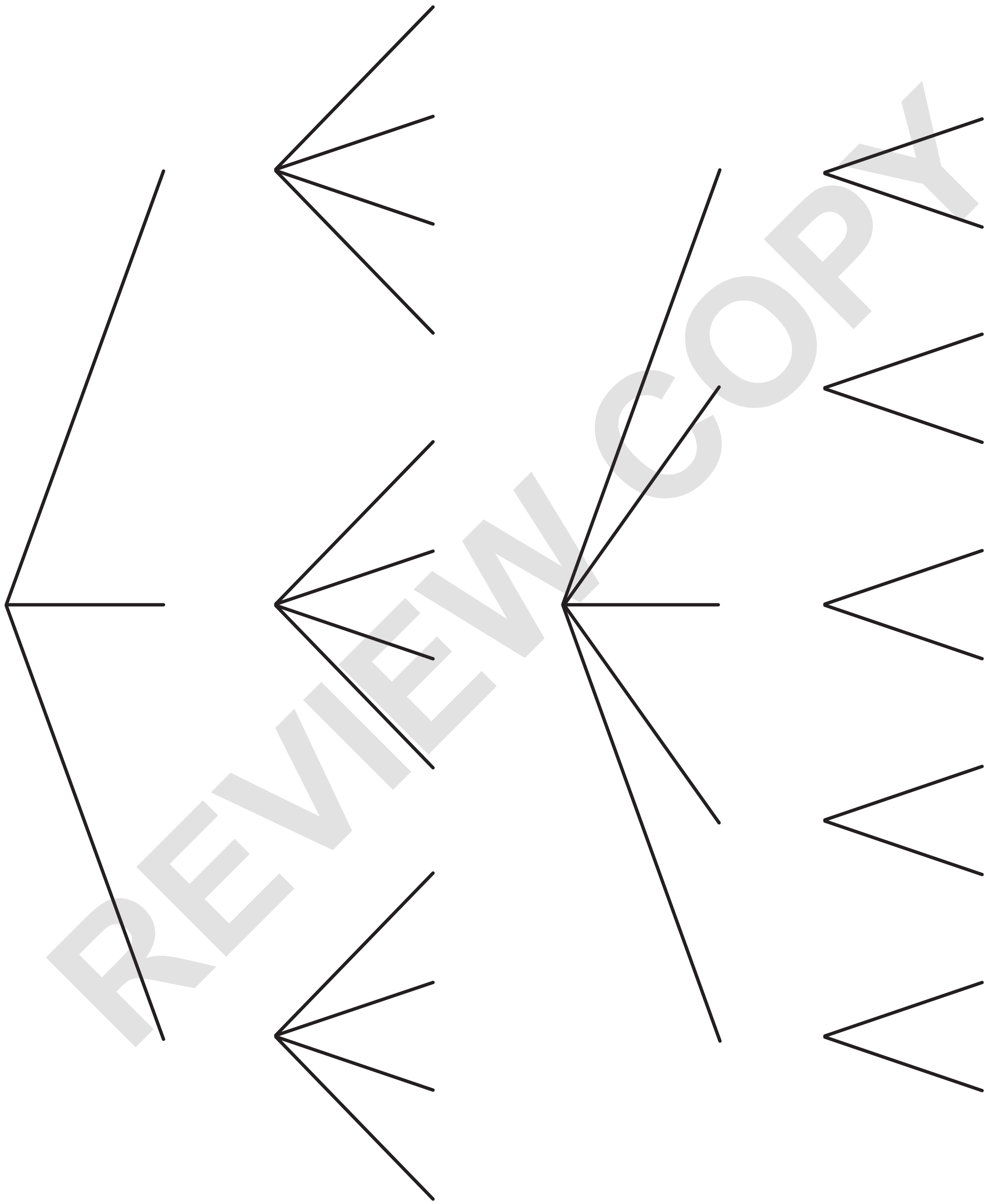


Axis title ____________________

Bar Chart/Histogram



Probability Trees



ALGEBRA

General	
(x, y)	Ordered pair
$(x, 0)$	x-intercept
$(0, y)$	y-intercept

Symbols	
$\approx$	Approximately equal to
$\neq$	Is not equal to
$\lvert a \rvert$	Absolute value of a
$\sqrt{a}$	Square root of a

Linear Equations	
$m=\dfrac{y_2-y_1}{x_2-x_1}$	Slope
$ax + b = c$	One variable
$y = mx + b$	Slope-intercept form
$ax + by = c$	General form
$y - y_1 = m(x - x_1)$	Point-slope form

Arithmetic Sequences	
$a_n = a_1 + (n - 1)d$	Explicit formula
$a_n = a_{n-1} + d$	Recursive formula

Geometric Sequences	
$a_n = a_1 \bullet r^{n-1}$	Explicit formula
$a_n = a_{n-1} \bullet r$	Recursive formula

Exponential Equations	
$y = ab^x$	General form
$y=ab^{\frac{x}{t}}$	Exponential equation
$y = a(1 + r)^t$	Exponential growth
$y = a(1 - r)^t$	Exponential decay

$A=P\left(1+\dfrac{r}{n}\right)^{nt}$	Compounded interest formula
Compounded...	n (number of times per year)
Yearly/annually	1
Semiannually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

Functions	
$f(x)$	Notation, "f of x"
$f(x) = mx + b$	Linear function
$f(x) = b^x + k$	Exponential function
$(f + g)(x) = f(x) + g(x)$	Addition
$(f - g)(x) = f(x) - g(x)$	Subtraction
$(f \bullet g)(x) = f(x) \bullet g(x)$	Multiplication
$(f \div g)(x) = f(x) \div g(x)$	Division

Properties of Equality	
Property	**In symbols**
Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$ and $c \neq 0$, then $a \bullet c = b \bullet c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Properties of Operations	
Property	**General rule**
Commutative property of addition	$a + b = b + a$
Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of multiplication	$a \bullet b = b \bullet a$
Associative property of multiplication	$(a \bullet b) \bullet c = a \bullet (b \bullet c)$
Distributive property of multiplication over addition	$a \bullet (b + c) = a \bullet b + a \bullet c$

Properties of Inequality
Property
If $a > b$ and $b > c$, then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \bullet c > b \bullet c$.
If $a > b$ and $c < 0$, then $a \bullet c < b \bullet c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Laws of Exponents	
Law	**General rule**
Multiplication of exponents	$b^m \bullet b^n = b^{m+n}$
Power of exponents	$\left(b^m\right)^n = b^{mn}$ $\left(bc\right)^n = b^n c^n$
Division of exponents	$\dfrac{b^m}{b^n} = b^{m-n}$
Exponents of zero	$b^0 = 1$
Negative exponents	$b^{-n} = \dfrac{1}{b^n}$ and $\dfrac{1}{b^{-n}} = b^n$

DATA ANALYSIS

$IQR = Q_3 - Q_1$	Interquartile range
$Q_1 - 1.5(IQR)$	Lower outlier formula
$Q_3 + 1.5(IQR)$	Upper outlier formula
$y - y_0$	Residual formula

GEOMETRY

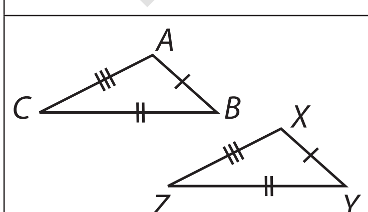
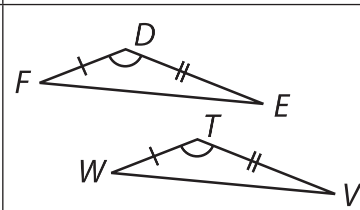
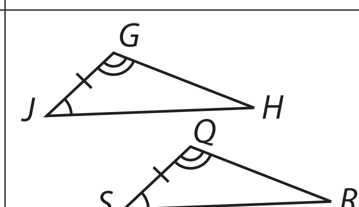
Symbols	
$d\left(\overset{\frown}{ABC}\right)$	Arc length
$\angle$	Angle
$\odot$	Circle
$\cong$	Congruent
$\overleftrightarrow{PQ}$	Line
$\overline{PQ}$	Line Segment
$\overrightarrow{PQ}$	Ray
$\parallel$	Parallel
$\perp$	Perpendicular
$\bullet$	Point
$\triangle$	Triangle
A'	Prime
$\circ$	Degrees

Translations	
$T_{(h,\,k)} = (x + h, y + k)$	Translation

Reflections	
$r_{x\text{-axis}}(x, y) = (x, -y)$	Through the x-axis
$r_{y\text{-axis}}(x, y) = (-x, y)$	Through the y-axis
$r_{y=x}(x, y) = (y, x)$	Through the line $y = x$

Rotations	
$R_{90}(x, y) = (-y, x)$	Counterclockwise 90° about the origin
$R_{180}(x, y) = (-x, -y)$	Counterclockwise 180° about the origin
$R_{270}(x, y) = (y, -x)$	Counterclockwise 270° about the origin

Congruent Triangle Statements

Side-Side-Side (SSS)	Side-Angle-Side (SAS)	Angle-Side-Angle (ASA)
		
$\triangle ABC \cong \triangle XYZ$	$\triangle DEF \cong \triangle TVW$	$\triangle GHJ \cong \triangle QRS$

Pythagorean Theorem
$a^2 + b^2 = c^2$

Distance Formula	
$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Distance formula

Area	
$A = lw$	Rectangle
$A=\frac{1}{2}bh$	Triangle

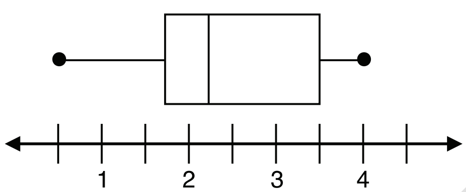
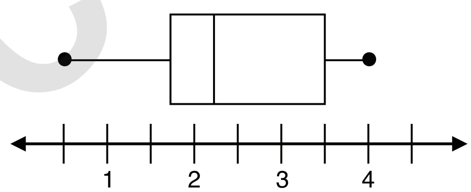
MEASUREMENTS

Length
Metric
1 kilometer (km) = 1000 meters (m)
1 meter (m) = 100 centimeters (cm)
1 centimeter (cm) = 10 millimeters (mm)
Customary
1 mile (mi) = 1760 yards (yd)
1 mile (mi) = 5280 feet (ft)
1 yard (yd) = 3 feet (ft)
1 foot (ft) = 12 inches (in)

Volume and Capacity
Metric
1 liter (L) = 1000 milliliters (mL)
Customary
1 gallon (gal) = 4 quarts (qt)
1 quart (qt) = 2 pints (pt)
1 pint (pt) = 2 cups (c)
1 cup (c) = 8 fluid ounces (fl oz)

Weight and Mass
Metric
1 kilogram (kg) = 1000 grams (g)
1 gram (g) = 1000 milligrams (mg)
1 metric ton (MT) = 1000 kilograms (kg)
Customary
1 ton (T) = 2000 pounds (lb)
1 pound (lb) = 16 ounces (oz)

English	Español
A	
accuracy closeness of a measurement to the actual value of the dimension being measured. For example, a measurement of 1.99999 cm for an object that is 2 cm wide has a high level of accuracy.	**exactitud** la proximidad de una medida al valor real de la dimensión que se está midiendo. Por ejemplo, una medida de 1.99999 cm para un objeto de 2 cm de ancho tiene un alto nivel de precisión.
algebraic expression a mathematical statement that includes numbers, operations, and variables to represent a number or quantity	**expresión algebraica** declaración matemática que incluye números, operaciones y variables para representar un número o una cantidad
area the amount of space inside the boundary of a two-dimensional figure	**área** cantidad de espacio dentro del límite de una figura bidimensional
arithmetic sequence a linear function with a domain of positive consecutive integers in which the difference between any two consecutive terms is equal	**secuencia aritmética** función lineal con dominio de enteros consecutivos positivos, en la que la diferencia entre dos términos consecutivos es equivalente
asymptote a line that a function gets closer and closer to as one of the variables increases or decreases without bound	**asíntota** una línea que una función se acerca cada vez más cerca de una de las variables aumenta o disminuye sin límite
average rate of change the ratio of the difference of output values to the difference of the corresponding input values: $\frac{f(b)-f(a)}{b-a}$; a measure of how a quantity changes over some interval	**tasa de cambio promedio** proporción de la diferencia de valores de salida a la diferencia de valores correspondientes de entrada: $\frac{f(b)-f(a)}{b-a}$; medida de cuánto cambia una cantidad en cierto intervalo
axis of symmetry of a parabola the line through the vertex of a parabola about which the parabola is symmetric. The equation of the axis of symmetry is $x=\frac{-b}{2a}$.	**eje de simetría de una parábola** línea que atraviesa el vértice de una parábola sobre la que la parábola es simétrica. La ecuación del eje de simetría es $x=\frac{-b}{2a}$.

English	Español
B	
binomial a polynomial with two terms	**binomio** polinomio con dos términos
boundary line the graph of the line that represents a linear inequality and that divides the coordinate plane into two half planes, one of which contains all the solutions of the inequality	**línea de límite** la gráfica de la línea que representa una desigualdad lineal y que divide el plano de coordenadas en dos medios planos, uno de los cuales contiene todas las soluciones de la desigualdad
box plot a plot showing the minimum, maximum, first quartile, median, and third quartile of a data set; the middle 50% of the data is indicated by a box. Example: 	**diagrama de caja** diagrama que muestra el mínimo, máximo, primer cuartil, mediana y tercer cuartil de un conjunto de datos; se indica con una caja el 50% medio de los datos. Ejemplo: 
C	
causation a relationship between two events where a change in one event is responsible for a change in the second event	**causalidad** relación entre dos eventos en la que un cambio en un evento es responsable por un cambio en el segundo evento
closure a system is closed, or shows closure, under an operation if the result of the operation is within the system	**cierre** un sistema es cerrado, o tiene cierre, en una operación si el resultado de la misma está dentro del sistema
common difference the number added to each consecutive term in an arithmetic sequence	**diferencia común** número sumado a cada término consecutivo en una secuencia aritmética
common ratio the number that each consecutive term is multiplied by in a geometric sequence	**proporción constante** el número que cada término esta multiplicado por en una secuencia geométrica
compound interest interest earned on both the initial amount and on previously earned interest; the formula is $A = P\left(1+\frac{r}{n}\right)^{nt}$	**interés compuesto** interés devengado tanto de la cantidad inicial como del interés previamente devengado; la fórmula es $A = P\left(1+\frac{r}{n}\right)^{nt}$

English	Español
concave down a graph of a curve that is bent downward, such as a quadratic function with a maximum value	**cóncavo hacia abajo** gráfico de una curva que se inclina hacia abajo, tal como una función cuadrática con un valor máximo
concave up a graph of a curve that is bent upward, such as a quadratic function with a minimum value	**cóncavo hacia arriba** gráfico de una curva que se inclina hacia arriba, tal como una función cuadrática con un valor mínimo
concavity with respect to a curve, the property of being arched upward or downward. A quadratic with positive concavity will increase on either side of the vertex, meaning that the vertex is the minimum or lowest point of the curve. A quadratic with negative concavity will decrease on either side of the vertex, meaning that the vertex is the maximum or highest point of the curve.	**concavidad** con respecto a una curva, la propiedad de ser arqueado hacia arriba o hacia abajo. Una función cuadrática con concavidad positiva se incrementará en ambos lados del vértice, lo que significa que el vértice es el punto mínimo o más bajo de la curva. Una función cuadrática con concavidad negativa disminuirá a cada lado del vértice, lo que significa que el vértice es el punto máximo o más alto de la curva.
congruent having the same shape, size, lines, and angles; the symbol for congruent is $\cong$.	**congruente** tiene la misma forma, tamaño, líneas y anglos; el símbolo paracongruente es $\cong$.
constraint a restriction or limitation on any of the variables in an equation or inequality	**limitación** una restricción o limitación de cualquiera de las variables en una ecuación o desigualdad
continuous having no breaks	**continuo** sin interrupciones
conversion factor a ratio of quantities given in different units that are equivalent. For example, the ratio $\frac{12\text{ inches}}{1\text{ foot}}$ is a conversion factor.	**factor de conversión** una relación de cantidades dadas en diferentes unidades que son equivalentes. Por ejemplo, la relación $\frac{12\text{ inches}}{1\text{ foot}}$ es un factor de conversión.
coordinate plane a plane determined by a set of two number lines, called the axes, that intersect at right angles	**plano de coordenadas** un plano determinado por un conjunto de dos líneas numéricas, llamadas los ejes, que se cruzan en ángulos rectos

English	Español
correlation a relationship between two events, where a change in one event is related to a change in the second event. A correlation between two events does not imply that the first event is responsible for the change in the second event; the correlation only shows how likely it is that a change also took place in the second event.	**correlación** relación entre dos eventos en la que el cambio en un evento se relaciona con un cambio en el segundo evento. Una correlación entre dos eventos no implica que el primero sea responsable del cambio en el segundo; la correlación sólo demuestra cuán probable es que también se produzca un cambio en el segundo evento.
correlation coefficient a quantity that assesses the strength of a linear relationship between two variables, ranging from –1 to 1; a correlation coefficient of –1 indicates a strong negative correlation, a correlation coefficient of 1 indicates a strong positive correlation, and a correlation coefficient of 0 indicates a very weak or no linear correlation	**coeficiente de correlación** cantidad que evalúa la fuerza de una relación lineal entre dos variables, que varía de –1 a 1; un coeficiente de correlación de –1 indica una fuerte correlación negativa, un coeficiente de correlación de 1 indica una fuerte correlación positiva, y un coeficiente de correlación de 0 indica una correlación muy débil o no lineal
cube root function a function that contains the cube root of a variable. The general form is $f(x)=a\sqrt[3]{(x-h)-k}$, where a, h, and k are real numbers.	**función raíz cúbica** función que contiene la raíz cúbica de una variable. La forma general es $y=a\sqrt[3]{(x-h)}+k$, donde a, h, y k son números reales.
curve the graphical representation of the solution set for $y = f(x)$; in the special case of a linear equation, the curve will be a line	**curva** representación gráfica del conjunto de soluciones para $y = f(x)$; en el caso especial de una ecuación lineal, la curva será una recta

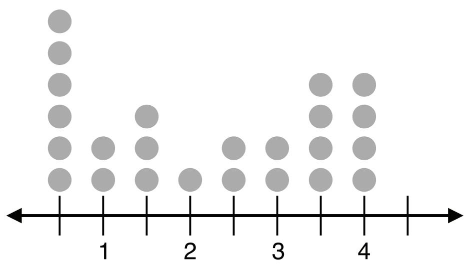
D

English	Español
decreasing the interval of a function for which the output values are becoming smaller as the input values are becoming larger	**decreciente** intervalo de una función por el que los valores de salida se hacen más pequeños a medida que los valores de entrada se hacen más grandes
dependent variable 1. generally labeled on the y-axis; the quantity that is based on the input values of the independent variable 2. a variable whose value is dependent on another variable	**variable dependiente** 1. generalmente designada en el eje y; cantidad que se basa en los valores de entrada de la variable independiente 2. una variable cuyo valor depende de otra variable

English	Español
difference of two squares a pattern for factoring a binomial that consists of two perfect squares that are being subtracted; for example, $x^2 - y^2 = (x + y)(x - y)$	**diferencia de dos cuadrados** un patrón para factorizar un binomio que consiste en dos cuadrados perfectos que se están restando; por ejemplo
dilation 1. a function transformation in which a figure is either stretched or compressed along an axis 2. a transformation that stretches or compresses a function either vertically or horizontally. A function that is being vertically dilated is characterized by $g(x) = k \bullet f(x)$, while a function that is being horizontally dilated is characterized by $g(x) = f(kx)$.	**dilatación** 1. una transformación de función en la que una figura es estirada o comprimida a lo largo de un eje 2. una transformación que estira o comprime una función ya sea vertical u horizontalmente. Una función que está siendo dilatada verticalmente se caracteriza por $g(x) = k \bullet f(x)$, mientras que una función que está siendo dilatada horizontalmente se caracteriza por $g(x) = f(kx)$.
discrete individually separate and distinct	**discreto** individualmente aparte y distinto
discriminant an expression whose solved value indicates the number and types of solutions for a quadratic. For a quadratic equation in standard form ($ax^2 + bx + c = 0$), the discriminant is $b^2 - 4ac$.	**discriminante** expresión cuyo valor resuelto indica la cantidad y los tipos de soluciones para una ecuación cuadrática. En una ecuación cuadrática en forma estándar ($ax^2 + bx + c = 0$), el discriminante es $b^2 - 4ac$.
distance formula a formula that states the distance between points (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	**fórmula de distancia** fórmula que establece la distancia entre los puntos (x_1, y_1) y (x_2, y_2) equivale a $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
domain 1. the set of all inputs values for which a function is defined; the set of x-values that are valid for a relation or function 2. the set of all input values (x-values) that satisfy the given function without restriction	**dominio** 1. el conjunto de todos los valores de entradas para los que se define una función; el conjunto de valores x que son válidos para una relación o función 2. conjunto de todos los valores de entrada (valores de x) que satisfacen la función dada sin restricciones

English

dot plot a frequency plot that shows the number of times a response occurred in a data set, where each data value is represented by a dot. Example:



E

end behavior the behavior of the graph as x approaches positive infinity and as x approaches negative infinity

equation a mathematical sentence that uses an equal sign (=) to show that two quantities are equal

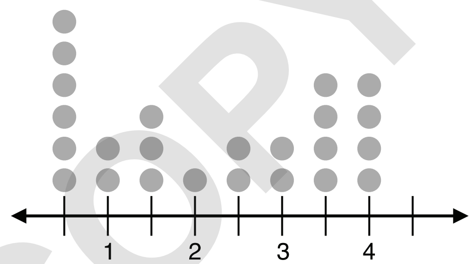
even function 1. a function that, when evaluated for $-x$, results in a function that is the same as the original function; $f(-x) = f(x)$
2. a function that is symmetric across the y-axis. Symbolically, $f(x) = f(-x)$.

explicit formula a formula used to find the nth term of a sequence; if n is the term number, then the explicit formula for the nth term of an arithmetic sequence is $a_n = a_1 + (n - 1)d$, and the explicit formula for the nth term of a geometric sequence is $a_n = a_1 \bullet r^{n-1}$

explicit function a function in which the dependent variable can be written in terms of the independent variable; $f(x) = 2x$ is an explicit function, where x is the independent variable and $f(x)$ is the dependent variable

Español

diagrama de puntos diagrama de frecuencia que muestra la cantidad de veces que se produjo una respuesta en un conjunto de datos, en el que cada valor de dato está representado por un punto. Ejemplo:



comportamiento final el comportamiento de la gráfica al aproximarse x a infinito positivo o a infinito negativo

ecuación declaración matemática que utiliza el signo igual (=) para demostrar que dos cantidades son equivalentes

función par 1. función que, cuando se la evalúa para $-x$, tiene como resultado una función que es igual a la original; $f(-x) = f(x)$
2. una función que es simétrica a través del eje y. Simbólicamente, $f(x) = f(-x)$.

fórmula explícita una fórmula usada para encontrar el término n de una secuencia; si n es el número del término, entonces la fórmula explícita para el término n de una secuencia aritmética es $a_n = a_1 + (n - 1)d$, y la fórmula explícita para el término n de una secuencia geométrica es $a_n = a_1 \bullet r^{n-1}$

función explícita una función en la que la variable dependiente se puede escribir en términos de la variable independiente; $f(x) = 2x$ es una función explícita, donde x es la variable independiente y $f(x)$ es la variable dependiente

English	Español
exponent the number of times a factor is being multiplied together in an exponential expression; in the expression a^b, b is the exponent	**exponente** cantidad de veces que se multiplica un factor en forma conjunta en una expresión exponencial; en la expresión a^b, b es el exponente
exponential decay 1. a decreasing exponential function 2. an exponential equation with a base, b, that is between 0 and 1 exclusive (that is, $0 < b < 1$); an example is the formula $y = a(1 - r)^t$, where a is the initial value, $(1 - r)$ is the base (with $0 < r < 1$), t is the variable exponent, and y is the final value	**decaimiento exponencial** 1. una función exponencial decreciente 2. ecuación exponencial con una base, b, que está entre 0 y 1 en forma exclusiva (es decir, $0 < b < 1$); un ejemplo es la fórmula $y = a(1 - r)^t$, en la que a es el valor inicial, $(1 - r)$ es la base (con $0 < r < 1$), t es el exponente variable, y y es el valor final
exponential equation an equation whose independent variable is in the exponent; the general form of its equation is $f(x) = ab^x + k$, where a is the initial value, b is the base, x is the input value, k is the vertical shift, and $f(x)$ is the output. Another form is $y = ab^{\frac{x}{t}}$, where t is the interval over which y changes by a factor of b, and x is measured in the same units as t.	**ecuación exponencial** ecuación cuya variable independiente es en el exponente; la forma general de su ecuación es $f(x) = ab^x + k$, donde a es el valor inicial, b es la base, x es el valor de entrada, k es el desplazamiento vertical y $f(x)$ es el valor de salida. Otra forma es $y = ab^{\frac{x}{t}}$, donde t es el intervalo en el que y cambia por un factor de b, y x se mide en las mismas unidades como t.
exponential function a function whose independent variable is in the exponent; the general form of its equation is $f(x) = ab^x + k$, where a is the initial value, b is the base, x is the input value, k is the vertical shift, and $f(x)$ is the output	**función exponencial** una función cuya variable independiente es en el exponente; la forma general de su ecuación es $f(x) = ab^x + k$, donde a es el valor inicial, b es la base, x es el valor de entrada, k es el desplazamiento vertical y $f(x)$ es el valor de salida
exponential growth an exponential equation with a base, b, greater than 1 ($b > 1$); an example is the formula $y = a(1 + r)^t$, where a is the initial value, $(1 + r)$ is the base (with $r > 1$), t is the variable exponent, and y is the final value	**crecimiento exponencial** una ecuación exponencial con una base, b, mayor que 1 ($b > 1$); un ejemplo es la fórmula $y = a(1 + r)^t$, donde a es el valor inicial, $(1 + r)$ es la base (con $r > 1$), t es el exponente variable ey y es el valor final

English	Español
expression a combination of variables, quantities, and mathematical operations; 4, $8x$, and $b + 10^2$ are all expressions	**expresión** combinación de variables, cantidades y operaciones matemáticas; 4, $8x$ y $b + 10^2$ son todas expresiones
extrema the minima or maxima of a function	**extremos** los mínimos o máximos de una función
F	
factor (noun) one of two or more numbers or expressions that when multiplied produce a given product	**factor (sustantivo)** uno de dos o más números o expresiones que al multiplicarse dan un producto determinado
factor (verb) to write an expression as the product of its factors	**factorizar (verbo)** escribir una expresión como el producto de sus factores
factored form of a quadratic function the intercept form of a quadratic equation, written as $f(x) = a(x - p)(x - q)$, where p and q are the x-intercepts of the function; also known as the *intercept form of a quadratic function*	**forma factorizada de una función cuadrática** forma de intercepto de una ecuación cuadrática, se expresa como $f(x) = a(x - p)(x - q)$, en la que p y q son los interceptos de x de la función; también se conoce como la *forma de intercepto de una función cuadrática*
first difference the difference between one dependent value and the next	**primera diferencia** la diferencia entre un valor dependiente y el siguiente
first quartile the value that identifies the lower 25% of the data; the median of the lower half of the data set; written as Q_1	**primer cuartil** valor que identifica el 25% inferior de los datos; mediana de la mitad inferior del conjunto de datos; se expresa Q_1
function a relation in which each element in the domain is mapped onto exactly one element in the range; that is, for every value of x, there is exactly one value of y	**función** relación en la que cada elemento de un dominio se combina con exactamente un elemento del rango; es decir, para cada valor de x, existe exactamente un valor de y
function notation a way to name a function using $f(x)$ to represent the dependent variable instead of y	**notación de función** forma de nombrar una función con el uso de $f(x)$ como la variable dependiente en lugar de y
function transformation a movement or stretching of the graph of the function, caused by adding or multiplying a constant to the function	**transformación de funcion** un movimiento o estiramiento de la gráfica de la función, causada por la adición o multiplicación de una constante a la función

English	Español

G

geometric sequence an exponential function that results in a sequence of numbers separated by a common ratio

secuencia geométrica una función exponencial que produce como resultado una secuencia de números separados por una relación común

greatest common factor (GCF) the largest factor that two or more terms share

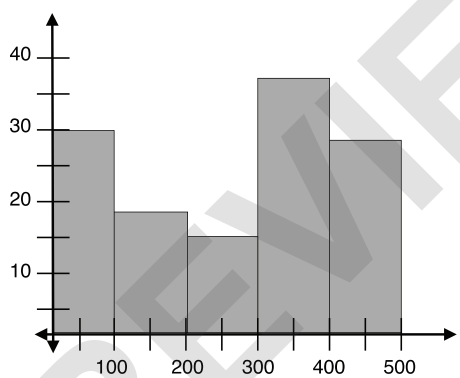
máximo común divisor (GCF) el factor más grande que comparten dos o más términos

H

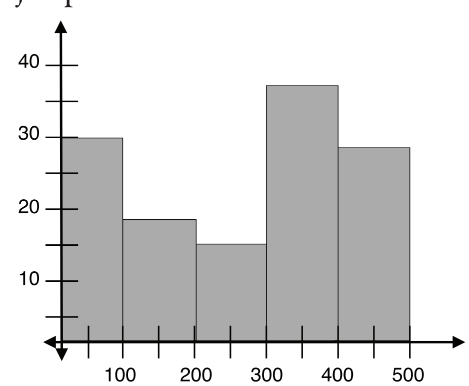
half plane a planar region containing all points that lie on one side of a boundary line; one-half of a plane

semiplano una región plana que contiene todos los puntos que se encuentran en un lado de una línea de límite; la mitad de un avión

histogram a frequency plot that shows the number of times a response or range of responses occurred in a data set. Example:



histograma una diagrama de frecuencia que muestra la cantidad de veces que se produce una respuesta o rango de respuestas en un conjunto de datos. Ejemplo:



horizontal asymptote a line defined as follows: The line $y = b$ is a horizontal asymptote of the graph of a function f if $f(x)$ gets closer to b as x either increases or decreases without bound.

asíntota horizontal línea recta que se define de la siguiente manera: La línea $y = b$ es una asíntota horizontal del gráfico de una función f si $f(x)$ se acerca a b a medida que x aumenta o disminuye sin límites.

horizontal compression squeezing of the parabola toward the y-axis

compresión horizontal contracción de la parábola hacia el eje y

horizontal stretch pulling of the parabola and stretching it away from the y-axis

estiramiento horizontal jalar de la parábola y estirarla lejos del eje y

English	Español
I	
inclusive when the points in a plane that lie along the boundary line of an inequality are included in the solution	**inclusivo** cuando los puntos en un plano que se encuentran a lo largo de la línea de límite de una desigualdad se incluyen en la solución
increasing the interval of a function for which the output values are becoming larger as the input values are becoming larger	**creciente** intervalo de una función para el que los valores de salida se hacen más grandes a medida que los valores de entrada también se vuelven más grandes
independent variable 1. generally labeled on the x-axis; the quantity that changes based on values chosen 2. a variable whose value can be freely chosen and not without considering the value of other variables	**variable independiente** 1. generalmente designada en el eje x; cantidad que cambia según valores seleccionados 2. una variable cuyo valor puede ser elegido libremente y no sin considerar el valor de otras variables
inequality a mathematical sentence that shows the relationship between quantities that may or may not be equivalent. An inequality contains one or more of the following symbols: $<$, $>$, $\le$, $\ge$, or $\ne$.	**desigualdad** enunciado matemático que demuestra la relación entre cantidades que pueden ser o no equivalentes. Una desigualdad contiene uno o más de los siguientes símbolos: $<$, $>$, $\le$, $\ge$ o $\ne$.
integers the set of positive and negative whole numbers and 0; the set $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$	**enteros** el conjunto de números enteros positivos y negativos y 0; el conjunto $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$
intercept 1. the value of the x- or y-coordinate where a line or curve intersects the x- or y-axis, respectively 2. the point at which a line intercepts the x- or y-axis	**intersección** 1. valor de la coordenada x o y donde una línea o curva interseca el eje x o y, respectivamente 2. punto en el que una línea intercepta el eje x o y
intercept form of a quadratic function the factored form of a quadratic equation, written as $f(x) = a(x - p)(x - q)$, where p and q are the x-intercepts of the function; also known as the *factored form of a quadratic function*	**forma de intercepto de una función cuadrática** forma factorizada de una ecuación cuadrática, expresada como $f(x) = a(x - p)(x - q)$, donde p y q son los interceptos de x de la función; también se conoce como la *forma factorizada de una ecuación cuadrática*

English	Español
interquartile range the difference between the third and first quartiles; 50% of the data is contained within this range	**rango intercuartílico** diferencia entre el tercer y primer cuartil; el 50% de los datos está contenido dentro de este rango
interval 1. the continuous set of real numbers between two given numbers 2. the set of all real numbers between two given numbers. The two numbers on the ends are the endpoints. The endpoints might or might not be included in the interval depending on whether the interval is open, closed, or half-open/half-closed.	**intervalo** 1. conjunto continuo de números reales entre dos números dados 2. conjunto de todos los números reales entre dos números dados. Los dos números en los finales son los extremos. Los extremos podrían o no estar incluidos en el intervalo, según si el intervalo está abierto, cerrado, o medio abierto o medio cerrado.
irrational number a real number that cannot be written as $\frac{m}{n}$, where both m and n are integers and $n \neq 0$; a non-terminating or non-repeating decimal	**número irracional** un número real que no puede ser escrito como $\frac{m}{n}$, donde m y n son números enteros y $n \neq 0$; un no-terminación o no repetitivo decimal
irreducible radical a radical whose radicand contains no perfect square factors. In other words, the radical cannot be further reduced. For example, $\sqrt{7}$ is an irreducible radical because the radicand, 7, does not have any perfect square factors.	**radical irredutíble** un radical cuya radicand no contiene factores cuadrados perfectos. En otras palabras, el radical no puede ser más reducido. Por ejemplo, $\sqrt{7}$ es un radical irreducible porque la radicand, 7, no tiene ningún factor cuadrado perfecto.
K	
key features of a quadratic function the x-intercepts, y-intercept, where the function is increasing and decreasing, where the function is positive and negative, relative minimums and maximums, symmetries, and end behavior of the function used to describe, draw, and compare quadratic functions	**características clave de una función cuadrática** interceptos de x, intercepto de y, donde la función aumenta y disminuye, donde la función es positiva y negativa, máximos y mínimos relativos, simetrías y comportamiento final de la función utilizado para describir, dibujar y comparar las funciones cuadráticas
L	
laws of exponents rules that must be followed when working with exponents	**leyes de los exponentes** normas que deben cumplirse cuando se trabaja con exponentes

English	Español
leading coefficient the coefficient of the term with the highest power. For a quadratic equation in standard form ($y = ax^2 + bx + c$), the leading coefficient is a.	**coeficiente líder** coeficiente del término con la mayor potencia. En una ecuación cuadrática en forma estándar ($y = ax^2 + bx + c$), el coeficiente líder es a.
like terms terms that contain the same variables raised to the same power	**términos semejantes** términos que contienen las mismas variables elevadas a la misma potencia
line segment a part of a line that is between two endpoints and that includes the endpoints; written as $\overline{PQ}$	**segmento de línea** parte de una línea que se encuentra entre dos puntos finales y que incluye los puntos finales; escrito como $\overline{PQ}$
linear equation a first-degree equation that can be written in the form $ax + by = c$, where a, b, and c are rational numbers; when written as $y = mx + b$, m is the slope of the line, and b is its y-intercept. The graph of a linear equation is a straight line.	**ecuación lineal** ecuación de primer grado que puede expresarse en la forma $ax + by = c$, donde a, b y c son números racionales; cuando se expresa como $y = mx + b$, m es la pendiente de la recta y b es el intercepto de y. La representación gráfica de una ecuación lineal es una línea recta.
linear fit (or linear model) an approximation of data using a linear function	**ajuste lineal (o modelo lineal)** aproximación de datos con el uso de una función lineal
linear function a first-degree equation that can be written in the form $f(x) = mx + b$, in which m is the slope of the line and b is the y-intercept. The graph of a linear function is a straight line.	**función lineal** una ecuación de primer grado que puede expresarse en la forma $f(x) = mx + b$, en la que m es la pendiente de la recta y b es el intercepto de y. El gráfico de una función lineal es una línea recta.
linear model an approximation of data using a linear function	**modelo lineal** una aproximación de datos con el uso de una función lineal
M	
mathematical model a way of representing a real-world situation, a pattern, or a concept using mathematical tools and language	**modelo matemático** una forma de representar una situación del mundo real, un patrón o un concepto utilizando herramientas y lenguaje matemáticos
maximum the largest y-value of a quadratic equation	**máximo** el mayor valor de y de una ecuación cuadrática

English	Español
mean the average value of a data set, found by summing all values and dividing by the number of data points	**media** valor promedio de un conjunto de datos, que se determina al sumar todos los valores y dividirlos por la cantidad de puntos de datos
mean absolute deviation the average distance between each data point and the mean; found by summing the absolute values of the difference between each data point and the mean, then dividing this sum by the total number of data points	**desviación media absoluta** distancia promedio entre cada punto de datos y la media; se determina al sumar los valores absolutos de la diferencia entre cada punto de datos y la media y luego dividir esta suma por la cantidad total de puntos de datos
measures of center values that describe expected and repeated data values in a data set; the mean and median are two measures of center	**medidas de centro** valores que describen los valores de datos esperados y repetidos de un conjunto de datos; la media y la mediana son dos medidas de centro
measures of variability values that describe how far apart data points are from one another in a data set. Higher measures of variability indicate the data points are more scattered apart from the mean, while smaller measures indicate they are more closely clustered about the mean. Measures of variability include the range, the interquartile range, the mean absolute deviation, and standard deviation.	**medidas de variabilidad** valores que describen qué tan separados están los puntos de datos entre sí en un conjunto de datos. Las medidas más altas de variabilidad indican que los puntos de datos están más dispersos aparte de la media, mientras que las medidas más pequeñas indican que están más agrupados alrededor de la media. Las medidas de variabilidad incluyen el rango, el rango intercuartílico, la desviación absoluta media y la desviación estándar.
median the middle-most value of a data set; 50% of the data is less than this value, and 50% is greater than it	**mediana** valor medio exacto de un conjunto de datos; el 50% de los datos es menor que ese valor, y el otro 50% es mayor
midpoint a point on a line segment that divides the segment into two equal parts	**punto medio** punto en un segmento de recta que lo divide en dos partes iguales
midpoint formula a formula that states the midpoint of a segment created by connecting (x_1, y_1) and (x_2, y_2) is given by the formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$	**fórmula de punto medio** fórmula que establece el punto medio de un segmento creado al conectar (x_1, y_1) con (x_2, y_2) está dado por la fórmula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

English	Español
minimum the smallest y-value of a quadratic equation	**mínimo** el menor valor de y en una ecuación cuadrática
model a mathematical equation that expresses a real-world situation	**modelo** una ecuación matemática que expresa una situación real
monomial an expression with one term, consisting of a number, a variable, or the product of a number and variable(s)	**monomio** expresión con un solo término, que consiste en un número, una variable, o el producto de un número y una o más variables
N	
natural numbers the set of positive integers {1, 2, 3, ...}	**números naturales** conjunto de enteros positivos {1, 2, 3, ...}
negative function a function or a portion of a function where the y-values are less than 0 for all x-values	**función negativa** función o porción de una función en la que los valores y son menores que 0 para todos los valores x
neither describes a function that, when evaluated for $-x$, does not result in the opposite of the original function (odd) or the original function (even)	**ni** describe una función que, cuando se evalúa para $-x$, no tiene como resultado lo opuesto de la función original (impar) ni la función original (par)
non-inclusive when the points in a plane that lie along the boundary line of an inequality are not included in the solution	**no inclusivo** cuando los puntos en un plano que se encuentran a lo largo de la línea de límite de una desigualdad no están incluidos en la solución
O	
odd function 1. a function that, when evaluated for $-x$, results in a function that is the opposite of the original function; $f(-x) = -f(x)$ 2. a function that is symmetric across the origin, $f(-x) = -f(x)$	**función impar** 1. función que, cuando se evalúa para $-x$, tiene como resultado una función que es lo opuesto a la función original; $f(-x) = -f(x)$ 2. una función que es simétrica a través del origen, $f(-x) = -f(x)$
ordered pair the coordinates of a point in a coordinate plane, (x, y) where the order is significant	**par ordenado** coordenadas de un punto en un plano de coordenadas, (x, y), en los que el orden es significativo

English	Español
outlier a data value that is much greater than or much less than the rest of the data in a data set; mathematically, any data less than $Q_1 - 1.5(IQR)$ or greater than $Q_3 + 1.5(IQR)$ is an outlier	**valor atípico** valor de datos que es mucho mayor o mucho menor que el resto de los datos de un conjunto de datos; en matemática, cualquier dato menor que $Q_1 - 1{,}5(IQR)$ o mayor que $Q_3 + 1{,}5(IQR)$ es un valor atípico
P	
parabola the U-shaped graph of a quadratic equation; the set of all points that are equidistant from a fixed line, called the directrix, and a fixed point not on that line, called the focus. The parabola, directrix, and focus are all in the same plane. The vertex of the parabola is the point on the parabola that is closest to the directrix.	**parábola** gráfico de una ecuación cuadrática en forma de U; conjunto de todos los puntos equidistantes de una línea fija denominada directriz y un punto fijo que no está en esa línea, llamado foco. La parábola, la directriz y el foco están todos en el mismo plano. El vértice de la parábola es el punto más cercano a la directriz.
parabolic curve the graph of a quadratic function	**curva parabólica** la gráfica de una función cuadrática
parallel lines lines in a plane that do not share any points and never intersect; written as $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$; line segments and rays can also be parallel	**líneas paralelas** líneas en un plano que no comparten ningún punto y nunca se cortan; se expresan como $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$; segmentos de línea y los rayos también pueden ser paralelos
parallelogram a special type of quadrilateral with two pairs of opposite sides that are parallel; denoted by the symbol ▱	**paralelogramo** un tipo especial de cuadrilátero con dos pares de lados opuestos paralelos; se expresa con el símbolo ▱
parameter numerical value(s) representing the data in a set, including proportion, mean, and variance	**parámetro** valores numéricos que representan los datos en un conjunto, incluyendo la proporción, la media y la varianza
perfect square 1. the product of an integer and itself. For example, 9 is a perfect square because $9 = 3^2$. 2. an expression that is produced by multiplying a value by itself	**cuadrado perfecto** 1. el producto de un número entero multiplicado por sí mismo. Por ejemplo, 9 es un cuadrado perfecto porque $9 = 3^2$. 2. una expresión que se produce multiplicando un valor por sí mismo

English	Español
perfect square trinomial a trinomial of the form $x^2+bx+\left(\frac{b}{2}\right)^2$ that can be written as the square of a binomial	**trinomio cuadrado perfecto** trinomio de la forma $x^2+bx+\left(\frac{b}{2}\right)^2$ que puede expresarse como el cuadrado de un binomio
perimeter the distance around a two-dimensional figure	**perímetro** distancia alrededor de una figura bidimensional
perpendicular lines two lines that intersect at a right angle (90˚); written as $\overleftrightarrow{AB}\perp\overleftrightarrow{PQ}$; line segments and rays can also be perpendicular	**líneas perpendiculares** dos líneas que se cortan en ángulo recto (90˚); se expresan como $\overleftrightarrow{AB}\perp\overleftrightarrow{PQ}$; segmentos de línea y los rayos también pueden ser perpendicular
polygon two-dimensional figure with at least three sides	**polígono** figura bidimensional con al menos tres lados
polynomial a monomial or a sum of monomials	**polinomio** monomio o suma de monomios
positive function a function or a portion of a function where the *y*-values are greater than 0 for all *x*-values	**función positiva** una función o porción de una función en la que los valores *y* son mayores que 0 para todos los valores *x*
precision the degree to which the accuracy of a measurement is known. In measurement systems, precision also refers to the reproducibility of a result upon repetition.	**precisión** el grado en que se conoce la exactitud de una medición. En los sistemas de medición, la precisión también se refiere a la reproducibilidad de un resultado en la repetición.
prime an expression that cannot be factored	**número primo** expresión que no puede ser factorizada
prime factor a factor that is prime	**factor primario** un factor que es primo
prime number a whole number that can only be evenly divided by itself	**número primo** un número entero que sólo puede ser dividido por sí mismo por sí mismo

Q

English	Español
quadratic equation an equation that can be written in the form $ax^2 + bx + c = 0$, where x is the variable, a, b, and c are constants, and $a \neq 0$	**ecuación cuadrática** ecuación que se puede expresar en la forma $ax^2 + bx + c = 0$, donde x es la variable, a, b, y c son constantes, y $a \neq 0$

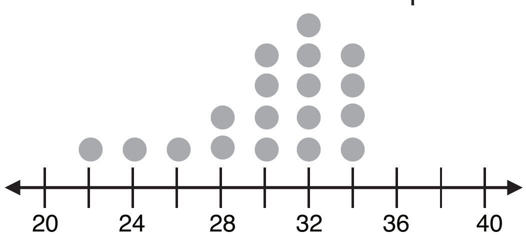
English	Español
quadratic expression an algebraic expression that can be written in the form $ax^2 + bx + c$, where x is the variable, a, b, and c are real numbers, and $a \neq 0$	**expresión cuadrática** expresión algebraica que se puede expresar en la forma $ax^2 + bx + c$, donde x es la variable, a, b, y c son constantes, y $a \neq 0$
quadratic formula a formula that states the solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. A quadratic equation in this form can have no real solutions, one real solution, or two real solutions.	**fórmula cuadrática** fórmula que establece que las soluciones de una ecuación cuadrática de la forma $ax^2 + bx + c = 0$ están dadas por $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Una ecuación cuadrática en esta forma tener ningún solución real, o tener una solución real, o dos soluciones reales.
quadratic function 1. a function that can be written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of any quadratic function is a parabola. 2. a function that can be written in the general form $f(x) = ax^2 + bx + c$, where c is the y-intercept, $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ is the maximum or minimum, and its solutions when $f(x) = 0$ are the x-intercepts; the graph is a parabola	**función cuadrática** 1. función que puede expresarse en la forma $f(x) = ax^2 + bx + c$, donde $a \neq 0$. El gráfico de cualquier función cuadrática es una parábola. 2. una función que se puede escribir en la forma general $f(x) = ax^2 + bx + c$, donde c es la intersección y, $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ es el máximo o mínimo, y sus soluciones cuando $f(x) = 0$ son las intercepciones x; el gráfico es una parábola
quadrilateral a polygon with four sides	**cuadrilátero** polígono con cuatro lados
quantify to find, describe, or measure the total amount or number of something	**cuantificar** para encontrar, describir o medir la cantidad total o el número de algo
quantity 1. a number that describes the total amount or number of something 2. a value or expression that may be expressed in numbers	**cantidad** 1. un número que describe el monto total o el número de algo 2. valor o expresión que puede expresarse en números

English	Español
R	
radical expression an expression containing a root, such as $\sqrt{5}$	**expresión radical** expresión que contiene una raíz, tal como $\sqrt{5}$
radicand in a radical expression, the number under the root sign; in the expression $\sqrt{5}$, the radicand is 5	**radicand** en una expresión radical, el número bajo el signo de la raíz; en la expresión $\sqrt{5}$, la radicand es 5
range the set of all outputs of a relation or function; the set of *y*-values for which a function is defined	**rango** conjunto de todas las salidas de una función; conjunto de valores de *y* para el que se define una función
rate of change a ratio that describes how much one quantity changes with respect to the change in another quantity; also known as the slope of a line	**tasa de cambio** proporción que describe cuánto cambia una cantidad con respecto al cambio de otra cantidad; también se la conoce como pendiente de una recta
rational number a real number that can be written as $\frac{m}{n}$, where both m and n are integers and $n \neq 0$; a terminating or repeating decimal	**número racional** un número que puede expresarse como $\frac{m}{n}$, en los que m y n son enteros y $n \neq 0$; cualquier número que puede escribirse como decimal finito o periódico
real numbers the set of all rational and irrational numbers	**números reales** conjunto de todos los números racionales e irracionales
rectangle a special parallelogram with four right angles	**rectángulo** paralelogramo especial con cuatro ángulos rectos
recursive formula a formula used to find the next term of a sequence when the previous term or terms are known; the recursive formula for an arithmetic sequence is $a_n = a_{n-1} + d$; the recursive formula for a geometric sequence is $a_n = a_{n-1} \bullet r$	**fórmula recursiva** fórmula que se utiliza para encontrar el término siguiente de una secuencia cuando se conoce el o los términos anteriores; la fórmula recursiva de una secuencia aritmética es $a_n = a_{n-1} + d$; la fórmula recursiva para una secuencia geométrica es $a_n = a_{n-1} \bullet r$

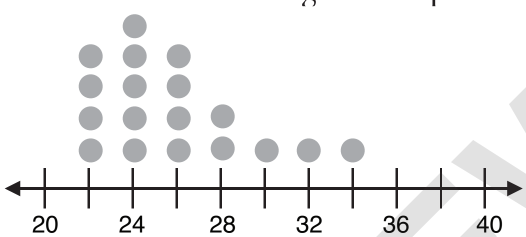
English	Español
reflection 1. a function transformation in which a mirror image is created across a line on the coordinate plane 2. a transformation that creates a mirror image of a function across one of the axes. A reflection across the x-axis is $g(x) = -f(x)$, while a reflection across the y-axis is $g(x) = f(-x)$.	**reflexión** 1. una transformación de función en la que se crea una imagen especular a través de una línea en el plano de coordenadas 2. una transformación que crea una imagen especular de una función a través de uno de los ejes. Una reflexión a través del eje x es $g(x) = -f(x)$, mientras que una reflexión a través del eje y es $g(x) = f(-x)$.
relation a set of ordered pairs	**relación** un conjunto de pares ordenados
relative maximum the greatest value of a function for a particular interval of the function	**máximo relativo** el mayor valor de una función para un intervalo particular de la función
relative minimum the least value of a function for a particular interval of the function	**mínimo relativo** el menor valor de una función para un intervalo particular de la función
rhombus a special parallelogram with all four sides congruent	**rombo** paralelogramo especial con sus cuatro lados congruentes
root(s) solution(s) of a quadratic equation	**raíces** soluciones de una ecuación cuadrática
S	
scatter plot a graph of data in two variables on a coordinate plane, where each data pair is represented by a point	**diagrama de dispersión** gráfica de datos en dos variables en un plano de coordenadas, en la que cada par de datos está representado por un punto
second difference 1. the difference between the first differences of a set of values 2. the difference of successive rates of change of a function. If the second difference is constant, the function is quadratic.	**segunda diferencia** 1. la diferencia entre las primeras diferencias de un conjunto de valores 2. la diferencia de tasas sucesivas de cambio de una función. Si la segunda diferencia es constante, la función es cuadrática.
sequence an ordered list of numbers	**secuencia** lista ordenada de números

English

skewed to the left data concentrated on the higher values in the data set, which has a tail to the left. Example:



skewed to the right data concentrated on the lower values in the data set, which has a tail to the right. Example:

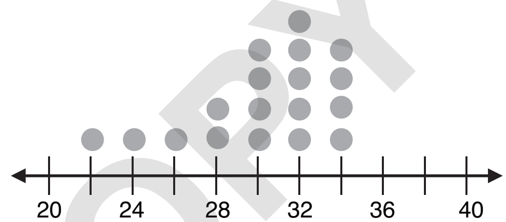


slope 1. the average rate of change of a linear function
2. the measure of the rate of change of one variable with respect to another variable; slope = $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$; the slope in the equation $y = mx + b$ is m

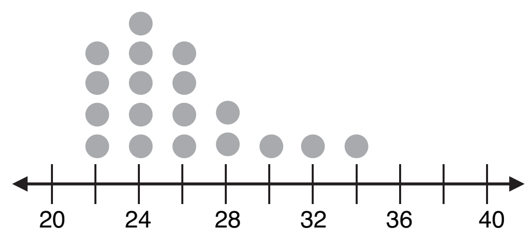
slope formula a formula that states the slope of the line through (or the line segment connecting) $A\ (x_1, y_1)$ and $B\ (x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$

Español

desviados hacia la izquierda datos concentrados en los valores más altos del conjunto de datos, que tiene una cola hacia la izquierda. Ejemplo:



desviados hacia la derecha datos concentrados en los valores más bajos del conjunto de datos, que tiene una cola hacia la derecha. Ejemplo:



pendiente 1. la tasa media de cambio de una función lineal
2. medida de la tasa de cambio de una variable con respecto a otra; $\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$; la pendiente en la ecuación $y = mx + b$ es m.

fórmula de pendiente fórmula que determina la pendiente de la línea que atraviesa (o el segmento de recta que conecta) $A\ (x_1, y_1)$ y $B\ (x_2, y_2)$ es $\frac{y_2 - y_1}{x_2 - x_1}$

English

slope-intercept form of a linear equation the form $y = mx + b$, where m is the slope of the line and b is the y-intercept

solution set the value or values that make a sentence or statement true; the set of ordered pairs that represent all of the solutions to an equation or a system of equations

solution to a system of linear inequalities the set of points that lie in the intersection of the half planes of the inequalities and which may also lie on the boundary lines; the solution set is the set of all points that satisfy the inequalities in the system

square a special parallelogram with four congruent sides and four right angles

square root For any real numbers a and b, if $a^2 = b$, then a is a square root of b. The square root of b is written using a radical: $\sqrt{b}$.

square root function a function that contains a square root of a variable. The general form is $f(x)=\sqrt{ax^2+bx+c}$, where a, b, and c are real numbers.

standard form of a quadratic equation a quadratic equation written as $ax^2 + bx + c = 0$, where x is the variable, a, b, and c are constants, and $a \neq 0$

standard form of a quadratic function a quadratic function written as $f(x) = ax^2 + bx + c$, where a is the coefficient of the quadratic term, b is the coefficient of the linear term, and c is the constant term

Español

forma pendiente-intersección de una ecuación lineal la forma $y = mx + b$, donde m es la pendiente y b es el punto de intersección con el eje y

conjunto de soluciones valor o valores que hacen verdadera una afirmación o declaración; conjunto de pares ordenados que representa todas las soluciones para una ecuación o sistema de ecuaciones

solución a un sistema de desigualdades lineales el conjunto de puntos que se encuentran en la intersección de los planos de la mitad de las desigualdades y que también pueden situarse en las líneas de contorno; el conjunto solución es el conjunto de todos los puntos que satisfacen las desigualdades en el sistema

cuadrado paralelogramo especial con cuatro lados congruentes y cuatro ángulos rectos

raíz cuadrada Para cualquier número real a y b, si $a^2 = b$, entonces a es la raíz cuadrada de b. La raíz cuadrada de b se expresa con un radical: $\sqrt{b}$.

función raíz cuadrada función que contiene la raíz cuadrada de una variable. La forma general es $f(x)=\sqrt{ax^2+bx+c}$, donde a, b y c son números reales.

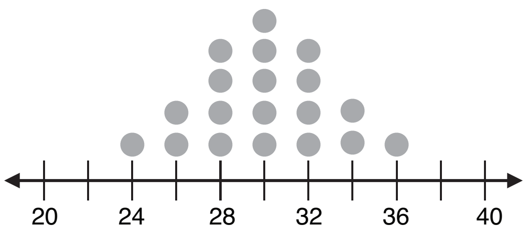
forma estándar de función cuadrática una ecuación cuadrática expresada como $ax^2 + bx + c = 0$, donde x es la variable, a, b, y c son constantes, y $a \neq 0$

forma estándar de función cuadrática función cuadrática expresada como $f(x) = ax^2 + bx + c$, donde a es el coeficiente del término cuadrático, b es el coeficiente del término lineal, y c es el término constante

English

standard units a widely accepted unit of measurement. Standard units are usually defined by law.

symmetric situation in which data is concentrated toward the middle of the range of data; data values are distributed in the same way above and below the middle of the sample. Example:



system of equations a set of equations with the same unknowns

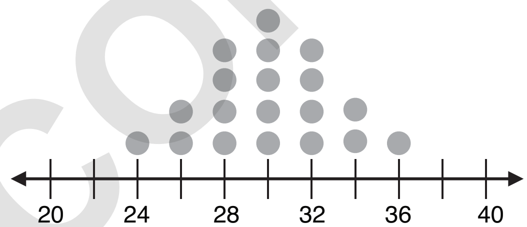
system of inequalities a set of two or more inequalities with the same unknowns

system of measurement a collection of units of measurement, with rules relating the measurements to each other. The metric system, or SI, is an example of a system of measurement.

Español

unidades estándar una unidad de medida ampliamente aceptada. Normalmente, las unidades estándar se definen por ley.

simétrico situación en la que los datos se concentran hacia el medio del rango de datos; los valores de datos se distribuyen de la misma manera por encima y por debajo del medio de la muestra. Ejemplo:



sistema de ecuaciones un conjunto de ecuaciones con las mismas incógnitas

sistema de desigualdades un conjunto de dos o más desigualdades con las mismas incógnitas

sistema de medida una colección de unidades de medida, con reglas que relacionan las mediciones entre sí. El sistema métrico, o SI, es un ejemplo de un sistema de medición.

T

third quartile value that identifies the upper 25% of the data; the median of the upper half of the data set; 75% of all data is less than this value; written as Q_3

transformation 1. adding or multiplying a constant to a function that changes the function's position and/or shape
2. an operation that changes a function through processes such as translation, reflection, and dilation

tercer cuartil valor que identifica el 25% superior de los datos; mediana de la mitad superior del conjunto de datos; el 75% de los datos es menor que este valor; se expresa como Q_3

transformación 1. suma o multiplicación de una constante con una función que cambia la posición y/o forma de la función
2. una operación que cambia una función a través de procesos como la traducción, la reflexión y la dilatación

English	Español
translation 1. transforming a function where the shape and size of the function remain the same but the function moves horizontally and/or vertically; adding a constant to the independent or dependent variable 2. the shifting of a function or graph in the coordinate plane by adding a constant. For a vertical shift, $g(x) = f(x) + k$. For a horizontal shift, $g(x) = f(x + k)$.	**traslación** 1. transformación de una función en la que la forma y el tamaño de la función permanecen iguales pero la función se traslada en sentido horizontal y/o vertical; suma de una constante a la variable independiente o dependiente 2. el desplazamiento de una función o gráfico en el plano de coordenadas mediante la adición de una constante. Para un desplazamiento vertical, $g(x) = f(x) + k$. Para un desplazamiento horizontal, $g(x) = f(x + k)$.
trinomial a polynomial with three terms	**trinomio** polinomio con tres términos
U	
unit of measurement a defined quantity of the subject being measured. For example, the current formal definition of a meter is "the length of the path traveled by light in a vacuum during a time interval of $\frac{1}{299{,}792{,}458}$ of a second."	**unidad de medida** una cantidad definida del sujeto que se está midiendo. Por ejemplo, la definición formal actual de un metro es "la longitud del camino recorrido por la luz en un vacío durante un intervalo de tiempo de $\frac{1}{299{,}792{,}458}$ de un segundo".
V	
variable a letter used to represent a value or unknown quantity that can change or vary	**variable** letra utilizada para representar un valor o una cantidad desconocida que puede cambiar o variar
vertex form of a quadratic function a quadratic function written as $f(x) = a(x - h)^2 + k$, where the vertex of the parabola is the point (h, k); the form of a quadratic equation where the vertex can be read directly from the equation	**fórmula de vértice de función cuadrática** función cuadrática que se expresa como $f(x) = a(x - h)^2 + k$, donde el vértice de la parábola es el punto (h, k); forma de una ecuación cuadrática en la que el vértice se puede leer directamente de la ecuación

English	Español
vertex of a parabola the point on a parabola that is closest to the directrix and lies on the axis of symmetry; the point at which the curve changes direction; the maximum or minimum	**vértice de una parábola** punto en una parábola que está más cercano a la directriz y se ubica sobre el eje de simetría; punto en el que la curva cambia de dirección; el máximo o mínimo
vertical compression squeezing of the parabola toward the *x*-axis	**compresión vertical** contracción de la parábola hacia el eje *x*
vertical stretch pulling of the parabola and stretching it away from the *x*-axis	**estiramiento vertical** jalar y estirar la parábola lejos del eje *x*
W	
whole numbers the set of positive integers and 0: {0, 1, 2, 3, …}	**números enteros** conjunto de enteros positivos que incluye el 0: {0, 1, 2, 3, …}
X	
***x*-intercept** 1. the point at which a line intersects the *x*-axis; written as (*x*, 0) 2. the *x*-coordinate of the point where a line or a curve intersects the *x*-axis	**intersección *x*** 1. punto en el que una línea cruza el eje *x*; se expresa como (*x*, 0) 2. coordenada *x* del punto en que una recta o curva corta el eje *x*
Y	
***y*-intercept** 1. the point at which a line intersects the *y*-axis; written as (0, *y*) 2. the *y*-coordinate of the point where a line or a curve intersects the *y*-axis	**intersección *y*** 1. punto en el que una línea cruza el eje *y*; se expresa como (0, *y*) 2. coordenada *y* del punto en que una recta o curva corta el eje *y*
Z	
Zero Product Property If the product of two factors is 0, then at least one of the factors is 0.	**Propiedad de producto cero** Si el producto de dos factores es 0, entonces al menos uno de los factores es 0.
zeros the *x*-values of a function for which the function value is 0	**ceros** valores de *x* de una función para la que el valor de la función es 0